

LAST NAME (Please Print): _____

FIRST NAME (Please Print): _____

HONOR PLEDGE (Please Sign): _____

Statistics 111

Midterm 1

- This is a closed book exam.
- You may use your calculator and a single page of notes.
- The room is crowded. Please be careful to look only at your own exam. Try to sit one seat apart; the proctors may ask you to randomize your seating a bit.
- **Report all numerical answers to at least two correct decimal places** or (when appropriate) write them as a fraction.
- All question parts count for 1 point.

1. Springfield Elementary has 500 students; 200 are girls and 300 are boys. For a school trip, the children could sign up to visit either the circus or the zoo. 150 girls and 50 boys went to the zoo; the rest saw the circus. Predictably, some students behaved badly on the field trip. Specifically, 40% of the children at the zoo misbehaved, of whom 5 were boys; but only 25% of the children at the circus misbehaved, and of them 50 were boys. Principal Skinner is planning to cancel the trip to the zoo next year, on the grounds that it promotes mischief, but Lisa opposes him.

_____ What is the misbehavior rate at the circus?

_____ What is the misbehavior rate for girls at the zoo?

_____ What is the misbehavior rate for girls at the circus?

Assume that boys behave better at the zoo than at the circus. What is Lisa's explanation to Principal Skinner?

_____ What is the corrected misbehavior rate at the circus?

_____ 2. In a class of 105 students, what is the approximate probability that 14 or fewer were born on Monday? (Assume all days are equally likely.)

_____ 3. On average, there are 0.8 bank robberies in North Carolina per day. What is the probability that there are more than 2 robberies tomorrow?

_____ 4. Consider the density function $f(x) = c|x|$ on the interval $[-2, 2]$, 0 else. What is c ?

_____ 5. Let $F(x) = x^4/16$ on $[0, 2]$. What is the standard deviation of X ?

6. Consider the following numbers (a sample from a population):

10, 0, 9, -10, 4, 8, 4, 2

_____ What is the IQR?

_____ What is the mode?

_____ What is the sample variance?

_____ Suppose a sample has IQR equal to 4. You add 4 to each number and multiply by -2. What is the new IQR?

_____ Suppose a set of numbers has mean equal to 4. You subtract 3 from each number and multiply by -2. What is the new mean?

7. A box of chocolate contains dark, milk and white chocolate candies. For each type of chocolate, there is one that contains a cherry, one that contains a peanut, one that contains coconut, and one that contains nougat.

_____ You eat one at random. What is the probability that it is dark chocolate or contains a cherry?

_____ You eat three at random. What is the probability that none are dark chocolate?

_____ You eat two at random. What is the probability that the second is white chocolate if the first was not dark chocolate?

_____ 8. Suppose 20% of Duke students major in Econ, 30% of students are from NC, and 40% are male. If you lose your ID and a Duke student returns it, what is the probability that the person is an Econ major or from NC or male? (Hint: Venn diagram.)

_____ 9. A university has 15 residence halls, all of equal size. If 6 students are drawn at random, what is the probability that two or more are in the same residence hall?

10. Let $f(x, y) = 3x/8$ for $0 \leq y \leq x \leq 2$.

What is the marginal density of X ? Indicate the support.

What is the conditional density of Y given $X = x$? Indicate the support.

_____ What is the probability that $Y \leq 1$ given that X is 1.5?

11. You toss a coin that has probability $1/4$ of coming up Heads. If it comes up Heads you draw a random value from a Poisson distribution with $\lambda_1 = 1.5$. If it comes up tails you draw a random value from an exponential with $\lambda_2 = 3.5$.

_____ What is the expected value of your random variable?

_____ If you observe $X > \frac{1}{2}$, what is the probability that you threw Tails?

_____ 12. Suppose 20% of Duke students major in Econ. You go on six random dates. Let X be the number of Econ majors in the first four dates, and let Y be the total number of Econ majors among all the dates. What is $\text{Cov}(X, Y)$?

_____ 13. Suppose the height of a Frisian male is normally distributed with mean 78 inches and an sd of 3 inches. (Dante thought they were the tallest people in the world.) What is the probability that Henk, a Frisian, is more than 80 inches tall?

_____ 14. At least what proportion of the data from any distribution must be within 1.6 standard deviations of the mean?

15. Suppose $f(x, y) = c$ for (x, y) inside the diamond bounded by $(-1, 3)$, $(1, 5)$, $(3, 3)$ and $(1, 0)$, and it is zero elsewhere.

_____ What is c ?

_____ Are X and Y independent?

What is $f_1(x)$? Indicate the support.

_____ What is $\mathbb{E}[X]$?

What is $g_1(x|y)$? Indicate the support.

_____ What is the correlation between X and Y ?

_____ 16. Suppose X has the normal distribution with mean 6 and standard deviation 3, and that Y has the normal distribution with mean -3 and variance 4. Suppose their correlation is -0.8. What is their covariance?

_____ 17. Suppose Duke required the sumo wrestling team to take STA 111 in the spring. You measure all the weights in the class. Which will be lower—the mean or the median?

_____ 18. A class consists entirely of econ majors and statistics majors (and no double majors). There are 6 econ majors and 7 statistics majors. You choose four students at random without replacement. What is the probability that you have exactly two of each?