LAST NAME (Please Print): _____

FIRST NAME (Please Print): _____

HONOR PLEDGE (Please Sign): _____

Statistics 111

Midterm 2

- This is a closed book exam.
- You may use your calculator and a single page of notes.
- The room is crowded. Please be careful to look only at your own exam. Try to sit one seat apart; the proctors may ask you to randomize your seating a bit.
- Report all numerical answers to at least two correct decimal places or (when appropriate) write them as a fraction.
- All question parts count for 1 point.

The mean and variance of a Poisson r.v. are both λ.

The mean and standard deviation of an Exponential r.v. are both $1/\lambda$.

The mean of a Beta r.v. is $\frac{\alpha}{\alpha+\beta}$ and the variance is $(\alpha\beta)/(\alpha+\beta)^2(\alpha+\beta+1)$.

The mean of a Gamma r.v. is $\frac{\alpha}{\beta}$ and the variance is $\frac{\alpha}{\beta^2}$.

1. You go to a restaurant and get a two-course dinner. The first course lasts for X hours, where X is exponential with parameter 3. The second course takes Y hours, where Y is independent of X and exponential with parameter 2. Let Z be the duration of the meal.

   _____ What is the expected value of Z?

   _____ What is the standard deviation of Z?

   _____ What is the covariance between X and Z?

   _____ What is the correlation between X and Z?

2. Let $X_1, \ldots, X_n$ be a random sample from the distribution with probability mass function

$$f(x;\theta) = \frac{\theta^x}{x!}\exp(-\theta) \text{ for } x = 0, 1, \ldots$$

   and 0 else, where $\theta \geq 0$.

   Write the likelihood function.

   _____ Find the maximum likelihood estimate of θ.

   _____ What is the bias of the estimate $\hat{\theta} = 3\bar{X}$?

   _____ What is the variance of $\hat{\theta} = 3\bar{X}$?

3. _____ The number of days Mary takes to read a novel has Gamma distribution with parameters $\alpha = 6$ and $\beta = 2$. What is the approximate probability that she reads more than 10 or more books in January?

4. _____ Consider the linear congruential generator $X_{n+1} \equiv 11X_n \pmod 7$ with seed (X_0) 6. What is X_2?

5. _____ A statistics class has three lab sections, each with 25 students. I give a quiz on confidence intervals to the first section, and 20 students answer correctly. Set a 90% lower confidence bound on the proportion of students in the class who understand confidence intervals.

6. Mary gets a Poisson number of phone calls in an hour. I think she is quite popular, and my belief about the average number of calls she receives is Gamma(5,2).

   _____ I observe that she receives 3, 2, and 1 phone calls over the next three hours. What distribution reflects my new belief about her average? (Include parameter values.)

   _____ What is my best guess for the mean number of phone calls per hour, if my penalty is proportional to the square of my error.

7. You want to place a confidence interval (CI) on the median number of hours slept by Duke students on the night before a statistics exam. You sample 50 random students; each reports the hours they slept and the median of that sample was 7.25 You resample 20 times from that sample (with replacement) and the medians of the 20 resamples are:

   5, 2, 8.5, 7, 4.5, 6, 5.5, 5.5, 7.5, 9, 6, 2.5, 5, 3.5, 8, 3, 4, 6.5, 7, 6.5

   _____ Set a 95% one-sided lower CI with the percentile bootstrap.

   Set a 90% CI with the pivot bootstrap: L = _____ U = _____

8. Let $X_1, \ldots X_n$ be a random sample from the exponential distribution with parameter $\lambda = 4$ Write the density function for the sample maximum. (Indicate support.)

9. Suppose $f(x, y) = c$ on the region $x^2 + y^2 \leq 1$ with $y > 0$.

_____ What is c?

What is $f_2(y)$? Indicate support.

What is $g_2(y \mid x = 1/2)$? Indicate support.

_____ What is $\mathbb{E}[X]$?

_____ What is the correlation between X and Y?

_____ Are X and Y independent?

10. Wally West is pretty fast—I think his probability of winning a race has the Beta(3,1) distribution. He participates in five races and wins each.

    _____ What is my new belief about p, his probability of winning?

    _____ What is my best estimate of p if I pay a penalty proportional to the square of my error?

    _____ What is my best estimate of p if I pay a penalty proportional to the absolute value of my error?

    _____ How much weight does the posterior mean place upon the sample mean?

11. _____ The number of alcoholic beverages a Duke freshman consumes in a week has the Poisson distribution with parameter $\lambda = 3.5$. If you sample 20 students, what is the approximate probabilty that $\bar{X}$ is greater than 3.6?

12. _____ Dean Kostyu wants a 95% upper confidence bound on the mean number of ounces of alcohol that Duke freshman consume in a week. A sample of 12 students has average 7.8 and standard deviation 4.2.

13. List all, and only, the true statements. _____

    A. Response bias concerns how a survey question is worded.
    B. The continuum of non-response method assumes those who are never reached in a survey are similar to those who are difficult to reach.
    C. Household bias says that large households are over-represented.
    D. Dr. John Snow stopped a smallpox epidemic in London.
    E. William Playfair made a graph showing Napoleon's attack upon and retreat from Moscow.
    F. An age pyramid for Sierra Leone would be tall and narrow.
    G. If X and Y are negatively correlated, then $\text{Var}[X+Y] > \text{Var}[X-Y]$.
    H. The $\text{Var}[\sum a_i X_i] = \sum a_i^2 \sigma_i^2 + 2\sum_{i<j} a_i a_j \text{Cov}(X_i, X_j)$.
    I. As confidence increases, so does the width of the confidence interval on μ.
    J. As the sample size increases, so does the width of the confidence interval on μ.