LAST NAME (Please Print): ______________________________

FIRST NAME (Please Print): ______________________________

HONOR PLEDGE (Please Sign): ______________________________

Statistics 111

Midterm 1

- This is a closed book exam.
- You may use your calculator and a single page of notes.
- The room is crowded. Please be careful to look only at your own exam. Try to sit one seat apart; the proctors may ask you to randomize your seating a bit.
- **Report all numerical answers to at least two correct decimal places** or (when appropriate) write them as a fraction.
- All question parts count for 1 point.

The Gamma(α, β) distribution has mean α/β, variance α/β^2, and density function

$$f(x) = \frac{\beta^\alpha}{(\alpha-1)!} x^{\alpha-1} \exp(-\beta x) \text{ for } 0 \leq x \text{ and } \alpha > 0, \beta > 0.$$

The Beta(α, β) has mean $\alpha/(\alpha+\beta)$, variance $\alpha\beta/[(\alpha+\beta)^2(\alpha+\beta+1)]$, and density function

$$f(x) = \frac{(\alpha+\beta-1)!}{(\alpha-1)!(\beta-1)!} x^{\alpha-1}(1-x)^{\beta-1} \text{ for } 0 \leq x \leq 1 \text{ and } \alpha > 0, \beta > 0.$$

The mean and variance of a Poisson r.v. are both λ.

The mean and standard deviation of an Exponential r.v. are both $1/\lambda$.

The mean of a Unif(0, θ) r.v. is $\theta/2$ and the variance is $\theta^2/12$.

1. A student takes calculus, statistics and creative writing. On a 4-point scale, their calculus and statistics grades are uniformly distributed between 2 and 4, and their poetry grade is uniformly distributed between 0 and 3. There is a positive correlation of 0.5 between the quantitative grades, and those are independent of the poetry grade. What are the mean and variance of that semester's GPA?

   ____________ mean   ____________ variance

2. ____________ Consider the linear congruential generator $X_n \equiv 7X_{n-1} + 3 \pmod 5$. Let $X_0 = 2$. What is X_2?

3. Let 0.8 and 0.75 be a random sample from the Beta distribution with unknown α. However, you know that $\beta = 1$.

   ____________ What is the maximum likelihood estimate (MLE) of α?

   ____________ What is the MLE of the mean of this Beta distribution?

4. You estimate the probability of Heads on a coin by tossing it n times, counting the number of Heads, and dividing by $n-1$. What is the bias, variance, and mean squared error in your estimator?

   bias = ____________   variance = ____________

   mean squared error = ____________

5. ____________ You are drunk and you limp, so that each step you take has probability 1/3 of being to the left and 2/3 of being to the right. If your steps are independent, what is the approximate probabilty that after 100 steps you are at least 42 steps to the right of your starting point?

6. ____________ Scores on a midterm from a class of 120 people have mean 80 and standard deviation 8. What is the approximate probability that the average of 60 random students exceeds 85?

7. You have a random sample $X_1, \ldots X_n$ where each observation has density

$$f(x) = \begin{cases} \lambda \exp[-\lambda(x-\theta)] & \text{if } x > \theta \\ 0 & \text{otherwise} \end{cases}$$

Assume θ is known. What is the maximum likelihood estimate for λ?

____

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____

8. The number of people who show up for a performance of the *The Condemned of Altona* has a Poisson distribution with unknown parameter λ. You express your uncertainty about λ by saying that it has a Gamma distribution with $\alpha = 100$ and $\beta = 0.5$. On three successive nights you observe that the attendance is 180, 160 and 170.

What is your posterior distribution on λ? ____________

You are a scalper, buying tickets in advance for resale at higher prices. If you buy too many or two few tickets, you lose an amount of money of your overestimate or underestimate. In words, what is your best guess on the number of tickets to buy?

The ____________ of the posterior density.

9. You have a $Beta(3, 2)$ prior on p_w, the proportion of women at Duke. A random sample of 20 people finds that 14 are women.

What is your posterior distribution for p_w? ______________________________

____________ If you write the posterior mean as $ca + (1-c)b$ where a is your prior mean and b is your sample mean, what is c?

10. Let $f(x, y) = 6(y - x)$ on $1 < x < y < 2$, and it is 0 otherwise.

What is the marginal density for X? (Remember to indicate where it is non-zero.)

______________ What is the expected value of X?

What is the conditional density of Y given that $X = 1.5$? (Indicate the support.)

11. Suppose that Y_i has Poisson distribution with mean λx_i where the x_i are fixed, known positive values. You observe a random sample $Y_1, \ldots, Y_n$.

Write the likelihood function.

$$f(y_1, \ldots y_n; \lambda) =$$

______________ What is the maximum likelihood estimator $\hat{\lambda}$?

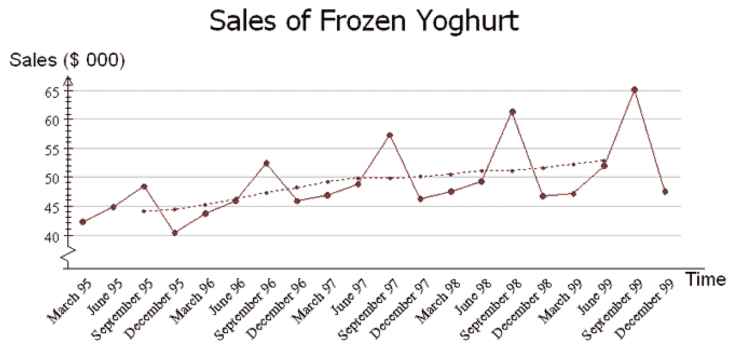
______________ What is the bias of this estimator?

______________ What is the standard deviation of this estimator?

12. You observe $X = x$ from a Pois(λ) distribution. Your prior on λ is exponenetial with parameter 3. Write an expression (with an integral denominator) for your posterior on λ.

$$\pi^*(\lambda|x) =$$

13. The figure below show frozen yogurt sales. The dotted line is the six-month average of the sales, and the solid are quarterly sales. The graph makes two main points. What are they?



(1) ____

(2) ____

14. List all, and only, the true statements. ______________________

A. Asymptotically, maximum likelihood estimators have minimum mean squared error.

B. Linear combinations of Poisson random variables have the Poisson distribution.

C. If X and Y are negatively correlated, then $\mathrm{Var}[X+Y] > \mathrm{Var}[X-Y]$.

D. The continuum of nonresponse method assumes that those who were never reached in a survey are like those who were difficult to reach.

E. If every person is equally likely to be drawn, one has a simple random sample.

F. Nonresponse bias occurs if an interviewee refuses to answer your question.

G. An example of response bias arises from the ordering of candidates' names on a ballot.

H. Florence Nightingale plotted cholera deaths on a street map of London.

I. Charles Joseph Minard made a statistical graphic showing Napoleon's casualties in his attack upon Poland.

J. Linear congruential generators produce random numbers.

K. An unbiased estimate of the standard deviation is $(n-1)^{-1}\sum(X_i - \bar{X})^2$.

L. The finite population correction factor adjusts for drawing without replacement.