

LAST NAME (Print): **KEY**

Statistics 111 **Quiz 10**

- 0.05** 1. Assume that the IQs of college students follow a Rayleigh distribution with mean 120 and variance 9. What is the probability that a class of 16 has a mean IQ greater than 121.25?

Use the CLT.  $\mathbb{P}[\bar{X} > 121] = \mathbb{P}[(\bar{X} - \mu)/se > (121.25 - 120)/(3/\sqrt{16})] \doteq \mathbb{P}[Z > 1.67] = 0.0475$ .

2. Consider the joint probability mass function defined by the following table:

	x=1	x=2	x=3
y=0	0.2	0.1	0.1
y=1	0.1	0.1	0.1
y=2	0.1	0.1	c

**0.1** What is the value of  $c$ ?

**2.4** What is the expected value of the product  $XY$ ?

$$1 * 0 * (0.2) + 1 * 1 * (0.1) + \dots + 3 * 2 * (0.2) = 1.8$$

**1/2** Consider the conditional distribution of  $X$  given that  $y = 0$ . What is  $p(X = 1 | y = 0)$ ?

$\mathbb{P}[X = 1 \text{ and } Y = 0] = 0.2$ , and  $\mathbb{P}[Y = 0] = 0.2 + 0.1 + 0.1$ . So  $\mathbb{P}[X = 1 | Y = 0] = \mathbb{P}[X = 1 \text{ and } Y = 0] / \mathbb{P}[Y = 0] = 0.5$ .

**No** Are  $X$  and  $Y$  independent?

Knowing that  $Y = 0$  implies the chance of  $X = 1$  is 0.2; but knowing  $Y = 1$  implies that the chance of  $X = 1$  is 0.1. So information about  $Y$  changes belief about  $X$ .

3. Let  $X_1$  have mean 7 and variance 10. Let  $X_2$  have mean 8 and variance 4. Suppose the covariance between them is -2.

**-0.32** What is the correlation between  $X_1$  and  $X_2$ ?

$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}[X]} * \sqrt{\text{Var}[Y]}} = \frac{-2}{\sqrt{10} * \sqrt{4}} = -0.3162.$$

**23** What is the mean of  $Y = X_1 + 2X_2$ ?

$$7 + 2 * 8 = 23.$$

**18** What is the variance of  $Y = X_1 + 2X_2$ ?

$$1 * 10 + 4 * 4 + 2 * (1 * 2) * (-2) = 18.$$

4. What is  $\mathbb{E}[\hat{\theta}] - \theta$ ? **bias**

5. **0.74** Let  $X_1, X_2, X_3$  be a random sample from the distribution with density  $f(x) = 2x$  for  $0 \leq x \leq 1$ . What is the probability the the sample maximum is less than 0.95?

$\mathbb{P}[X_1 \leq z] = z^2$  so the probability that  $X_1$  and  $X_2$  and  $X_3$  are less than  $z$  is  $z^6$ , so  $(0.95)^6 = 0.7351$

6. You measure the IQs of 12 children in a class of 20 (without replacement). If the true mean in the classroom is 105 and the variance is 121, what is the approximate distribution of your average?

Normal with mean 105 and standard deviation 2.06

It is normal because of the CLT. The mean is 105, since the sample mean is unbiased. It has standard deviation  $[\sqrt{121}/\sqrt{12}] * [\sqrt{20-12}/\sqrt{20-1}] = 2.0605$ .