LAST NAME (Print): KEY

Statistics 111 **Quiz 11**

1. Let $X_1, \ldots, X_n$ be a random sample with density $f(x) = \theta x^{\theta-1}$ for $0 \leq x \leq 1$ with $\theta > 0$. Find the MLE of θ.

The likelihood function is

$$f(x_1, \ldots, x_n; \theta) = \prod_{i=1}^{n} \theta x_i^{\theta-1} = \theta^n \prod_{i=1}^{n} x_i^{\theta-1}$$

so the log-likelihood is

$$\ell(\theta) = n \ln \theta + (\theta - 1) \sum_{i=1}^{n} lnx_i.$$

Take the derivative of this wrt θ, set it to 0 and solve:

$$\frac{d\ell(\theta)}{d\theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \ln x_i = 0$$

implies $\hat{\theta} = -n/[\sum \ln x_i]$.

2. Let X_1, X_2, X_3 be independent and suppose X_i is normally distributed with mean i and standard deviation i^2. Let $Y = 2X_1 - 3X_2 + X_3$.

-1 = $\mathbb{E}[Y]$ 229 = Var[Y]

This uses the formulæ for the mean and variance of a linear combination of random variables. The mean of Y is just the linear combination of the means of X_i, or 2*1 - 3*2 + 1*3 = -1.

The variance is the sum of the squares of the coefficients times the variances of X_i, so $2^2 * 1^4 + (-3)^2 * 2^4 + 1^2 * 3^4 = 229$.

3. You estimate the mean of distribution by summing the n random observations and dividing by $n - 1$. What is (a) the bias in this estimator, (b) the variance of this estimator, and (c) the mean squared error of this estimator?

(a) $\frac{1}{n-1}\mu$ (b) $\left(\frac{n}{n-1}\right)^2 \frac{\sigma^2}{n}$ (c) $\left(\frac{n}{n-1}\right)^2 \frac{\sigma^2}{n} n + \left(\frac{1}{n-1}\right)^2 \mu^2$

In this problem your estimate of the mean is $\hat{\mu} = \frac{1}{n-1}\sum X_i = \frac{n}{n-1}\bar{X}$. We know that $\mathbb{E}[\bar{X}] = \mu$, so the bias is $\mathbb{E}[\frac{n}{n-1}\bar{X}] - \mu = \frac{n}{n-1}\mu - mu = \frac{1}{n-1}\mu$.

Similarly, the variance is $\text{Var}\,[\frac{n}{n-1}\bar{X}] = \left(\frac{n}{n-1}\right)^2 \frac{\sigma^2}{n}$.

The MSE is variance + the square of the bias.

0.21 to 0.23 4. An elevator fails if the total weight exceeds 2000 pounds. Suppose weights have a gamma distribution with mean 160 and sd 30. If 12 people get on the elevator, what is the approximate probability that it fails?

CLT for sums. The total weight is approximately normal with mean 12*160 = 1920 and sd $\sqrt{12}$*30 = 103.923. The z-transformation gives $z = 0.77$, and from the table the area above this is 0.22.

This is the beta-binomial. So, from the lectures, the posterior distribution is Beta(2+8, 5+2), which has mean (2+8)/[(2+8)+(5+2)].

5 List all, and only, the true statements. (5 pts) B, D, E, F

A. Maximum likelihood estimates are unbiased.

B. As the sample size gets large, MLEs have minimum variance.

C. The transformation of an unbiased estimator is an unbiased estimate of the transformation.

D. Alan Turing worked on the Central Limit Theorem.

E. Sir Ronald Fisher invented MLEs.

F. A linear combination of normal random variables has a normal distribution.