LAST NAME (Print): KEY

Statistics 111 **Quiz 12**

1. A student lines up 12 campus interviews with Wall Street firms. She does not know the probability p that an employer will make her an offer, but she thinks p has a Beta(3,1) distribution. After the round of interviews she gets two offers. What is (a) her new distribution for p, and (b) what is the mean of that distribution?

   (a) Beta(5, 11) (b) 0.31

   Beta-binomial family. The posteror is Beta$(3+x, 1+(n-x))$ = Beta(5, 11). The mean is $\alpha^*/(\alpha^*+\beta^*) = 5/16 = 0.3125$.

2. Assume that the number of times that a Duke student falls in love during his four years at the university has a Poisson distribution with parameter λ. Dean Kostyu does not know λ, but she thinks it has the Gamma(2,1) distribution. She observes that Abelard falls in love twice, Balthazar three times, and Clytemnestra seventeen times. What is (a) her new distribution for λ, and (b) what is the mean of that distribution?

   (a) Gamma(24, 4) (b) 6

   Gamma-Poisson family. The posterior is Gamma$(\alpha + \sum x_i, \beta + n)$ = Gamma(24, 4) which has mean $\alpha^*/\beta^* = 24/4 = 6$.

3. You estimate the mean of a distribution by summing the n random observations and dividing by $n-2$. What is (a) the bias in this estimator, (b) the variance of this estimator, and (c) the mean squared error of this estimator?

   (a) $\frac{2}{n-2}\mu$ (b) $\left(\frac{n}{n-2}\right)^2 \frac{\sigma^2}{n}$ (c) $\left(\frac{n}{n-2}\right)^2 \frac{\sigma^2}{n} + \left(\frac{2}{n-2}\right)^2 \mu^2$

   In this problem your estimate of the mean is $\hat{\mu} = \frac{1}{n-2} \sum X_i = \frac{n}{n-2}\bar{X}$. We know that $\mathbb{E}[\bar{X}] = \mu$, so the bias is $\mathbb{E}[\frac{n}{n-2}\bar{X}] - \mu = \frac{n}{n-2}\mu - mu = \frac{1}{n-2}\mu$.

Similarly, the variance is $\text{Var}[\frac{n}{n-2}\bar{X}] = \left(\frac{n}{n-2}\right)^2 \frac{\sigma^2}{n}$.

The MSE is variance + the square of the bias.

6. List all, and only, the true statements. (6 points) B, D, E

A. The transformation of an unbiased estimator is an unbiased estimate of the transformation.

B. Bayesian estimates take account of prior knowledge.

C. If the prior is binomial and the model is beta, there is a simple solution for the posterior.

D. A Bayesian expresses personal uncertainty as a probability.

E. Thomas Bayes was a minister.

F. If the penalty you pay for being wrong is proportional to the square of your error, then your best guess about a parameter is the median of its posterior distribution.