

LAST NAME (Print): **KEY**

Statistics 111      **Quiz 13**

1. Assume that the number of driving accidents a person has in their life follows a Poisson distribution with unknown parameter  $\lambda$ . You think  $\lambda$  has a Gamma(1, 3) distribution. Among four elderly people who have stopped driving, they report 0, 2, 3, and 0 accidents. What is your posterior distribution for  $\lambda$ ?

**Gamma(6, 7)**

**This is the Bayes Gamma-Poisson case. From the lectures, we know that the posterior is Gamma with  $\alpha = 1 + (0 + 2 + 3 + 0)$  and  $\beta = 3 + 4$ .**

2. What is the value from a  $t$ -table with 5 degrees of freedom which has (a) area 0.05 under the curve and to the right, (b) area 0.1 under the curve and to the left, and (c) area 0.025 in each tail?

(a) **2.01 or 2.02**

(b) **-1.48**

(c) **2.57**

- 3.07** 3. A sample of 100 Duke students have sample mean GPA 3.00 and sample standard deviation 0.4. Set a one-sided 95% upper confidence interval on the mean GPA of all Duke students.

**This is a large sample case, so we use a  $z$ -table. And the population of Duke is large, so we don't worry about sampling with replacement. Thus  $U = 3.00 + \frac{0.4}{\sqrt{100}} * z_{0.95}$  where  $z_{0.95}$  is either 1.65 or 1.64, depending on how you round. Both round the answer to 3.07.**

- 3.23** 4. A sample of 10 Duke students have sample mean GPA 3.00 and sample standard deviation 0.4. Set a one-sided 95% upper confidence interval on the mean GPA of all Duke students.

**Here we have a small sample, and must use the  $t$ -table. Thus  $U = 3.00 + \frac{0.4}{\sqrt{10}} * t_{10-1,0.95}$  where  $t_{9,0.95}$  is 1.833.**

- 3.17** 5. A sample of 10 Duke students from a class of 20 have sample mean GPA 3.00 and sample standard deviation 0.4. Set a one-sided 95% upper confidence interval on the mean GPA of the class.

This is a small sample, but it is a large fraction of the population, so we need to use the finite population correction factor (FPCF). Thus  $U = 3.00 + \frac{0.4}{\sqrt{100}} * FPCF * t_{10-1,0.95}$  where  $t_{9,0.95}$  is 1.833 and the FPCF is  $\sqrt{\frac{20-10}{20-1}} = 0.7255$ .

6. List all, and only, the true statements. (6 pts) **C, F**
- A.** As  $1 - \alpha$  increases, the width of the interval decreases.
  - B.** As  $\sigma$  increases, the width of the interval decreases.
  - C.** As  $n$  increases, the width of the interval decreases.
  - D.** As the FPCF increases, the width of the interval decreases.
  - E.** If a 95% CI on mean GPA is  $[3, 3.2]$ , then you expect that 95% of the students will have GPAs in this range.
  - F.** MLEs are asymptotically unbiased and have minimum variance.