

LAST NAME (Print): Key

Statistics 111 Quiz 15

1. 2 Let 1, 2, 3 be a random sample from a Poisson(λ) distribution. Find the MLE of λ .

The likelihood function is $\prod_{i=1}^n \frac{\lambda^{x_i}}{x_i!} \exp(-\lambda)$, so the log-likelihood is $-n\lambda + \ln \lambda \sum x_i + \ln \prod \frac{1}{x_i!}$. Take the derivative wrt λ , getting $-n + \frac{1}{\lambda} \sum x_i$. Set this to 0 and solve to get $\hat{\lambda} = \bar{X}_i$. For the data here, the average is 2.

2. 1.27 Let 0.25, 0.5, and 0.75 be a random sample from a Beta($\alpha, 1$) distribution. Find the MLE of α .

The Beta density is $\alpha x^{\alpha-1}$ so the likelihood is $\prod_{i=1}^n \alpha x_i^{\alpha-1}$ so the log-likelihood is $n \ln \alpha + (\alpha - 1) \sum \ln x_i$. Take the derivative wrt α to get $\frac{n}{\alpha} + \sum \ln x_i$. Set this to 0 and solve, so $\hat{\alpha} = \frac{-n}{\sum \ln x_i}$. For these data, it is 1.2674.

3. Let X_i be normally distributed with mean i and standard deviation i for $i = 1, 2, 3$. Suppose the covariance between X_1 and X_2 is 2, and X_3 is independent of both. Let $Y = X_1 + 2X_2 - 3X_3$.

Normal What is the name for the distribution of Y ?

A linear combination of normal random variables is normal.

-4 What is the mean of that distribution?

$$\mathbb{E}[Y] = \mathbb{E}[X_1 + 2X_2 - 3X_3] = \mathbb{E}[X_1] + 2\mathbb{E}[X_2] - 3\mathbb{E}[X_3] = 1 + 2 * 2 - 3 * 3 = -4.$$

10.3 What is the standard deviation of that distribution?

$$\text{Var}[Y] = \sum a_i^2 \text{Var}[X_i] + 2 \sum_{i < j} a_i a_j \text{Cov}(X_i, X_j) = 1^2 * 1^2 + 2^2 * 2^2 + (-3)^2 * 3^2 + 2 * 1 * 2 * 2 = 106.$$

The square root of 106 is 10.2956.

1 What is the correlation between X_1 and X_2 ?

$$\text{Corr}(X_1, X_2) = \text{Cov}(X_1, X_2) / \sqrt{\text{Var}[X_1] * \text{Var}[X_2]} = 2 / \sqrt{1^2 * 2^2} = 1.$$

0 What is the correlation between X_1 and X_3 ?

0, since these are independent.

Why is $\text{Var}[X_1 + X_2] > \text{Var}[X_1 - X_2]$?

Since X_1 and X_2 have positive correlation, then large values of one are likely to have large values for the other, and small values of one imply small values for the other. So the sum tends to spread out while the difference tends to be about the same.

4. List only the true statements. (3 points) **A, B, C**

- A. The pivot bootstrap is more accurate than the percentile bootstrap.
- B. Asymptotically, the bootstrap is never worse than the CLT.
- C. Brad Efron invented the bootstrap.