NAME (Please Print): KEY

Statistics 111 **Quiz 6**

1. The proportion of a day that a student is awake has the density $f(x) = cx(1-x)^2$ (so it takes a random value between 0 and 1).

_____ What is the value of c?

We find c so that the integral of the density is 1. Since

$$\int_0^1 x(1-x)^2\,dx = \int_0^1 x - 2x^2 + x^3\,dx = \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{4}x^4\,|_0^1$$

we see that the integral is 1/12 and thus the constant must be 12.

To avoid propagation of error, in the following questions assume the same situation but that the density is $f(x) = 4x^3$.

≈ 1 What is the probability that she is awake more than 6 hours?

$1 - F(1/4) = 1 - 0.25^4 = 0.9961$.

4/5 What is the average proportion of a day that she is awake?

$\int_0^1 x * 4x^3\,dx = 4/5$.

0.03 What is the variance in that proportion?

Since $\mathrm{E}[X^2] = \int_0^1 x^2 * 4x^3\,dx = 2/3$, then the variance is $2/3 - (4/5)^2 = 0.02667$.

0.16 What is the standard deviation in the proportion?

The square root of the variance is 0.163.

0.30 2. You manufacture 50 chips. Each chip has probability of failing equal to 1/5. What is the approximate probability that 12 or more fail?

This is the normal approximation to the binomial, with a continuity correction. The mean of this binomial is $np = 10$ and the standard deviation is $\sqrt{np(1-p)} = 2.8284$. So the z-transformation, with the continuity correction, is $z = (11.5 - 10)/2.8284 = 0.53$. From our table, the probability of a larger value is 1 -0.7019 = 0.2981.

0.61 3. Assume that the lifetime of an insect in days is exponential with parameter $\lambda = 2$. What is the probability that the insect lives more than 6 hours?

The probability the insect lives less than 6 hours is $1 - \exp(-2 * 0.25) = 0.393$, so the probability it lives more than 6 hours is 1 - 0.393 = 0.607.

4. A pizza parlor has five meat toppings and six vegetable toppings. A coupon allows you to get a free pizza with one meat and two vegetable toppings. In how many ways can you do this, if the three toppings must be distinct? In how many ways can you do it if they need not be distinct (e.g., "double pepperoni").

Distinct 75 Not Distinct 105

For distinct toppings, it is $\begin{pmatrix} 5 \\ 1 \end{pmatrix} * \begin{pmatrix} 6 \\ 2 \end{pmatrix} = 75$. If they need not be distinct, it is $5 * [\begin{pmatrix} 6 \\ 2 \end{pmatrix} + 6] = 105$.

5. Find $\int_2^3 x \ln x dx$. 2.31

Integration by parts. Set $u(x) = \ln x$ and $v'(x) = x$, so

$$\ln x * x^2/2|_2^3 - \int_2^3 x^2/2 * (1/x) dx = 2.3075.$$