LAST NAME (Print): KEY

Statistics 111 **Quiz 7**

1. Suppose $f(x, y) = c$ for all (x, y) that lie on the perimeter of the circle $(x-4)^2 + (y-2)^2 \leq 3$. The following questions can be answered with minimal calculation, just geometry.

   0.09 What is the value of c?

   The perimeter has length $2\pi\sqrt{3}$, so c is the inverse of that, or 0.0919.

   4 What is the value of μ_X?

   By symmetry, 4.

   8 What is the expected value of the product XY?

   8, since the correlation has to be 0. The correlation is unchanged by recentering the circle to (0, 0), and there the symmetery shows that every positive product is cancelled by a negative product.

   0 What is the correlation?

   0

   No Are X and Y indpendent?

   Explain your answer regarding independence.

   Knowing X gives lots of information about Y. If you are the largest possible x-value, then y has to 2.

   What is $g_2(y|x)$?

It is equally likely to be $2 + \sqrt{3 - (x-4)^2}$ or $2 - \sqrt{3 - (x-4)^2}$.

2. Suppose $f(x, y) = 6x$ for $x + y \leq 1$ with both x and y restricted to be between 0 and 1.

What is the marginal density of X? (Indicate support.)

The region of support is the triangle with vertices (0, 0), (1, 0), and (0, 1). So

$$f_1(x) = \int_0^{1-x} 6x \, dy = 6x(1-x)$$

for $0 \leq x \leq 1$.

$$f_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ 6x(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$$

0.5 What is the expected value of X?

$\mathbb{E}[X] = \int_0^1 x f_1(x) \, dx = \int_0^1 6x^2(1-x) \, dx = 1/2.$

What is the conditional density of Y given $X = x$? (Indicate support.)

$g_2(y \mid x) = f(x, y)/f_1(x) = 6x/6x(1-x) = (1-x)^{-1}$ for $0 \leq y \leq 1 - x$.

$$g_2(y \mid x) = \begin{cases} 0 & \text{if } y < 0 \\ \frac{1}{(1-x)} & \text{if } 0 \leq y \leq 1 - x \\ 0 & \text{if } y > 1 - x \end{cases}$$

0.1 What is $\mathbb{E}[XY]$?

$$\int_0^1 \int_0^{1-y} xy f(x, y) \, dx dy \quad = \quad \int_0^1 y[2x^3]|_0^{1-y} dy$$

$$
\begin{aligned}
&= \int_0^1 y * 2(1-y)^3 \, dy \\
&= 0.1
\end{aligned}
$$