Statistics 111   **Quiz 9**

1. Suppose $f(x, y) = 24x$ for all (x, y) such that $0 < x < 1$, $0 < y < 1$, and $2x < y$.

   $1/8$ What is the probability that both X and Y are between 0 and $1/2$?

   This is the integral of the density in the triangle with vertices (0,0), (0, 1/2), (1/4, 1/2). Thus

   $$\mathbb{P}[0 \leq X, Y \leq 1/2] = 24 * \int_0^{1/4} \int_{2x}^{1/2} x \, dy dx = 24 * \int_0^{1/4} x(\frac{1}{2} - 2x) \, dx = 24 * (1/64) * (1/3) = 0.125.$$

   0.25 What is the value of μ_x?

   Integrate the joint density with respect to y to find $f_1(x) = 24 * (x - 2x^2)$ for $0 \leq x \leq 1/2$. Then $\int_0^{1/2} x f_1(x) \, dx = 24 * [(1/24) - (1/32)] = 0.25$.

   6 What is the expected value of $1/x$?

   Here $\int_0^{1/2} x^{-1} f_1(x) \, dx = \int_0^{1/2} (1/x)(x - 2x^2) \, dx = 6$.

   0.75 What is the expected value of Y when $x = 0.25$?

   Since $g_2(y \mid x) = f(x, y)/f_1(x) = 24x/24(x - 2x^2) = 1/(1 - 2x)$ for $x = 0.25$, or 2, for $1/2 \leq y \leq 1$. So $\int_{1/2}^1 y * 2 \, dy = 1 - (1/4) = 0.75$.

2. Write the Central Limit Theorem for sums. Use S to denote the random sum, and let n be the number of terms, each of which has mean μ and variance σ^2.

   $S \sim N(n\mu, \sqrt{n}\sigma)$

3. 0.69, 0.7 Make 50 draws, $X_1, \ldots, X_n$ from a binomial distribution with $n = 8$ and $p = 0.4$. What is the approximate probability that $\bar{X} \leq 3.3$?

The mean of the binomial is $\mu = pn = 3.2$ and the standard deviation is $\sigma = \sqrt{np(1-p)} = 1.3856$. Since the CLT says that $\bar{X} \dot\sim \mathrm{N}(\mu, \sigma/\sqrt{n})$, then the z-transformation is $z = (3.3 - 3.2)/(1.3856/\sqrt{50} = 0.5103$. From the table, this is 0.695.

3. List all, and only, the true statements. A, C, F

   A. The standard error is the standard deviation of an estimate of a parameter.

   B. The Law of Averages is more specific than the Central Limit Theorem.

   C. Alan Turing worked on the Central Limit Theorem.

   D. If every person in the population is equally likely to be chosen, one has a simple random sample.

   E. Nonresponse bias occurs when people refuse to answer a survey.

   F. The FPCF is used when sampling without replacement.

   G. Response bias occurs when people who respond are different from those who do not.