OPERATIONS ANALYSIS DURING THE UNDERWATER SEARCH FOR SCORPION*

Henry R. Richardson
and
Lawrence D. Stone

Daniel H. Wagner, Associates
Paoli, Pennsylvania

ABSTRACT

This paper discusses the operations analysis in the underwater search for the remains of the submarine *Scorpion.*

The a priori target location probability distribution for the search was obtained by monte-carlo procedures based upon nine different scenarios concerning the *Scorpion* loss and associated credibility weights. These scenarios and weights were postulated by others. *Scorpion* was found within 260 yards of the search grid cell having the largest a priori probability.

Frequent computations of local effectiveness probabilities (*LEP*s) were carried out on scene during the search and were used to determine an updated (a posteriori) target location distribution. This distribution formed the basis for recommendation of the current high probability areas for search.

The sum of *LEP*s weighted by the a priori target location probabilities is called search effectiveness probability (*SEP*) and was used as the overall measure of effectiveness for the operation. *SEP* and *LEP*s were used previously in the Mediterranean H-bomb search.

On-scene and stateside operations analysis are discussed and the progress of the search is indicated by values of *SEP* for various periods during the operation.

This paper discusses the operations analysis effort made by the authors' firm, on scene and stateside, during the search for the lost submarine *Scorpion,* June–October 1968. We present an outline of the methods employed and advice given during the search, as well as the impact of the operations analysis on the conduct of the search. An appraisal of the analysis effort is also presented. Because of estimates made of sensor capabilities after *Scorpion* was found, we are able to present a review of the effectiveness of the search effort as it developed in time using the after-search estimates of sensor capabilities and compare these estimates of effectiveness to those made during the search.

In general, the operations analysis provided a quantitative framework for the planning and documentation of the search effort; of itself this substantially enhanced the continuity of the search in the face of considerable turnover of leadership and other personnel. The computation of the a priori distribution of target location proved to be highly successful both as a predictor of location and as a basis for estimation of total required search effort to be expected. The objective of providing good guidance for allocation of search effort was reasonably achieved, although various factors, principally overestimation of sensor capabilities, detracted from the quality of this guidance. The single substantial

*The analysis reported here was largely performed by Daniel H. Wagner, Associates under Ocean Systems, Incorporated Purchase Order No. 863-3539 as a subcontract to Contract No. N00024-69-C-0201 with the Supervisor of Salvage, Naval Ship Systems Command.

contact investigation effort was guided by operations analysis, which also provided a basis for concluding this effort.

The first section provides general background on the operation. Sensors, navigation, the search grid, and the a priori distribution are discussed. The primary measure of effectiveness used, search effectiveness probability (*SEP*), is mentioned with a more detailed explanation included in appendix A. The next two sections discuss separately the operations analysis as it actually took place on scene and stateside. The on-scene analysis of the investigation of contact M8/3 is given in appendix B. After-search estimates of search effectiveness are presented in the fourth section. These values are based upon estimates of sensor sweep width obtained using data collected on scene after *Scorpion* was found. An appraisal of the operations analysis effort is given in the final section.

General Background

After the emergency search phase which stretched across the Atlantic Ocean, the search effort was confined to an area located approximately 400 miles southwest of the Azores.

The search operation was under the tactical command of a succession of six commodores, each an Atlantic Fleet submarine squadron commander. Most of the search in this area was conducted by the *USNS Mizar* with which the embarked ocean engineering team from NRL has had a continuing working relationship; the instruments on board *Mizar* were manned by NRL and Navy Oceanographic Office personnel with help from industry personnel who were on board until August 14. The scientific personnel were divided into two shifts each of which worked 12 hours a day allowing the search to continue around the clock 7 days a week.

The search phases can be conveniently described in terms of *Mizar's* five cruises: (1) June 10–28, (2) July 10–August 6, (3) August 14–September 8, (4) September 20–October 7, and (5) October 16–November 2. These are dates out of the Azores which were 1.5 days sailing from the search scene.

On-scene operations analysis was supplied by the authors (Richardson on cruise (1) and Stone on the second half of cruise (2) and the first half of cruise (3)) and by student colleagues of theirs (S. G. Simpson, a doctoral candidate in mathematics at MIT, on the latter half of cruise (3) and J. A. Rosenberg, a Drexel University cooperative student, cruises (4) and (5)). During cruises (4) and (5), primary responsibility for operations analysis was assigned to Lt David Brummersted, USN and Lcdr James Finlen, USN of Submarine Development Group One (which has a deep submergence mission) who had been briefed on the main techniques presented herein—they were assisted by Rosenberg.

Great difficulties were involved in this search because of the depth of the ocean, the remoteness of this region, and, most of all, the lack of direct information as to the location of *Scorpion*. Despite these problems, *Scorpion* was located on October 28.

Search sensors. The search equipment consisted of a towed platform (called the sled) upon which were mounted cameras, magnetometers, and sonars. This platform operated as much as 2 miles below the surface of the ocean, creating difficult maneuverability problems.

The visual sensors consisted of wide angle lens cameras some of which could be loaded with enough film to take pictures every 30 seconds for as long as 30 hours.

The magnetometer which was employed throughout the search was a proton precession type which measured the strength, but not the direction of the magnetic field. It, too, was activated by a signal from *Mizar*. Despite the uneven magnetic background in the search area, the magnetometer was the first sensor to detect *Scorpion*.

Two different side-looking sonars were used during the search. One of these could be used in either short range or long range mode; the short range mode providing the higher resolution. This was the sonar which was used at the beginning of the search and was replaced by the second sonar after August 4. The latter sonar operated in a single mode.

Navigation. Because the search area was 400 miles away from the nearest land, there was no useful land-based navigation system available. *Loran* and *Omega* although available were not precise enough for conducting this type of search. Thus, navigation throughout the search was done with the aide of satellite fixes available about 18 times a day at irregular intervals and with respect to transponders which were anchored to the ocean bottom. The Navy Oceanographic Office provided the equipment and personnel to obtain the satellite fixes. Inertial navigation was *not* available; this degraded the accuracy usually ascribed to the satellite navigation system.

During most of the search, navigation was performed with uncoded transponders. This meant that one had to guess which transponder or transponders were responding at a given time. This caused considerable confusion and introduced large navigational errors at times.

When coded transponders were available, a computer-transponder system could be used to calculate the position of the ship and sled. This system consisted of coded transponders anchored to the bottom, three transceivers mounted in the hull of *Mizar*, and a responder located on the sled which carried the search sensors. The coding of the transponders allowed *Mizar* to interrogate a particular transponder without interrogating the entire field of them.

Estimates made on scene indicated that the error in this tracking system had a standard deviation of approximately 300 feet in any direction. This was a considerable improvement over the other navigational system using satellites and uncoded transponders.

Search grid. In order to allocate search effort effectively, the search grid shown in Figure 1 was established. The cells are 1 mile in the north-south direction and 0.84 miles in the east-west direction. This figure indicates several points of interest to the analysis. These are:

(i) Location of the piece of metal which was found early in the search and which was subsequently determined to be part of the *Scorpion* debris.

(ii) Sonar contact—This was a contact found in June; however, the area in which this contact is located is quite mountainous.

(iii) M8/3—The location of a large magnetic anomaly found on August 3. This anomaly was investigated quite thoroughly and determined to be a large magnetic rock.

(iv) The location of the largest piece of the *Scorpion*, which was photographed on October 28. For convenience, we index these rectangles either by the integers $j = 1, \ldots, N$, or by a letter-numeral combination (e.g., C8).

A priori distribution. Preparation of the a priori distribution (see Figure 2) superseded earlier attempts to define the search area.

As soon as certain preliminary inquiries and investigations into the possible locations of *Scorpion* were completed (by others), a conference was held July 18 where Dr. J. P. Craven and Dr. F. A. Andrews postulated nine scenarios concerning the events attending the disaster and assigned credences to them. These were then converted by one of the authors to individual a priori probability distributions of the location of *Scorpion* corresponding to the respective scenarios, and these were combined as an average weighted by the assigned credences to obtain the overall distribution (see Figure 2). Craven had earlier

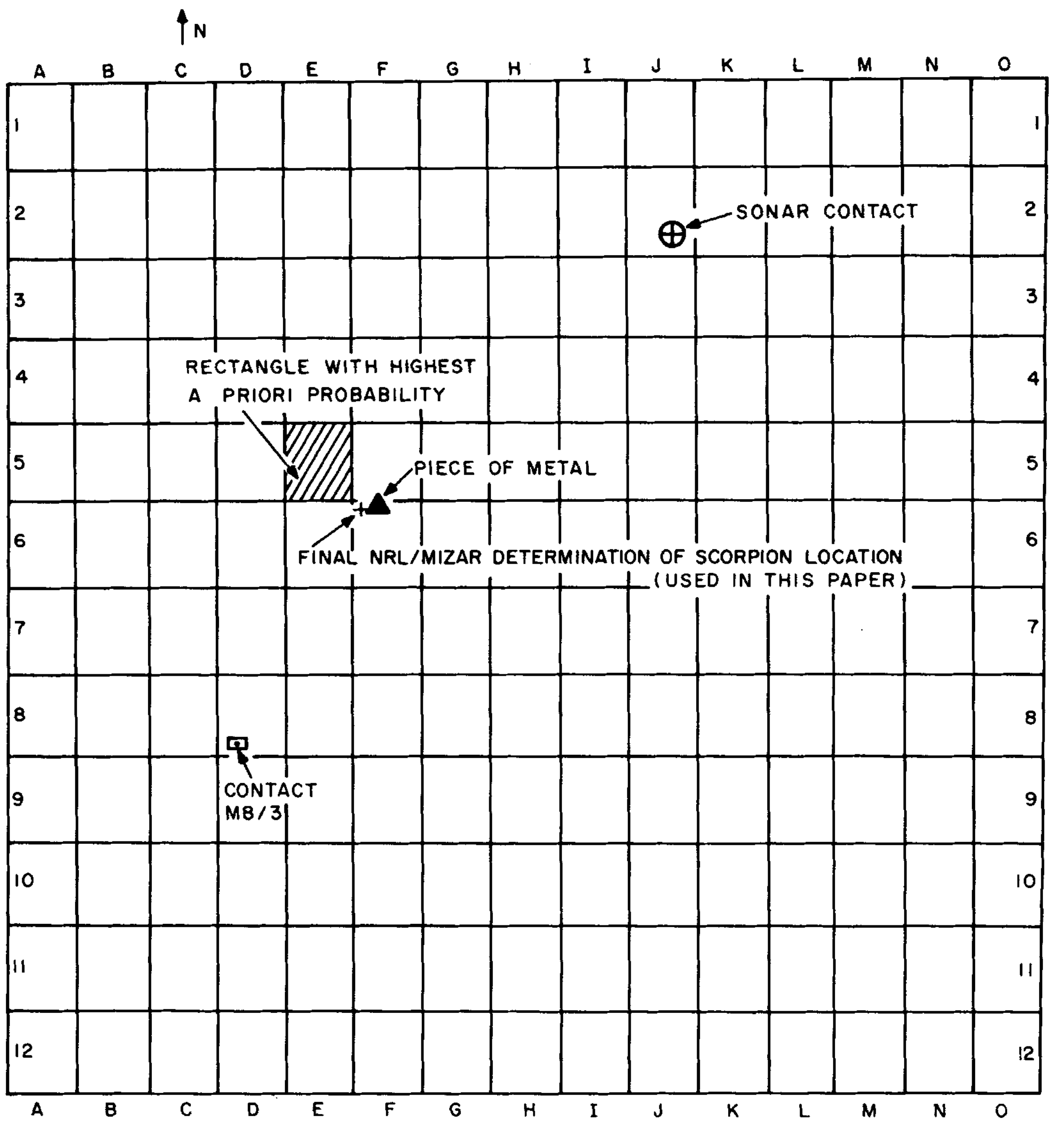

FIGURE 1. *Scorpion* search grid

originated this approach in the H-bomb search. The calculations were performed by monte-carlo procedures.

The credence (a number between zero and one) associated with each scenario reflected the scenario's plausibility relative to the others and incorporated (at least indirectly as interpreted by Craven and Andrews) the views and opinions of Navy operating personnel as well as the analysis of specialists in many scientific areas.

The monte-carlo computer program for generating the probability map begins a replication by causing a random number to be drawn in order to determine the choice of scenario. The scenarios are selected with frequency specified by the assigned credences.

The movement of the submarine as specified by the selected scenario is then simulated with random numbers drawn as required to represent the uncertainties in course, speed, and position, at the time the emergency occurred, as well as other variables. The final position of *Scorpion* is determined and the output of the replication is a "one" added to the appropriate search cell. The result of 10,000 replications is shown in Figure 2.

It is interesting to note that two search cells, E5 and B7, have 23 percent probability of enclosing *Scorpion*. It has now been determined that *Scorpion's* location is in rectangle F6 within 260 yards of the edge of rectangle E5, the rectangle having distinctly the highest a priori probability. Considering the size of the area covered by the a priori distribution, this is indeed a near miss.

Because of the efficiency associated with long sweeps, the piece of metal found early in the search, and other reasons, search effort was never assigned *solely* to cell E5, even when it was the current (a posteriori) high probability cell. Rather, search effort was assigned to the general vicinity of E5 with large amounts placed in neighboring cells, including F6.

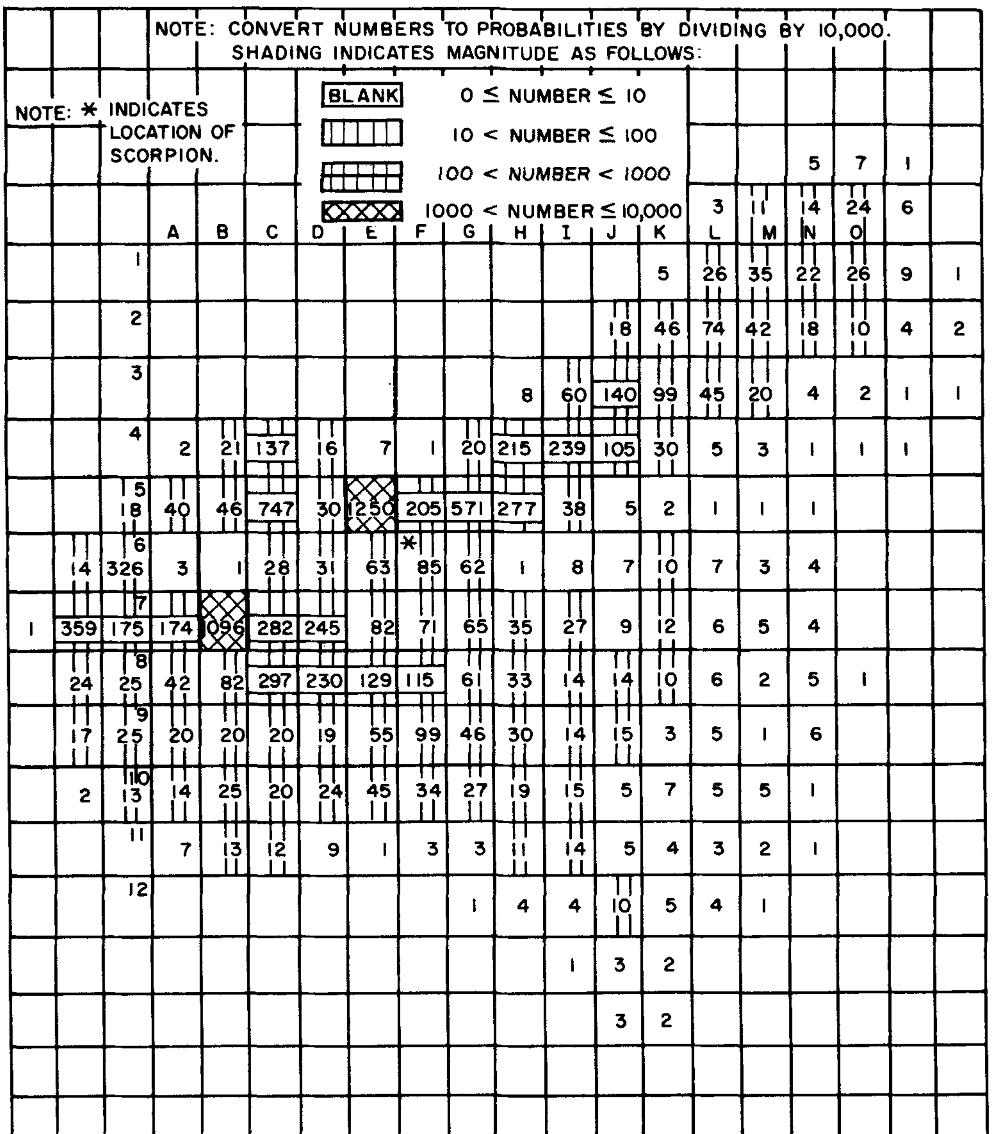

FIGURE 2. Overall A Priori distribution for *Scorpion* search

Calculations show, however, that even if effort had been confined to the search cells having highest a posteriori probability, then assuming a sweep width of 800 feet, the cells to be searched would have included F6 except for the first 153 miles of track length.

In the actual search, 1,026 miles of track were used for broad-area search (not including contact investigation); thus by comparison, 153 miles (6.4 days at 24 miles per day) does not seem too costly a delay in reaching the target's cell. Note that the sweep width of 800 feet assumed in the above calculation is actually smaller than the combined-sensor sweep width used in most of the search (see the second section to follow).

Measure of effectiveness. The measure of search effectiveness, *SEP*, used in this search is basically the same one as developed for the Mediterranean H-bomb search (see appendix A). Denote by L_j the a priori probability that *Scorpion* was in rectangle j and by E_j the probability that *Scorpion* would have been found and identified by the search effort *if Scorpion* were in rectangle j. The probability E_j is called local effectiveness probability (*LEP*), and search effectiveness probability (*SEP*) is defined as

$$\mathrm{SEP} = \sum_{j=1}^{N} L_j E_j.$$

From the *LEP*'s and the a priori probabilities, the a posteriori probabilities L_j' are computed by

$$L_j' = \frac{L_j(1-E_j)}{1-SEP};$$

this is the probability that *Scorpion* is in rectangle j considering the (unsuccessful) search from which the E_j's are computed.

Incremental maximization of *SEP* was the criterion used to recommend search procedures. In practice, this criterion usually resulted in the recommendation that broad area search be conducted in the search cells having the largest values of L_j'.

Few contacts were logged during the search and only two were considered important. One of these, contact *M*8/3, received an extensive investigation (August 14 to August 26) which is discussed in appendix B. Incremental maximization of *SEP* was not being carried out during this investigation.

The computation of *LEP* and *SEP* requires a knowledge of the sensor capabilities, the navigational uncertainties of the search system, and the amount of track length contributed by the sensor groups (i.e., combinations of sensors used simultaneously from the same towed platform). As in the Palomares H-bomb search and many other searches, knowledge of the capabilities of the sensors against the search target was uncertain.

Throughout the analysis we characterize a sensor by two numbers M and β; M is twice the *maximum* detection range of the sensor, and β is an average detection probability of the sensor given that the sensor passes the target at lateral range r, averaged with respect to r over $[0, M]$.

More precisely, let $f(x)$ denote the probability of detecting the target, given that the distance of closest point of approach to the sensor moving on a straight track (the "lateral range") is x. The "sweep width" W (see Reference [1]) of the sensor is defined by

$$W = \int_{-\infty}^{\infty} f(x)\,dx,$$

and β is given by the expression

$$W=\beta M,$$

where M is defined above.

Often several sensors are towed on the same platform. The sweep Ω of the "sensor group" is computed by the formula

$$\Omega=W_n+\sum_{j=1}^{n-1} W_j \prod_{k=j+1}^{n} (1-\beta_k), \tag{1}$$

where the n individual sensors in the group are characterized by the parameters (M_j, β_j) for $1 \leq j \leq n$; the sensors are ordered so that $M_{j+1} \geq M_j$. Formula (1) is based on the assumption that each sensor on the platform operates independently. "Correlation" is introduced by the fact that on a given sweep, the target's lateral range is the same for all sensors in the group.

Operations Analysis On Scene

On-scene operations analysis was used to document the search effort and give day-to-day advice on the conduct of the search. To this end records of the track length in each search cell were kept for each sensor group in Table A–1 and *SEP*'s and posterior target location probabilities were computed regularly.

The operations analysis during the first cruise established a search grid and evaluated the effectiveness of the search effort up to that time. At this time no formal a priori distribution was available.

From June 27 to July 23 there was a hiatus in the on-scene analysis effort aboard *Mizar*. During this time, two contacts were found. A shiny, bent piece of metal was photographed, near the border of *F*5 and *F*6, and a large sonar contact, approximately three hundred feet long was found in *J*2. Both of these contacts received cursory investigations which were not successful in regaining or rephotographing the contacts. We now know that the piece of metal photographed at this time was part of the *Scorpion* debris.

When the on-scene analysis resumed on July 23, during the latter half of the second cruise, the computation of effectiveness was resumed. During run 28 on August 3, a large magnetic anomaly was found in rectangle *D*8. This anomaly was designated *M*8/3. A brief and apparently successful attempt was made to regain this contact on the magnetometer and then *Mizar* returned to port.

In the third cruise, it was decided to investigate and identify the contact *M*8/3. The operations analysis provided assistance in devising search plans for investigating this contact, in evaluating the effectiveness of the contact investigation, and in recommending the allocation of investigation effort.

The investigation of *M*8/3 was discontinued on August 26 after run 52. At this time, it was calculated that the probability that if *M*8/3 were *Scorpion*, then it would have been identified, was 0.86. The details of this analysis are given in appendix B.

During the investigation of *M*8/3, estimates were made of the camera sweep width and of the uncertainty in the computer-transponder navigation system. From records kept during the investigation of the sonar contact in *J*2, is was determined that the probability that if the sonar contact were *Scorpion*, then it would have been identified, was 0.06.

Some of the previous runs had produced photographs of small apparently man-made objects (called artifacts) which might have come from *Scorpion*. These and subsequent artifacts were carefully logged. It was decided to try to determine the size and density of this field of artifacts. Thus although the a priori distribution had arrived on scene by this time and *LEP*'s and *SEP*'s were being computed by the on-scene analyst, these probabilistic considerations did not play a substantial part in determining the placement of search effort at this time.

In the fourth cruise, some effort was made to extend and further define the artifact field, however, this effort met with little success and was soon abandoned. During the remaining part of this phase, which lasted until October 9, the *LEP*'s played an important role in determining the allocation of search effort. This caused a significant amount of effort to be placed in the areas of high probability, notably in *E*5. One of the principal objectives of the search during this cruise was to incrementally maximize *SEP*; this usually resulted in recommendation to search in the cells of highest a posteriori probability. This procedure approximates the Koopman optimal allocation of search effort (Reference [1]) in the case where there are no false contacts. Recent research by the authors' firm has resulted in better search criteria when false contacts are present (see References [3] and [4]).

The fifth cruise was originally intended to be a research and development/search cruise to test sensor modifications, to refine the underwater tracking system, and to study bottom contours and attempt regular search patterns. Search effort was eventually applied to a region containing *F*6 (where *Scorpion* was found) because of proximity to the piece of metal and the fact that this was a region with high a priori probability. On run 74 the magnetometer registered several sharp anomalies. It was possible to return to the same region and on the next run to regain the original contacts and to photograph the *Scorpion* on October 28. On November 1 the on-scene analysis terminated with Rosenberg returning stateside as a courier of photographs of *Scorpion.*

Table 1 gives *SEP* as well as *LEP* in *F*6 as they were computed on scene for search terminating September 10, October 9, and October 28.

TABLE 1. *On-Scene SEP and LEP in F6*

	Runs			
	1–38[a]	1–63	1–71	1–74
	Termination Date			
	Aug. 6	Sept. 10	Oct. 9	Oct. 28
On-scene *SEP*	....	0.40	0.73	0.75
On-scene *LEP* in *F*6	....	0.65	0.94	0.99

[a] Individual sensor detection probabilities, rather than *LEP* and *SEP*, were computed on scene through run 38.

The *LEP*'s computed on scene for *F*6 are, to a surprising degree, higher than the after-search estimates (see Table 3 in the penultimate section), largely because of the optimistic sensor (particularly sonar) sweep width estimates then available to the analysts. Reference [2] and section 5 of [4] are addressed to optimal search with uncertain sweep width data and were motivated by problems arising in this search.

As noted *Scorpion* location is less than 260 yards from the edge of *E*5, which, despite the optimistic values of sweep width used, had the highest a posteriori probability, computed on scene, of containing *Scorpion* during most of the search. At the end of run 71, the last time on-scene a posteriori probabilities were computed, only 2 percent of the rectangles had higher a posteriori probabilities than $E5$. Since parts of *Scorpion* were found close to $E5$, the a posteriori probabilities provided good advice as to where to search, despite again the adverse influence of unrealistic sonar sweep width estimates then available.

Operations Analysis Stateside During the Search

Although the long lines of communication impeded coordination between on-scene and stateside analysis, during the search useful supporting operations analyses were being done stateside. The most important of these was the calculation of the a priori distribution (see Figure 2) of the location of *Scorpion* from scenarios and scenario credences supplied by Craven and Andrews.

The a priori distribution along with *LEP*'s provided a basis for an orderly allocation of search effort by the method described in the first section. The a priori distribution was completed and the estimates of required search time based upon it were transmitted to the Technical Analysis Group on August 1. The distribution was then hand-carried at the earliest date to the scene of operations arriving August 12.

Thus, it was only during the at-sea periods beginning with the third cruise that the a priori distribution had an opportunity to play a role in the conduct of the search.

Even though the a priori distribution was not available on-scene for the first two months of the search effort, it should be noted that the presence of an a priori distribution during the latter part of a search coupled with estimates of effectiveness during the initial part of the search can be used to bring the probability of detection eventually to the level which would have been achieved using an optimal allocation of search effort from the very beginning. This does not mean that time lost by non-optimal procedures at the beginning of the search can be recovered fully by subsequent search. It does mean that the effects of initial non-optimal allocation can be overcome as time progresses and the time penalties thereby minimized.

Estimates of the search effort required by various search systems to find *Scorpion* were made by determining the probability of detecting *Scorpion* as a function of time. Expected time to detection was also computed. Four alternative search systems were considered. These were one and two ships using camera only, and one and two ships using magnetometer and sonar simultaneously.

Figure 3 (submitted to the TAG August 1) shows probability of detection as a function of bottom time for one ship using magnetometer and sonar. Because of lack of information about the search area, expected time spent investigating false contacts was not included in this forecast. The extent of combined coverage is assumed to be 400 feet with probability of detection within that coverage of 0.5; track length is converted to time duration by assuming a speed of 1 knot. The cases of good and poor navigation are considered separately and the estimated expected time to detection is indicated.

It is interesting to note that the curve for poor navigation in Figure 3 (the case which corresponds closest to the actual search situation) indicates that the expected bottom time necessary to produce detection is between 35 and 45 days. The total length of track in the actual search including the research and development effort up to detection and excluding the identification effort around $M8/3$ is approximately 1026 miles or about 43 days of bottom time at the speed of 1 knot assumed in the computations.

Computer programs were developed to allow rapid and presumably more accurate computation of *SEP*, *LEP*, and a posteriori target location probabilities.

As mentioned above, throughout the search apparently man-made objects (artifacts) were photographed and careful records were made of their location. Estimates of the density of artifacts in various

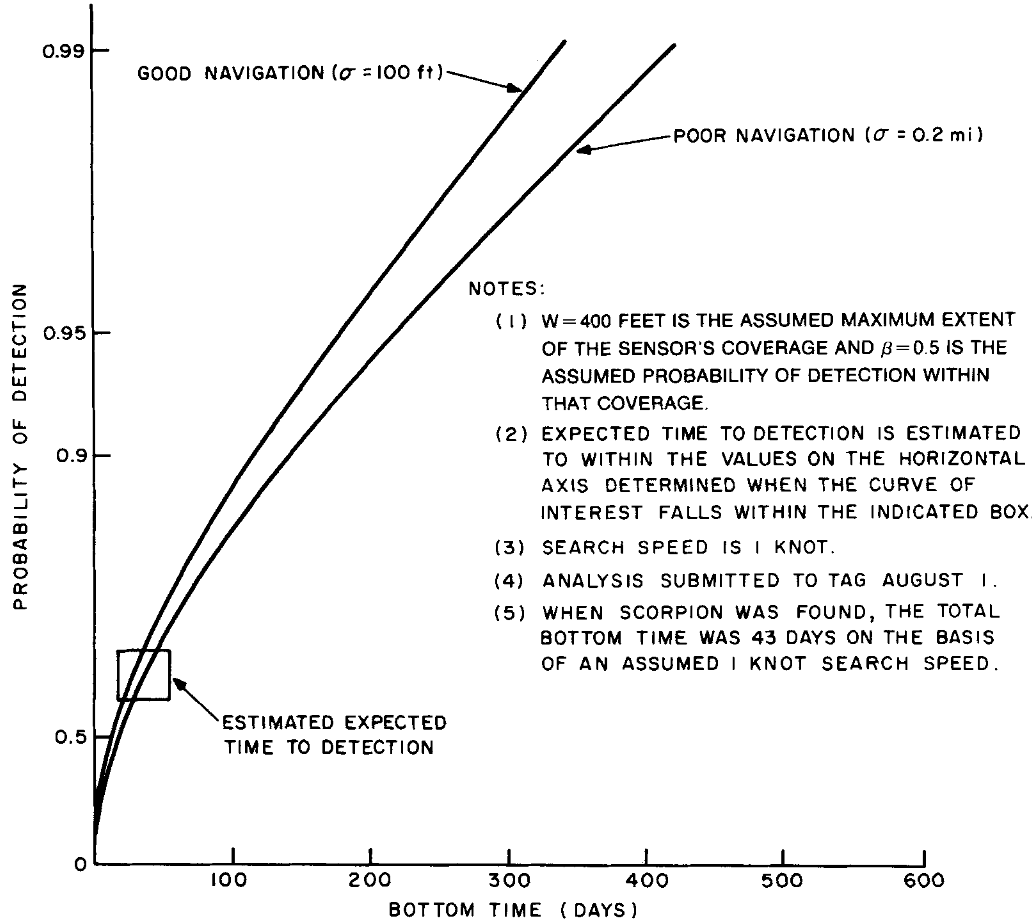

FIGURE 3. Probability of detection for one ship using magnetometer and sonar ($W = 400$ ft and $\beta = 0.5$)

rectangles were made for the purpose of determining whether a pattern was present which would require revision of the a priori distribution. These estimates and associated confidence bounds depended upon the value of visual detection probability in a cell as well as upon the number of artifacts actually observed.

In addition, the estimates were useful in determining the rectangles of highest artifact density; recommendations were presented for testing new camera techniques in areas most likely to contain objects of interest.

Because of the uncertainty in the capabilities of the nonvisual sensors and in particular, because of uncertainties in sweep width, analysis was performed to determine the penalties associated with incorrectly estimating sensor capabilities and to find optimal plans in the face of such uncertainty. It was shown that when no false contacts are present, finding an optimal plan for a sensor with uncertain sweep width (see Reference [2]) may be reduced to a standard separable allocation problem which may be attacked by the same types of methods used to solve this problem with deterministic sweep width, e.g., a Neyman-Pearson constrained extremal method (see Reference [5]). When false contacts are present, more general methods, such as those of [3] and [4], must be used to find optimal plans for search with uncertain sweep width and multiple sensors.

After-Search Estimates of Search Effectiveness

By examining the track charts for the runs during which *Scorpion* was detected and photographed, it is possible to make crude sensor sweep width estimates against the actual target of the search. With these estimates one may calculate, using hindsight, the effectiveness of the search effort as it developed in time. We emphasize that these estimates apply to *Scorpion* as it was actually found (i.e., partially buried). They may or may not be appropriate in another search.

The target of the search is taken as the largest piece of *Scorpion*. The sweep widths in Table 2 are used in our after-search assessment of effectiveness.

The after-search estimates of *SEP* for the search through runs 38, 63, 71, and 74 (*Scorpion* was photographed on run 75) are given in Table 3, based on the sweep widths of Table 2.

Scorpion's position is estimated to be in $F6$ and therefore the *LEP*'s in this rectangle are of particular interest. Table 3 presents these *LEP*'s throughout the search based on the after-search estimates of sensor sweep widths given in Table 2. It is interesting to compare these sweep widths with the estimates available to the on-scene analysts particularly for the non-visual sensor. The latter estimates were typically 800 feet and 400 feet for sonar and magnetometer, respectively.

TABLE 2. *After-Search Estimates of Sensor Sweep Widths*

	Largest-Piece Target		
	M[b]	β[b]	W[b]
Camera	250 ft	0.74	185 ft
Magnetometer	400 ft	0.55	220 ft
Sonar[a]	0	0	0

[a] The value 0 for the sonar sweep width against *Scorpion* is based on the failure of one of the two sonars to detect *Scorpion*. No data are available pertaining to the performance of the other sonar against the *Scorpion* as found.

TABLE 3. *After-Search Estimates of SEP and LEP in F6 Throughout the Search*

	Runs			
	1–38	1–63	1–71	1–74
	Termination Date			
	Aug. 6	Sept. 10	Oct. 9	Oct. 28
SEP	0.26	0.29	0.45	0.48
LEP in *F6*	0.46	0.47	0.65	0.74

[b] M indicates the maximum extent of sensor coverage and β indicates the probability of detection within the coverage; $W=\beta M$. The sweep width of the camera takes into account the length of the target.

Appraisal of Operations Analysis Effort

The a priori distribution is noted to be remarkably accurate with the location of *Scorpion* being placed 260 yards or less from the rectangle $E5$ which had the highest a priori probability of containing

Scorpion. Using the a priori distribution, estimates were made of the expected search effort required to find *Scorpion*. Although based on rather uncertain information, these estimates also proved to be quite accurate. Unfortunately, the a priori distribution was not available on scene until August 12.

As noted above, the a priori distribution combined with the *LEP*'s tended to give good advice as to where to search despite the high *LEP*'s resulting from the optimistic sweep width estimates.

The documentation and evaluation of search effort especially in the form of *LEP*'s and *SEP*'s provided a clear and concise method of describing the results of the search to the Technical Advisory Group.

Throughout the search there was no generally agreed-upon objective method of deciding when to investigate contacts nor was there any generally agreed upon criterion for defining a contact. This caused many possible contacts not to be recorded. It also appears in retrospect that one or more contacts in $F6$ should have been regarded as worthy of further investigation and treated as such in the *LEP* computation. This would have considerably reduced *LEP* in $F6$.

In view of the difficulties presented in the *Scorpion* search, lack of knowledge of the capabilities of the nonoptical sensors against various targets appears to be the major source of uncertainty in measuring search effectiveness and in optimal allocation of search effort. Because of the subsequent usefulness of the analysis of a few passes near the actual location of *Scorpion*, it is considered worth expending at least a modest amount of effort at the beginning of the search to make rough estimates of the sensor capabilities against objects which are similar to the target of the search.

The accuracy of the *SEP* computations was degraded to some extent by errors in the track charts introduced by uncertainty in the true locations of the navigational transponders. As the search progressed, the positions of these transponders became better known due to an increasing number of satellite cross checks. It was intended to revise the track charts during the planned winter operational pause and recompute the *LEP*'s and *SEP* accordingly. *Scorpion*, of course, was found before this effort could take place.

Improvement in communications and transmission of substantive information would help to improve search efficiency. Time lags of roughly three weeks to one month were experienced between the attainment of results stateside (e.g., the a priori distribution) and their application on scene.

One of the principal needs for improved analytic techniques arises in the area of contact investigation. In particular, it remains a problem to determine the proper allocation of effort between contact investigation and broad search in complicated searches involving many different types of sensors. Some progress on this problem has been made in [4].

In view of the uncertainties encountered regarding sensor capabilities in the *Scorpion* search and others (e.g., the H-bomb search), it is apparent that more consideration should be given to the problem of search optimization in the face of uncertainty in sensor sweep widths and other search parameters.

Appendix A

SEARCH EFFECTIVENESS PROBABILITY

This appendix discusses the computation of search effectiveness probability (*SEP*), the measure of effectiveness used during the *Scorpion* search. In contrast to the H-bomb search, where *SEP* was also used, performance of different sensors cannot be treated as statistically independent, since several sensors often were used from approximately the same physical location.

Let E_j, called local effectiveness probability (LEP), be the probability that if the target were in R_j (the j^{th} rectangle), it would have been found (detected and visually identified) by the search effort placed in R_j. (An exception to this definition occurs when no contacts have been generated and is discussed below.) The target might be found during the investigation of a contact generated by a nonvisual sensor or during the general visual search. Search effectiveness probability (SEP) is given by (N is the number of rectangles)

$$SEP = \sum_{j=1}^{N} L_j E_j,$$

where L_j is the a priori probability that the target is in the j^{th} rectangle.

In order to calculate the probabilities E_j, let D_j be the probability that if *Scorpion* were in rectangle j, it would have been detected either visually or nonvisually. Let P_j be the probability that if *Scorpion* were in rectangle j, it would have been detected (and it is assumed identified) by the general camera search (i.e., search not directed toward identifying a specific contact). Define C_j to be the probability that if the *Scorpion* were one of the nonvisual contacts in rectangle j, it would have been identified. If there are no nonvisual contacts in rectangle j, C_j is set equal to 1 by convention.

Then

(A–1) $$E_j = P_j + C_j(D_j - P_j).$$

Let L_j' be defined by

(A–2) $$L_j' = \frac{L_j(1-E_j)}{1-SEP},$$

so that L_j' gives the a posteriori probability that *Scorpion* was in the j^{th} rectangle given the search up to the time at which the E_j's were computed. These are the numbers, considering the search to date, which are used to guide the search to the areas of highest probability of containing the target.

When $C_j = 1$ in (A–1), $E_j = D_j$ and LEP is the probability that at least one sensor (visual or nonvisual) detects, rather than the probability of detection and visual identification. This convention for C_j leads to the correct formula for the a posteriori probability distribution (A–2), but makes it difficult to give an intuitively appealing definition of SEP.

It can be shown that

(A–3) $$\frac{SEP-Q}{1-Q} = Pr\left\{\begin{matrix}\text{Target is detected and}\\ \text{visually identified}\end{matrix} \;\middle|\; \begin{matrix}\text{Contacts at the locations}\\ \text{observed in the search}\end{matrix}\right\},$$

where for $S_N = \{j: \text{no contacts in } R_j\}$

$$Q = \sum_{j \in S_N} L_j Pr\left\{\begin{matrix}\text{Target is detected in } R_j \text{ by}\\ \text{a } \textit{non-visual} \text{ sensor}\end{matrix} \;\middle|\; \text{Target within } R_j\right\}.$$

The conditional probability of detection and identification given by (A–3) probably provides a better basis for terminating an unsuccessful search in the presence of false targets than does SEP. The derivation of (A–3) is omitted since it is rather lengthy and would take us beyond the expository nature of the present paper.

In order to compute D_j and P_j, track length records are used which give the amount of track of each sensor group in each search rectangle. A sensor group is a combination of sensors which are used simultaneously. A list of sensor groups used in the *Scorpion* search is given in Table A–1.

TABLE A–1. *Definition of Sensor Groups for Scorpion Search*

Sensor Group Designation [a] [b]	Description
Single Sensor 1	Camera
Single Sensor 2	Magnetometer
Single Sensor 3	First Sonar (Short Range—SR)
Single Sensor 4	First Sonar (Long Range—LR)
Single Sensor 5	Second Sonar
Single Sensor 6	....
7	Camera and Magnetometer
8	Camera and First Sonar (SR)
9	....
10	Camera and Second Sonar
11	....
12	Magnetometer and First Sonar (SR)
13	Magnetometer and First Sonar (LR)
14	Magnetometer and Second Sonar
15	....
16	Camera, Magnetometer, and First Sonar (SR)
17	....
18	Camera, Magnetometer, and Second Sonar

[a] Gaps in the table correspond to provision made for the introduction of new sensor groups.
[b] Only the sensors actually mentioned belong in the group.

Let Ω_i be the sweep width of the i^{th} sensor group and let Λ_{ij} be the amount of track length of the i^{th} sensor group in the j^{th} rectangle. Because of the large navigational uncertainties and lack of regular search plans, the probability $D_j^{(i)}$ of the i^{th} sensor group detecting the *Scorpion* given that it is the j^{th} rectangle is computed by the random search formula of Koopman [1]. That is,

$$D_j^{(i)}=1-\exp\left(\frac{-\Lambda_{ij}\Omega_j}{A_j}\right),$$

where A_j is the area of the j^{th} rectangle ($A_j=0.84$ square miles for the *Scorpion* search). The above formula assumes the effort is uniformly, but randomly, distributed in the area. To compute D_j, the probability that at least one of the sensor groups would detect *Scorpion* given that it is in the j^{th} rectangle, we assume that the sensor *groups* are mutually statistically independent. Thus

$$D_j=1-\prod_{i=1}^{18}(1-D_j^{(i)}).$$

(Note there are 18 sensor groups in Table A–1 not all of which were actually used in the search.)

The probabilities P_j are calculated as follows: Let $\Lambda_j^{(c)}$ be the amount of camera track length in the j^{th} grid rectangle and let W_c be the camera sweep width. Then P_j is given by

$$P_j=1-\exp\left(\frac{-\Lambda_j^{(c)}W_c}{A_j}\right).$$

The track length $\Lambda_j^{(c)}$ is computed by the equation

$$\Lambda_j^{(c)}=\Lambda_{1j}+\Lambda_{7j}+\Lambda_{8j}+\Lambda_{10j}+\Lambda_{16j}+\Lambda_{18j}.$$

Let K_j be the number of contacts in the j^{th} rectangle. For $K_j \neq 0$, let γ_{kj} be the conditional probability that the target would have been identified by contact investigation, given that the target is the k^{th} contact in the j^{th} rectangle (see the computation of γ_{kj} in the case of contact $M8/3$ discussed in appendix B).

The contact investigation probabilities C_j are computed by

$$C_j=\begin{cases} \dfrac{1}{K_j}\displaystyle\sum_{k=1}^{K_j}\gamma_{kj} & \text{for } K_j \neq 0 \\ 1 & \text{otherwise.}\end{cases}$$

Appendix B
INVESTIGATION OF CONTACT M8/3

This appendix discusses operations analysis connected with the investigation of the contact designated $M8/3$. This was the only contact other than the final contact actually produced by the *Scorpion* which was systematically investigated. Operations analysis provided a basis for allocating contact investigation effort and contributed to the decision to abandon the investigation.

A Priori Contact Location Distribution

In order to systematically investigate $M8/3$ a circular normal a priori contact location distribution with standard deviation $\sigma=1200$ ft in any direction was postulated. The value chosen for σ was based on the navigational uncertainty at the time the contact was made and on the uncertainty in returning to the charted location (i.e., posit) of $M8/3$.

Upon returning to the search area on August 14, coded transponders were planted in the vicinity of the posit of $M8/3$. The locations of these transponders were calculated by use of several satellite fixes. All navigation about $M8/3$ was then performed with respect to these transponders, and a check of the navigational errors for this system indicated that the navigational uncertainty had a standard deviation σ_N of about 300 ft in any direction.

Because of the improved navigational accuracy, a microgrid was constructed with center at the posit of $M8/3$. The grid was composed of 64 cells, 1000 ft on a side. The cells were indexed by the ordered pairs (i, j) $i=1, \ldots, 8$ and $j=1, \ldots, 8$. Using a circular normal distribution with $\sigma=1200$ ft for the location of $M8/3$, L_{ij}^{μ}, the probability that $M8/3$ was contained in the $(i, j)^{th}$ grid cell, was computed and used to approximate the circular normal distribution above by assuming that the location of $M8/3$ was contained in the $(i, j)^{th}$ grid cell with probability L_{ij}^{μ}.

Allocation of Magnetometer Effort

The magnetometer local detection probability $\tilde{D}_{ij}^{m}$ in the $(i, j)^{th}$ cell is the probability that the magnetometer would have detected $M8/3$ given that $M8/3$ was in that cell. The magnetometer was assumed to have a definite range law; that is, it would detect with probability $\beta=1$ if $M8/3$ were at lateral ranges of 200 feet or less and would not detect beyond that. This gave the magnetometer a sweep width W_m of 400 feet. The search within each cell was evaluated as random. That is, if Λ_{ij}^{μ} was the amount of track length placed in the $(i,j)^{th}$ cell and A^{μ}_{ij} the area of that cell, then

$$\tilde{D}^{m}_{ij}=1-\exp\left[-\left(\frac{W_m\Lambda^{\mu}_{ij}}{A^{\mu}_{ij}}\right)\right]. \tag{B-1}$$

Detection effectiveness probability (*DEP*) is the probability that if *M*8/3 were the *Scorpion*, it would have been detected by the magnetometer during the microsearch. That is,

$$DEP=\sum_{i=1}^{8}\sum_{j=1}^{8} L^{\mu}_{ij}\tilde{D}^{m}_{ij}. \tag{B-2}$$

Every two to three runs *DEP* was computed and posteriori contact location probabilities were calculated by

$$\tilde{L}^{\mu}_{ij}=\frac{L^{\mu}_{ij}\,(1-\tilde{D}^{m}_{ij})}{DEP}.$$

The magnetometer search effort was directed to the cells having the highest posterior probabilities. This provided a rough optimization of magnetometer detection effort.

The *DEP* resulting from runs 39–51 is 0.95. Run 52 which was cut short because of weak batteries in the bottom transponders near *M*8/3 is not included in the calculations of effectiveness.

Identification of Contacts Found During Runs 39–51

During the search about *M*8/3, all significant magnetometer contacts were found within a circle of radius 650 feet centered at *PIX* 42. *PIX* 42 is located approximately 1300 ft south and east of the posit of *M*8/3 and is so called because it is the location of a group of rocks photographed during run 42 at the same time a magnetic anomaly was detected.

An intensive camera search was performed inside this circle. The circle was divided into quadrants, and camera detection probabilities were calculated within each of these quadrants. In order to calculate these probabilities, it was assumed that the camera had a sweep width $W_c=75$ ft and that detection and identification occurs with probability 1 across this sweep width (i.e., $\beta=1$). The value of W_c that was used is an average sweep width derived from data available on scene. The effectiveness of the camera search was evaluated as random in each quadrant, and the appropriate analog of (B–1) was used to calculate the camera detection probabilities in each quadrant.

The camera detection probabilities for the four quadrants were averaged to compute $C^{\mu}=0.91$ the probability that if one of the micro-contacts were the *Scorpion*, it would have been identified.

Thus if *M*8/3 were *Scorpion*,

$C_j=Pr\{$*Scorpion* would have been identified $|$ *Scorpion* among micro contacts$\}$.

$Pr\{$ *Scorpion* among micro contacts$\}$

$=C^{\mu}\ DEP=0.91\times 0.95=0.86.$

It is this value of 0.86 which is used for C_j in *D*8 (the rectangle containing *M*8/3). This high value for C_j contributed to the commodore's decision to abandon the investigation of *M*8/3 after run 52.

ACKNOWLEDGMENTS

The authors wish to make several acknowledgments. Particular gratitude is expressed for the scientific leadership of Dr. John P. Craven, Director of the stateside Technical Advisory Group formed for the search; the general view is shared that his insight and perseverance kept the search going, and he contributed a vital (and highly successful) part of the operations analysis. It was a pleasure to work under the command of the search commodores—their approaches were varied, but in general it was their interest in the usefulness of search analysis that made the operations research efforts worthwhile. The splendid cooperation in logistical support, etc., received from the staff of Commander, Submarine Forces, Atlantic Fleet, under whose command this successful operation was entirely carried out, is very much appreciated. Submarine Development Group One officers did a very creditable job of rapid mastery of the relevant operations analysis techniques during stateside briefings and of carrying on the calculations on scene in the latter part of the search. Deep respect is expressed for the splendid ocean engineering work done by the Naval Research Laboratory (NRL) scientific teams working aboard *Mizar*—the feat of towing and locating the sensor sled close off the ocean bottom in some 10,000 feet of water for weeks on end over the five months search period, with rather low equipment downtime, reflects great skill and dedication. Moreover, their Chief Scientist, Mr. C. L. Buchanan, was directing the search during the research and development phase during which the *Scorpion* was finally found. Finally, it was a pleasure to cooperate with Dr. Frank A. Andrews of Catholic University who assisted the Technical Advisory Group as an Ocean Systems, Inc., consultant; having commanded the *Thresher* search as a Captain, USN, and having consulted to the H-bomb search, he has an unusually comprehensive knowledge of and experience in the operational engineering, and analytical aspects of deep submerged search.

REFERENCES

[1] Koopman, B. O., "The Theory of Search: III Optimum Distribution of Searching Effort," Operations Research **5**, 613–626 (1957).

[2] Richardson, H. R. and B. Belkin, "Optimal Search With Uncertain Sweep Width," submitted for publication.

[3] Stone, L. D. and J. A. Stanshine, "Optimal Search Using Uninterrupted Contact Investigation," SIAM Journal on Applied Mathematics, **20**, 241–263 (1971).

[4] Stone, L. D., J. A. Stanshine, and C. A. Persinger, "Optimal Search With Poisson Distributed False Targets," submitted for publication.

[5] Wagner, D. H., "Nonlinear Functional Versions of the Neyman-Pearson Lemma," SIAM Review **11**, 52–65 (1969).