Bayesian Analysis: A Look at Today and Thoughts of Tomorrow *

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Abstract

First, a snapshot is provided of the current state of Bayesian statistics. Included are entry points to the Bayesian literatures in various disciplines and various areas of statistics. Next, the status of the various approaches to Bayesian analysis are discussed; these approaches are termed the objective, subjective, robust, frequentist-Bayes, and quasi-Bayes approaches. Speculations about the future are sprinkled throughout this latter material. Finally, comments about computation and existing and future software are given.

Some key words and phrases: Subjective Bayes; Objective Bayes; Robust Bayes; Frequentist Bayes; Quasi-Bayes; Bayesian software.

1. Introduction

Life was simple when I became a Bayesian in the 70's; it was possible to track virtually all Bayesian activity. Preparing this paper on Bayesian statistics was humbling, because it caused me to realize that I have lately been aware of only about 10% of the ongoing activity in Bayesian analysis. One goal of this paper is thus to provide an overview of, and access to, a significant portion of this current activity. Necessarily, the overview will be extremely brief; indeed, an entire area of Bayesian activity might only be mentioned in one sentence and with a single reference. And, many areas of activity are ignored altogether, either due to ignorance on my part or because no single reference provides access to the literature.

A second goal of the paper will be to highlight issues or controversies that may shape the way that Bayesian analysis develops. This material is somewhat self-indulgent and should not to be taken too seriously; for instance, if I had been asked to write such an article ten years ago,

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I would have missed the mark by not anticipating the extensive development of MCMC and its enormous impact on Bayesian statistics.

Section 2 provides a brief snapshot of the existing Bayesian activity and emphasizes its dramatic growth in the 90's, both inside and outside statistics. I found myself simultaneously rejoicing and being disturbed at the level of Bayesian activity. As a Bayesian, I rejoiced to see the extensive utilization of the paradigm, especially among non-statisticians. As a statistician, I worried that our profession may not be adapting fast enough to this dramatic change; we may be in danger of 'losing' Bayesian analysis to other disciplines (as we have 'lost' other areas of statistics). In this regard, it is astonishing that most statistics and biostatistics departments in the U.S.A. do not even regularly offer a single Bayesian statistics course.

Section 3 is organized by *approaches to Bayesian analysis*, in particular the objective, subjective, robust, frequentist-Bayes, and what I term quasi-Bayes approaches. It is in this section that musings about the current and future state of Bayesian statistics primarily occur. Section 4 briefly discusses the critical issues of computation and software.

2. Bayesian Activity

2.1. Numbers and Organizations

The dramatically increasing level of Bayesian activity can be seen in part through the raw numbers. Harry Martz (personal communication) studied the SciSearch database at Los Alamos National Laboratories to determine the increase in frequency of articles involving Bayesian analysis over the last 25 years. From 1974 to 1994, the trend was linear, with roughly a doubling of articles every 10 years. In the last five years, however, there has been a very dramatic upswing in both the number and the rate of increase of Bayesian articles.

This same phenomenon is also visible by looking at the number of books written on Bayesian analysis. During the first 200 years of Bayesian analysis (1769 - 1969), there were perhaps 15 books written on Bayesian statistics. Over the next 20 years (1970-1989), a guess as to the number of Bayesian books produced is 30. Over the last 10 years (1990-1999), roughly 60 Bayesian books have been written, not counting the many dozens of Bayesian conference proceedings and collections of papers. Bayesian books in particular subject areas are given in Sections 2.2 and 2.3. A selection of general Bayesian books are listed in Appendix 1.

Another aspect of Bayesian activity is the diversity of existing organizations that are significantly Bayesian in nature, including the following (which are those with an active website): *International Society of Bayesian Analysis* (http://www.bayesian.org), *ASA Section on Bayesian Statistical Science* (http://www.stat.duke.edu/sbss/sbss.html), *Decision Analysis So-*

ciety of INFORMS (http://www.informs.org/society/da), and *ASA Section on Risk Analysis* (http://www.isds.duke.edu/riskanalysis/ras.html).

In addition to the activities and meetings of these societies, the following are long standing series of prominent Bayesian meetings that are not organized explicitly by societies: *Valencia Meetings on Bayesian Statistics* (http://www.uv.es/∼bernardo/valenciam.html), *Conferences on Maximum Entropy and Bayesian Methods* (http://omega.albany.edu:8008/maxent.html), *CMU Workshops on Bayesian Case Studies* (http://lib.stat.cmu.edu/bayesworkshop/), and *RSS Conferences on Practical Bayesian Statistics*. The average number of Bayesian meetings per year is now well over 10, with at least an equal number of meetings being held that have a strong Bayesian component.

2.2. Interdisciplinary Activities and Applications

Applications of Bayesian analysis in industry and government are rapidly increasing but hard to document, as they are often 'in-house' developments. It is far easier to document the extensive Bayesian activity occurring in other disciplines; indeed, in many fields of the sciences and engineering, there are now active groups of Bayesian researchers. Here we can do little more than list various fields that have seen a considerable amount of Bayesian activity, and present a few references to access the corresponding literature. Most of the given references are books on Bayesian statistics in the given field, emphasizing that the activity in the field has reached the level wherein books are being written. Indeed, this was the criterion for listing an area, although fields in which there is a commensurate amount of activity, but no book, are also listed. (It would be hard to find an area of human investigation in which there does not exist some level of Bayesian work, so many fields of application are being omitted.)

For **Archaeology**, see Buck, Cavanaugh, and Litton (1996); **Atmospheric Sciences**, see Berliner, Royle, Wikle, and Milliff (1999); **Economics and Econometrics**, see Cyert and DeGroot (1987), Poirier (1995), Perlman and Blaug (1997), Kim, Shephard and Chib (1998) and Geweke (1999); **Education**, see Johnson (1997); **Epidemiology**, see Greenland (1998); **Engineering**, see Godsill and Rayner (1998); **Genetics**, see Iversen, Parmigiani and Berry (1998), Dawid (1999) and Liu, Neuwald, and Lawrence (1999); **Hydrology**, see Parent, Hubert, Bobée and Miquel (1998); **Law**, see DeGroot, Fienberg, and Kadane (1986) and Kadane and Schuan (1996); **Measurement and Assay**, see Brown (1993), and http://www.pnl.gov/bayesian/; **Medicine**, see Berry and Stangl (1996) and Stangl and Berry (1998); **Physical Sciences**, see Bretthorst (1988), Jaynes (1999), and http://astrosun.tn.cornell.edu/staff/loredo/bayes/; **Quality Management**, see Moreno and Rios-Insua (1999); **Social Sciences**, see Pollard (1986) and Johnson and Albert (1999).

2.3. Areas of Bayesian Statistics

In this subsection, Bayesian activity is listed by statistical area. Again, the criterion for inclusion of an area is primarily the amount of Bayesian work being done in that area, as evidenced by books being written (or a corresponding level of papers).

For **Biostatistics**, see Berry and Stangl (1996), Carlin and Louis (1996), and Kadane (1996); **Causality**, see Spirtes, Glymour and Scheines (1993) and Glymour and Cooper (1999); **Classification, Discrimination, Neural Nets, etc.**, see Neal (1996, 1999), Müller and Rios-Insua (1998), and George (2000); **Contingency Tables**, see Fienberg (2000); **Decision Analysis and Decision Theory**, see Smith (1988), Robert(1994), Clemen (1996), and Brown (2000); **Design**, see Pilz (1991), Chaloner and Verdinelli (1995), and Müller (1999); **Empirical Bayes**, see Carlin and Louis (1996) and Carlin and Louis (2000); **Exchangeability and Other Foundations**, see Good (1983), Regazzini (1999), Kadane, Schervish and Seidenfeld (1999) and Robins and Wasserman (2000); **Finite Population Sampling**, see Bolfarine and Zacks (1992) and Mukhopadhyay (1998); **Generalized Linear Models**, see Dey, Ghosh and Mallick (2000); **Graphical Models and Bayesian Networks**, see Pearl (1988), Jensen (1986), Lauritzen (1996), Jordan (1998), and Cowell, Dawid, Lauritzen, and Spiegelhalter (1999). **Hierarchical (Multilevel) Modeling**, see Hobert (2000); **Image Processing**, see Fitzgerald, Godsill, Kokaram, and Stark (1999); **Information**, see Barron, Rissanen and Yu (1998) and Soofi (2000); **Missing Data**, see Rubin (1987) and Meng (2000); **Nonparametrics and Function Estimation**, see Dey, Müller and Sinha (1998), Müller and Vidakovic (1999), and Robins and Wasserman (2000); **Ordinal Data**, see Johnson and Albert (1999); **Predictive Inference and Model Averaging**, see Aitchison and Dunsmore (1975), Leamer (1978), Geisser (1993), Draper (1995), Clyde (1999), and the BMA website under software; **Reliability and Survival Analysis**, see Clarotti, Barlow and Spizzichino (1993) and Sinha and Dey (1999); **Sequential Analysis**, see Carlin, Kadane, and Gelfand (1998) and Qian and Brown (1999); **Signal Processing**, see Ó Ruanaidh and Fitzgerald (1996) and Fitzgerald, Godsill, Kokaram, and Stark (1999); **Spatial Statistics**, see Wolpert and Ickstadt (1998) and Besag and Higdon (1999); **Testing, Model Selection, and Variable Selection**, see Kass and Raftery (1995), O'Hagan (1995), Berger and Pericchi (1996), Berger (1998), Racugno (1998), Sellke, Bayarri and Berger (1999), Thiesson, Meek, Chickering, and Heckerman (1999) and George (2000); **Time Series**, see Pole, West and Harrison (1995), Kitagawa and Gersch (1996) and West and Harrison (1997).

3. Approaches to Bayesian Analysis

This section presents a rather personal view of the status and future of five approaches to Bayesian analysis, termed the objective, subjective, robust, frequentist-Bayes, and quasi-Bayes approaches. This is neither a complete list of the approaches to Bayesian analysis, nor a broad discussion of the considered approaches. The main purpose of the section is to emphasize that there exist a variety of different and viable Bayesian approaches to statistics, each of which can be of great value in certain situations and for certain users. We should be aware of the strengths and weaknesses of each approach, but all will be with us in the future and should be respected as part of the Bayesian paradigm.

3.1. Objective Bayesian Analysis

It is a common perception that Bayesian analysis is a subjective theory. This is neither true historically nor in practice. The first Bayesians, Thomas Bayes (see Bayes 1783) and Laplace (see Laplace 1812) performed Bayesian analysis using a constant prior distribution for unknown parameters. Indeed, this approach to statistics, then called "inverse probability" (see Dale 1991) was very prominent for most of the nineteenth century, and was highly influential in the early part of this century. Criticisms of use of a constant prior distribution caused Jeffreys to introduce significant refinements of this theory (see Jeffreys 1961). Most of the applied Bayesian analyses I see today follow the Laplace-Jeffreys objective school of Bayesian analysis, possibly with additional modern refinements. (Of course, others may more frequently see subjective Bayesian applications, depending on the area in which they work.)

Many Bayesians object to the label 'objective Bayes', claiming that it is misleading to say that any statistical analysis can truly be objective. While agreeing with this at a philosophical level (cf. Berger and Berry 1988), I feel that there are a host of practical and sociological reasons to use the label; Statistics must get over its aversion to calling good things by attractive names.

The most familiar element of the objective Bayesian school is the use of *noninformative* or *default* prior distributions. The most famous of these is the *Jeffreys prior* (see Jeffreys 1961). *Maximum entropy* priors are another well-known type of noninformative prior (although they often also reflect certain informative features of the system being analyzed). The more recent statistical literature emphasizes what are called *reference priors* (Bernardo, 1979, and Yang and Berger, 1997), which prove remarkably successful from both Bayesian and non-Bayesian perspectives. See Kass and Wasserman (1996) for a recent review of methods for selecting noninformative priors.

A quite different area of the objective Bayesian school is that concerned with techniques for default model selection and hypothesis testing. Successful developments in this direction are much more recent (cf. O'Hagan, 1995, Berger and Pericchi, 1996, and Sellke, Bayarri and Berger, 1999). Indeed, there is still considerable ongoing discussion as to which default methods are to be preferred for these problems (see Racugno 1998).

The main concern with objective Bayesian procedures is that they often utilize improper prior distributions, and so do not automatically have desirable Bayesian properties, such as coherency. Also, poor choice of improper priors can even lead to improper posteriors. Thus proposed objective Bayesian procedures are typically studied to ensure that such problems do not arise. Also, objective Bayesian procedures are often evaluated from non-Bayesian perspectives, and usually turn out to be stunningly effective from these perspectives.

3.2. Subjective Bayesian Analysis

Although comparatively new on the Bayesian scene, subjective Bayesian analysis is currently viewed by many Bayesians to be the 'soul' of Bayesian statistics. Its philosophical appeal is undeniable, and few statisticians would argue against its use when the needed inputs (models and subjective prior distributions) can be fully and accurately specified. The difficulty in such specification (cf. Kahneman, Slovic, and Tversky 1986) often limits application of the approach, but there has been a considerable research effort to further develop elicitation techniques for subjective Bayesian analysis (cf. French and Smith, 1997, and *The Statistician*, **47**, vol.1, 1998).

There are many problems in which use of subjective prior information is clearly essential and others in which it is readily available; use of subjective Bayesian analysis for such problems can provide dramatic gains. Even when a complete subjective analysis is not feasible, a judicious use of partly subjective and partly objective prior distributions is often attractive (cf. Andrews, Berger, and Smith 1993).

3.3. Robust Bayesian Analysis

Robust Bayesian analysis recognizes the impossibility of complete subjective specification of the model and prior distribution; after all, complete specification would involve an infinite number of assessments, even in the simplest situations. The idea is thus to work with classes of models and classes of prior distributions, with the classes reflecting the uncertainty remaining after the (finite) elicitation efforts. (Classes could also reflect the differing judgements of various individuals involved in the decision process.)

The foundational arguments for robust Bayesian analysis are compelling (cf. Kadane, 1984, and Walley, 1991) and there is an extensive literature on the development of robust Bayesian methodology, including Berger (1985), Rios Insua (1990), Berger (1994), and Berger et. al.

(1996). Routine practical implementation of robust Bayesian analysis will require development of appropriate software, however.

Robust Bayesian analysis is also an attractive technology for actually implementing a general subjective Bayesian elicitation program. Resources (time and money) for subjective elicitation are typically very limited in practice, and need to be optimally utilized. Robust Bayesian analysis can, in principle, be used to direct the elicitation effort, by first assessing if the current information (elicitations and data) is sufficient for solving the problem and, if not, determining which additional elicitations would be most valuable (cf. Liseo, Petrella and Salinetti 1996).

3.4. Frequentist-Bayes Analysis

It is hard to imagine that the current situation, with several competing foundations for statistics, will exist indefinitely. Assuming that a unified foundation is inevitable, what will it be? Today, an increasing number of statisticians envisage that this unified foundation will be a mix of Bayesian and frequentist ideas (with elements of the current likelihood theory - see Reid 2000 - thrown in). Here is my view of what this mixture will be.

First, the language of statistics will be Bayesian. Statistics is about measuring uncertainty, and over 50 years of efforts to prove otherwise have convincingly demonstrated that the only coherent language in which to discuss uncertainty is the Bayesian language. In addition, the Bayesian language is an order of magnitude easier to understand than the classical language (witness the P-value controversy; cf. Sellke, Bayarri, and Berger, 1999), so that a switch to the Bayesian language should considerably increase the attractiveness of statistics. Note that, as discussed earlier, this is not about subjectivity or objectivity; the Bayesian language can be used for either subjective or objective statistical analysis.

On the other hand, from a methodological perspective, it is becoming clear that both Bayesian and frequentist methodology is going to be important. For parametric problems, Bayesian analysis seems to have a clear methodological edge, but frequentist concepts can be very useful, especially in determining good objective Bayesian procedures (see, e.g., Reid 2000).

In nonparametric analysis, it has long been known (cf. Diaconis and Freedman 1986) that Bayesian procedures can behave poorly from a frequentist perspective. While poor frequentist performance is not necessarily damning to a Bayesian, it typically should be viewed as a warning sign that something is amiss, especially when the prior distribution being used contains more 'hidden' information than elicited information (as is virtually always the case with nonparametric priors).

Furthermore, there are an increasing number of examples in which frequentist arguments yield satisfactory answers quite directly, while Bayesian analysis requires a formidable amount of extra work. (The simplest such example is MCMC itself, in which one evaluates an integral by a sample average, and not by a formal Bayesian estimate; see Robins and Wasserman, 2000, for other examples). In such cases, I believe that the frequentist answer can be accepted by Bayesians as an approximate Bayesian answer, although it is not clear, in general, how this can be formally verified.

This discussion of unification has been primarily from a Bayesian perspective. From a frequentist perspective, unification also seems inevitable. It has long been known that 'optimal' unconditional frequentist procedures must be Bayesian (cf. Berger 1985), and there is growing evidence that this must be so even from a conditional frequentist perspective (cf. Berger, Boukai, and Wang 1997).

Note that I am *not* arguing for an eclectic attitude towards statistics here; indeed, I think the general refusal in our field to strive for a unified perspective has been the single biggest impediment to its advancement. I am simply saying that the unification that will be achieved will almost necessarily have frequentist components to it.

3.5. Quasi-Bayesian Analysis

There is another type of Bayesian analysis that one increasingly sees being performed, and that is unsettling to 'pure' Bayesians and most non-Bayesians. In this type of analysis, priors are chosen in a variety of ad hoc fashions including use of vague proper priors; choosing priors to 'span' the range of the likelihood; and choosing priors with tuning parameters that are adjusted until the answer 'looks nice'. I call such analyses *quasi-Bayes* because, while they utilize Bayesian machinery, they do not carry with them any of the guarantees of good performance that come with either true subjective analysis or (well-studied) objective Bayesian analysis. It is useful to briefly discuss the problem with each of these quasi-Bayes procedures.

Use of vague proper priors will work well when the vague proper prior is a good approximation to a good objective prior, but this often fails to be the case. For instance, in normal hierarchical models with a 'higher level' variance V, it is quite common to use the vague proper prior density $\pi(V) \propto V^{-(\varepsilon+1)} \exp(-\varepsilon'/V)$, with ε and ε' small. However, as $\varepsilon \to 0$ it is typically the case in these models that the posterior distribution for V will pile up its mass near 0, so that the answer can be ridiculous if ε is too small. An objective Bayesian, who incorrectly used the related prior $\pi(V) \propto V^{-1}$, would typically become aware of the problem, since the posterior would not converge (as it will with the vague proper prior). The common perception that using a vague proper prior is safer than using improper priors, or conveys some type of guarantee of good performance, is simply wrong.

The second common quasi-Bayes procedure is to choose priors that span the range of the likelihood function. For instance, one might choose a uniform prior over a range that includes most of the 'mass' of the likelihood function, but that does not extend too far (thus hopefully avoiding the problem of using a 'too vague' proper prior). Another version of this procedure is to use conjugate priors, with parameters chosen so that the prior is considerably more spread out than the likelihood function, but is roughly centered in the same region. The two obvious concerns with these strategies are that (i) the answer can still be quite sensitive to the spread of the rather arbitrarily chosen prior; and (ii) centering the prior on the likelihood is a problematical double use of the data. Also, in problems with complicated likelihoods, it can be very difficult to implement this strategy successfully.

Perhaps the most questionable of all the quasi-Bayes procedures is to write down proper (often conjugate) priors with unspecified parameters, and then to treat these parameters as 'tuning' parameters to be adjusted until the answer 'looks nice.' Unfortunately, one is often not even told that this has been done; i.e., the choice of the parameters is, after the fact, presented as 'natural.'

These issues are complicated by the fact that, in the hands of an expert Bayesian analyst, even the 'quasi-Bayes' procedures mentioned above can be reasonable, in that the expert may have the experience and skill to tell when the procedures are likely to be successful. Also, one must always consider the question: What is the alternative? I have seen many examples in which an answer was required, and in which I would trust the quasi-Bayes answer more than the answer from any feasible alternative analysis.

Finally, it is important to recognize that the genie cannot be put back into the bottle. The Bayesian 'machine', together with MCMC, is arguably the most powerful mechanism ever created for the processing of data and knowledge. The quasi-Bayes approach can rather easily create *procedures* of astonishing flexibility for data analysis, and its use to create such procedures should not be discouraged. However, it must be recognized that these procedures do not possess compelling Bayesian justifications, and so must be justified on extrinsic grounds (e.g., through extensive sensitivity studies, simulations, etc.).

4. Computation and Software

4.1. Computational Techniques

Even twenty years ago one often heard the refrain "Bayesian analysis is nice conceptually; too bad it is not possible to compute Bayesian answers in realistic situations." Today, truly complex models can often *only* be computationally handled by Bayesian techniques. This has attracted many newcomers to the Bayesian approach and has had the interesting effect of considerably reducing discussion of 'philosophical' arguments for and against the Bayesian position.

Although other goals are possible, most Bayesian computation is focused on calculation of posterior expectations, which are typically integrals of from one to thousands of dimensions. Another common type of Bayesian computation is calculation of the posterior mode (as in computation of MAP estimates in image processing).

The 'traditional' numerical methods for computing posterior expectations are numerical integration, Laplace approximation, and Monte Carlo importance sampling. Numerical integration can be effective in moderate (say, up to 10) dimensional problems. Modern developments in this direction are discussed in Monahan and Genz (1996). Laplace and other saddlepoint approximations are discussed in the vignette Strawderman (2000). Until recently, Monte Carlo importance sampling was the most commonly used traditional method of computing posterior expectations. The method can work in very large dimensions, and has the nice feature of producing reliable measures of the accuracy of the computation.

Today, MCMC has become the most popular method of Bayesian computation, in part because of its power in handling very complex situations and, in part, because it is comparatively easy to program. Since the vignettes Gelfand (2000) and Cappé and Robert (2000) both address this computational technique, we do not discuss it here. Recent books in the area include Tanner (1993), Gamerman (1997), Robert and Casella (1999), and Chen, Shao, and Ibrahim (2000). It is not strictly the case that MCMC is replacing the more traditional methods listed above. For instance, in some problems importance sampling will probably always remain the computational method of choice, as will standard numerical integration in low dimensions (especially when extreme accuracy is needed).

Availability of general user-friendly Bayesian software is clearly needed to advance the use of Bayesian methods. A number of software packages do exist, and are very useful for particular scenarios. A listing and description of pre-1990 Bayesian software can be found in Goel (1988) and Press (1989). A listing of some of the Bayesian software developed since 1990 is given in Appendix 2.

It would, of course, be wonderful to have a single general purpose Bayesian software package, but three of the major strengths of the modern Bayesian approach create difficulties in developing generic software. One difficulty is the extreme flexibility of Bayesian analysis, with virtually any constructed model being amenable to analysis. Most classical packages need to contend with only a relatively few well-defined models or scenarios for which a classical procedure has been determined. Another strength of Bayesian analysis is the possibility of extensive utilization of subjective prior information, and many Bayesians tend to feel that software should include an elaborate expert system for prior elicitation. Finally, implementing the modern computational techniques in a software package is extremely challenging, because it is difficult to codify the 'art' of finding a successful computational strategy in a complex situation.

Note that development of software implementing the objective Bayesian approach for 'standard' statistical models can avoid these difficulties. There would be no need for a subjective elicitation interface, and the package could incorporate specific computational techniques suited to the various standard models being considered. Since the vast majority of statistical analyses being done today use such 'automatic' software, having a Bayesian version would greatly impact the actual use of Bayesian methodology. Its creation should thus be a high priority for the profession.

Appendix 1. *General Bayesian References*

- *Historical and General Monographs:* Laplace (1812), Jeffreys (1961), Raiffa and Schlaifer (1961), de Finetti (1974, 1975), Savage (1972), Zellner (1971), Lindley (1972), Box and Tiao (1973), Hartigan (1983), Florens, Mouchart, and Roulin (1990).

- *Graduate Level Texts:* DeGroot (1970), Berger (1985), Press (1989), Bernardo and Smith (1994), O'Hagan (1994), Robert (1994), Gelman, Carlin, Stern, and Rubin (1995), Poirier (1995), Schervish (1995), Piccinato (1996).

- *Elementary Texts:* Winkler (1972), O'Hagan (1988), Albert (1996), Berry (1996), Sivia (1996), Antleman (1997), Lee (1997).

- *General Proceedings Volumes:* The International Valencia Conferences produce highly acclaimed proceedings, the last of which was Bernardo et. al. (1999). The Maximum Entropy and Bayesian Analysis conferences also have excellent proceedings volumes, the last of which was Erickson et. al. (1998). The CMU Bayesian Case Studies Workshops produce unique volumes of in-depth case studies in Bayesian analysis, the last volume being Gatsonis, et. al. (1998). The Section on Bayesian Statistical Science of the ASA has an annual JSM proceedings volume, produced by the ASA.

Appendix 2 *Available Bayesian Software*

- AutoClass is a Bayesian classification system (http://ic-www.arc.nasa.gov/ic/projects/bayes-group/group/autoclass/)

- BATS is designed for Bayesian time series analysis (http://www.stat.duke.edu/∼mw/bats.html)

- BAYDA is a Bayesian system for classification and discriminant analysis (http://www.cs.Helsinki.fi/research/cosco/Projects/NONE/SW/)
- BAYESPACK, etc., are numerical integration algorithms (http://www.math.wsu.edu/math/faculty/genz/homepage)
- Bayesian biopolymer sequencing software (http://www-stat.stanford.edu/~jliu/)
- B/D is a linear subjective Bayesian system (http://fourier.dur.ac.uk:8000/stats/bd/)
- BMA is software for Bayesian model averaging for predictive and other purposes (http://www.research.att.com/~volinsky/bma.html)
- Bayesian regression and classification software based on neural networks, Gaussian processes, and Bayesian mixture models (http://www.cs.utoronto.ca/~radford/fbm.software.html)
- Belief networks software (http://bayes.stat.washington.edu/almond/belief.html)
- BRCAPRO implements a Bayesian analysis for genetic counseling of women at high risk of hereditary breast and ovarian cancer (http://www.stat.duke.edu/~gp/brcapro.html)
- BUGS is designed to analyze general hierarchical models via MCMC (http://www.mrc-bsu.cam.ac.uk/bugs/)
- First Bayes is a Bayesian teaching package (http://www.shef.ac.uk/~st1ao/1b.html)
- Matlab and Minitab Bayesian computational algorithms for introductory Bayes and ordinal data (http://www-math.bgsu.edu/~albert/)
- Nuclear magnetic resonance Bayesian software; this is the manual (http://www.bayes.wustl.edu/glb/manual.pdf)
- StatLib is a repository for statistics software, much of it Bayesian (http://lib.stat.cmu.edu/)
- Time Series software for nonstationary time series and analysis with autoregressive component models (http://www.stat.duke.edu/~mw/books_software_data.html)
- LISP-STAT, an object-oriented environment for statistical computing and dynamic graphics with a variety of Bayesian capabilities (Tierney 1991)

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