Lecture 23 - Sensitivity, Specificity, and Decisions

Sta102 / BME102

Colin Rundel

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(An old) Example - *House*

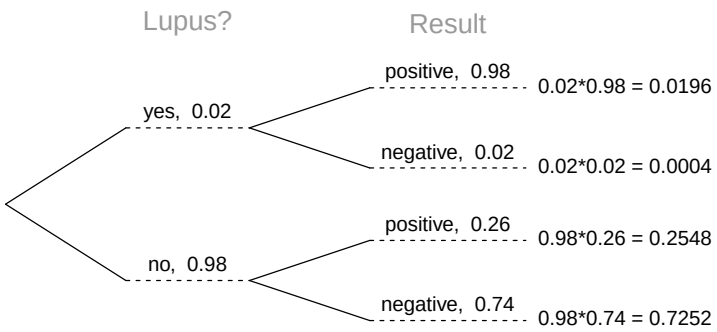
If you've ever watched the TV show *House* on Fox, you know that Dr. House regularly states, "It's never lupus."

Lupus is a medical phenomenon where antibodies that are supposed to attack foreign cells to prevent infections instead see plasma proteins as foreign bodies, leading to a high risk of blood clotting. It is believed that 2% of the population suffer from this disease.

The test for lupus is very accurate if the person actually has lupus, however is very inaccurate if the person does not. More specifically, the test is 98% accurate if a person actually has the disease. The test is 74% accurate if a person does not have the disease.

Is Dr. House correct even if someone tests positive for Lupus?

(An old) Example - *House*



$$P(\text{Lupus}|+) = \frac{P(+, \text{Lupus})}{P(+, \text{Lupus}) + P(+, \text{No Lupus})}$$
$$= \frac{0.0196}{0.0196 + 0.2548} = 0.0714$$

Testing for lupus

It turns out that testing for Lupus is actually quite complicated, a diagnosis usually relies on the outcome of multiple tests, often including: a complete blood count, an erythrocyte sedimentation rate, a kidney and liver assessment, a urinalysis, and or an antinuclear antibody (ANA) test.

It is important to think about what is involved in each of these tests (e.g. deciding if complete blood count is high or low) and how each of the individual tests and related decisions plays a role in the overall decision of diagnosing a patient with lupus.

Testing for lupus (cont.)

At some level we can view a diagnosis as a binary decision (lupus or no lupus) that involves the complex integration of various explanatory variables.

The example does not give us any information about how a diagnosis is made, but what it does give us is just as important - the *sensitivity* and the *specificity* of the test(s). These values are critical for our understanding of what a positive or negative test result actually means.

Sensitivity and Specificity

Sensitivity - measures a tests ability to identify positive results.

$$P(\text{Test } + \mid \text{Conditon } +) = P(+ \mid \text{lupus}) = 0.98$$

Specificity - measures a tests ability to identify negative results.

$$P(\text{Test } - \mid \text{Condition } -) = P(- \mid \text{no lupus}) = 0.74$$

It is illustrative to think about the extreme cases - what is the sensitivity and specificity of a test that always returns a positive result?

What about a test that always returns a negative result?

Sensitivity and Specificity (cont.)

	Condition Positive	Condition Negative
Test Positive	True Positive	False Positive (Type I error)
Test Negative	False Negative (Type II error)	True Negative

$$\textit{Sensitivity} = P(\text{Test } + \mid \text{Condition } +) = TP/(TP + FN)$$
$$\textit{Specificity} = P(\text{Test } - \mid \text{Condition } -) = TN/(FP + TN)$$
$$\textit{False negative rate } (\beta) = P(\text{Test } - \mid \text{Condition } +) = FN/(TP + FN)$$
$$\textit{False positive rate } (\alpha) = P(\text{Test } + \mid \text{Condition } -) = FP/(FP + TN)$$

$$\textit{Sensitivity} = 1 - \textit{False negative rate} = \text{Power}$$
$$\textit{Specificity} = 1 - \textit{False positive rate}$$

So what?

Clearly it is important to know the Sensitivity and Specificity of a test (and or the false positive and false negative rates). Along with the incidence of the disease, e.g. $P(\text{lupus})$, these values are necessary to calculate important quantities like $P(\text{lupus}|+)$.

Additionally, our foray into power analysis after the first midterm should also give you an idea about the trade offs that are inherent in minimizing false positive and false negative rates (increasing power required either increasing α or n).

How do we use this information when we are trying to come up with a decision?

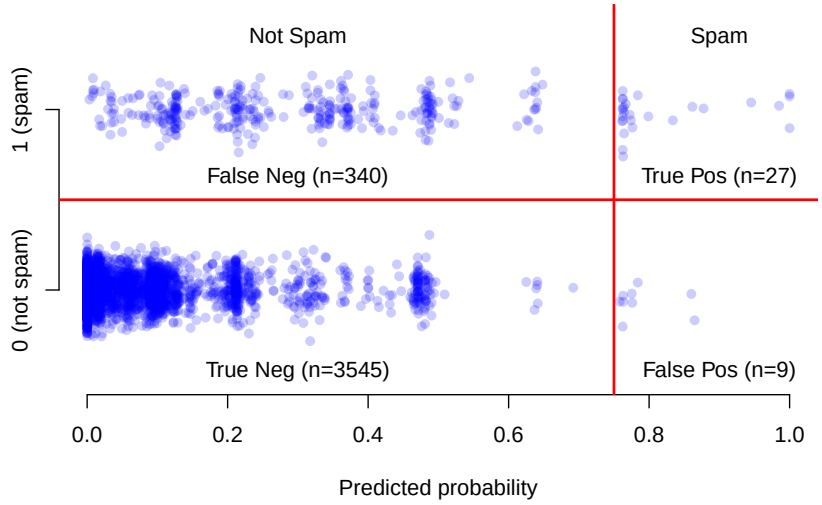
Back to Spam

In lab this week, we will examine a data set of emails where we are interested in identifying spam email messages. You will examine different logistic regression models to evaluate how different predictors influenced the probability of a message being spam.

These models can also be used to assign probabilities to incoming messages (this is equivalent to prediction in the case of SLR / MLR). However, if we were designing a spam filter this would only be half of the battle, we would also need to use these probabilities to make a decision about which emails get flagged as spam.

While not the only possible solution, we will consider a simple approach where we choose a threshold probability and any email that exceeds that probability is flagged as spam.

Picking a threshold



Lets see what happens if we pick our threshold to be *0.75*.

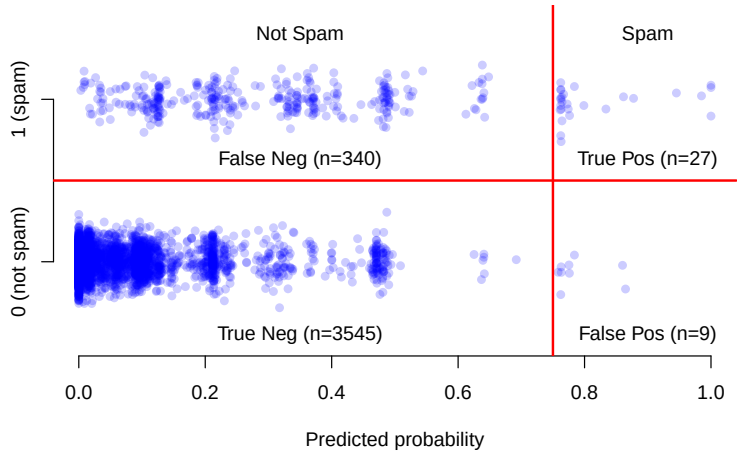
Consequences of picking a threshold

For our data set picking a threshold of 0.75 gives us the following results:

$$FN = 340 \qquad TP = 27$$
$$TN = 3545 \qquad FP = 9$$

What are the sensitivity and specificity for this particular decision rule?

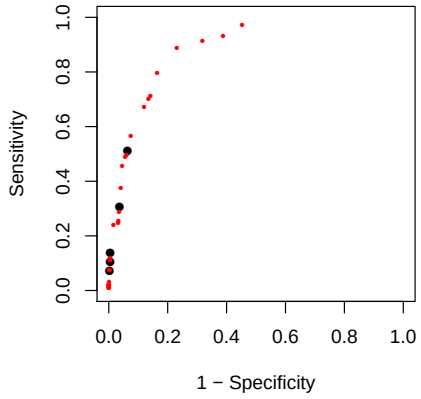
Trying other thresholds



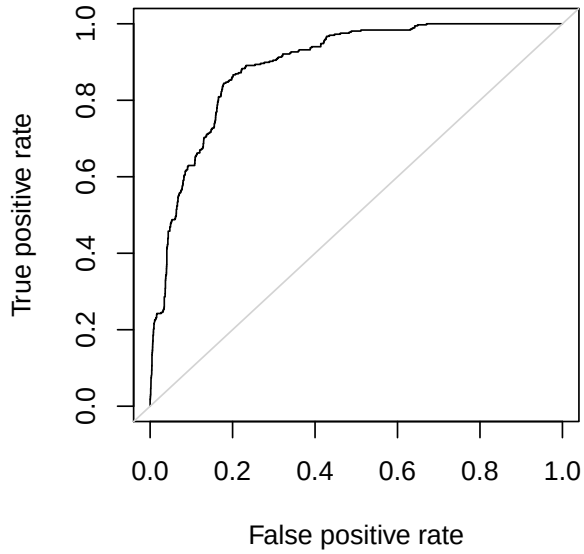
Threshold	0.75	0.625	0.5	0.375	0.25
Sensitivity	0.074	0.106	0.136	0.305	0.510
Specificity	0.997	0.995	0.995	0.963	0.936

Relationship between Sensitivity and Specificity

Threshold	0.75	0.625	0.5	0.375	0.25
Sensitivity	0.074	0.106	0.136	0.305	0.510
Specificity	0.997	0.995	0.995	0.963	0.936



Receiver operating characteristic (ROC) curve



Receiver operating characteristic (ROC) curve (cont.)

Why do we care about ROC curves?

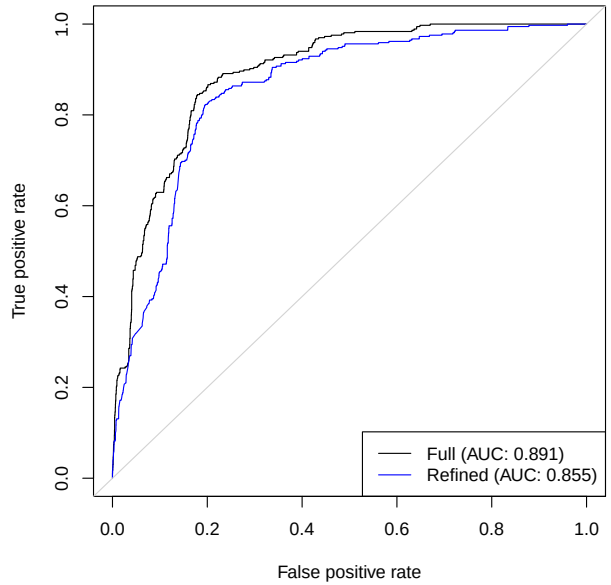
- Shows the trade off in sensitivity and specificity for all possible thresholds.
- Straight forward to compare performance vs. chance.
- Can use the area under the curve (AUC) as an assessment of the predictive ability of a model.

Refining the Spam model

```
refined = glm(spam ~ to_multiple+cc+image+attach+winner
                     +password+line_breaks+format+re_subj
                     +urgent_subj+exclaim_mess,
              data=email, family=binomial)
summary(refined)
```

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-1.7594	0.1177	-14.94	0.0000
to_multipleyes	-2.7368	0.3156	-8.67	0.0000
ccyes	-0.5358	0.3143	-1.71	0.0882
imageyes	-1.8585	0.7701	-2.41	0.0158
attachyes	1.2002	0.2391	5.02	0.0000
winneryes	2.0433	0.3528	5.79	0.0000
passwordyes	-1.5618	0.5354	-2.92	0.0035
line_breaks	-0.0031	0.0005	-6.33	0.0000
formatPlain	1.0130	0.1380	7.34	0.0000
re_subjyes	-2.9935	0.3778	-7.92	0.0000
urgent_subjyes	3.8830	1.0054	3.86	0.0001
exclaim_mess	0.0093	0.0016	5.71	0.0000

Comparing models



Utility Functions

There are many other reasonable quantitative approaches we can use to decide on what is the "best" threshold.

If you've taken an economics course you have probably heard of the idea of utility functions, we can assign costs and benefits to each of the possible outcomes and use those to calculate a utility for each circumstance.

Utility function for our spam filter

To write down a utility function for a spam filter we need to consider the costs / benefits of each outcome.

Outcome	Utility
True Positive	1
True Negative	1
False Positive	-50
False Negative	-5

$$U(p) = TP(p) + TN(p) - 50 \times FP(p) - 5 \times FN(p)$$

Utility for the 0.75 threshold

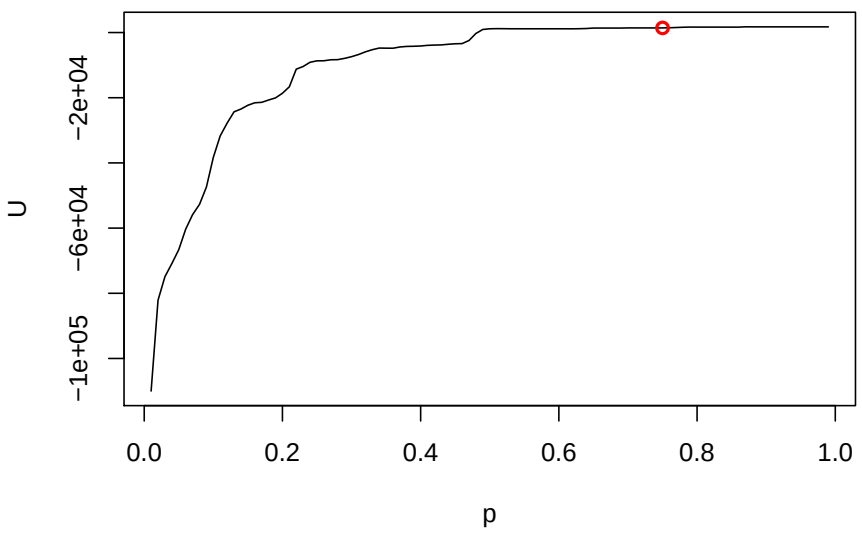
For the email data set picking a threshold of 0.75 gives us the following results:

$$FN = 340 \qquad TP = 27$$
$$TN = 3545 \qquad FP = 9$$

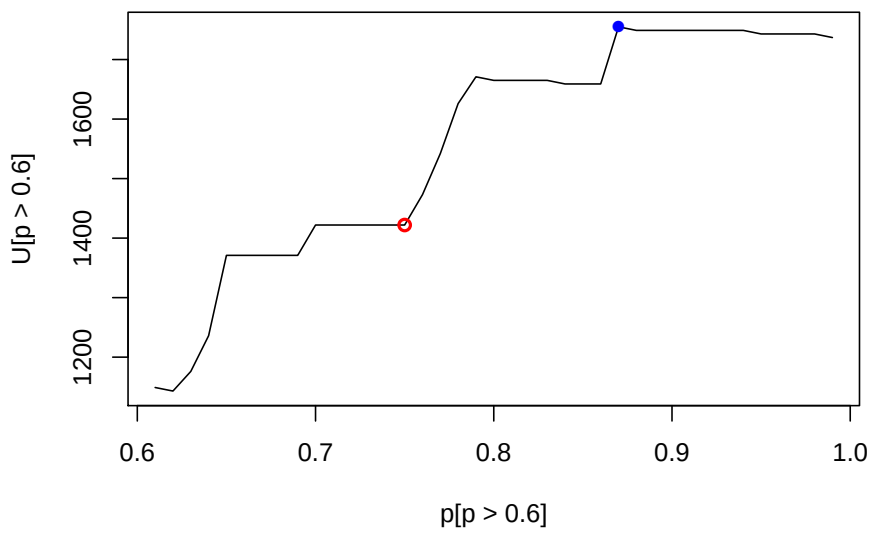
$$\begin{aligned} U(p) &= TP(p) + TN(p) - 50 \times FP(p) - 5 \times FN(p) \\ &= 27 + 3545 - 50 \times 9 - 5 \times 340 = 1422 \end{aligned}$$

Not useful by itself, but allows us to compare with other thresholds.

Utility curve



Utility curve (zoom)



Maximum Utility