Lecture 21 - Power Analysis

Sta230 / Mth230

Colin Rundel

April 21, 2014

Example - Blood Pressure

Blood pressure oscillates with the beating of the heart, and the systolic pressure is defined as the peak pressure when a person is at rest. The average systolic blood pressure for people in the U.S. is about 130 mmHg with a standard deviation of about 25 mmHg.

We are interested in finding out if the average blood pressure of employees at a certain company is greater than the national average, so we collect a random sample of 100 employees and measure their systolic blood pressure. What are the hypotheses?

We'll start with a very specific question – "What is the power of this hypothesis test to correctly detect an increase of 2 mmHg in average blood pressure?"

Calculating power

The preceeding question can be rephrased as – How likely is it that this test will reject H_0 when the true average systolic blood pressure for employees at this company is 132 mmHg?

Let's break this down intro two simpler problems:

1. Problem 1: Which values of $\bar{x}$ represent sufficient evidence to reject H_0?
2. Problem 2: What is the probability that we would reject H_0 if $\bar{x}$ had come from $N\left(mean = 132, SE = \frac{25}{\sqrt{100}} = 2.5\right)$, i.e. what is the probability that we can obtain such an $\bar{x}$ from this distribution?

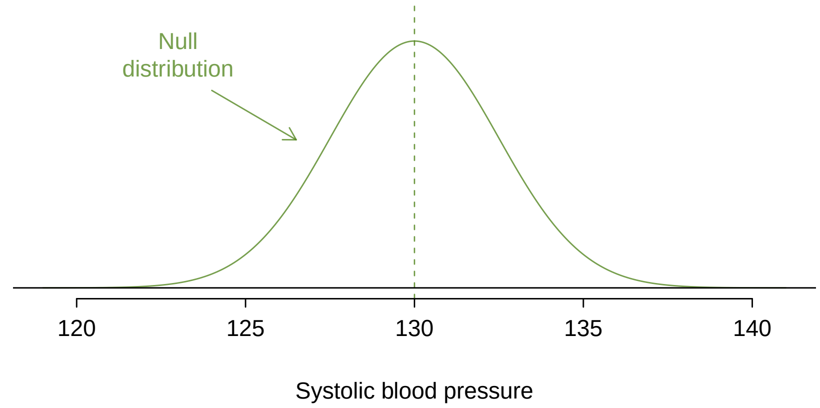
Problem 1

Which values of $\bar{x}$ represent sufficient evidence to reject H_0?
(Remember $H_0 : \mu = 130$, $H_A : \mu > 130$)

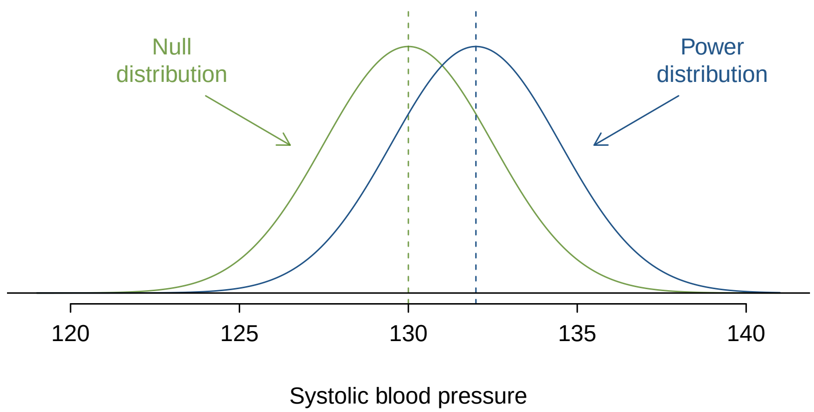
Problem 2

What is the probability that we would reject H_0 if $\bar{x}$ did come from $N(mean = 132, SE = 2.5)$.

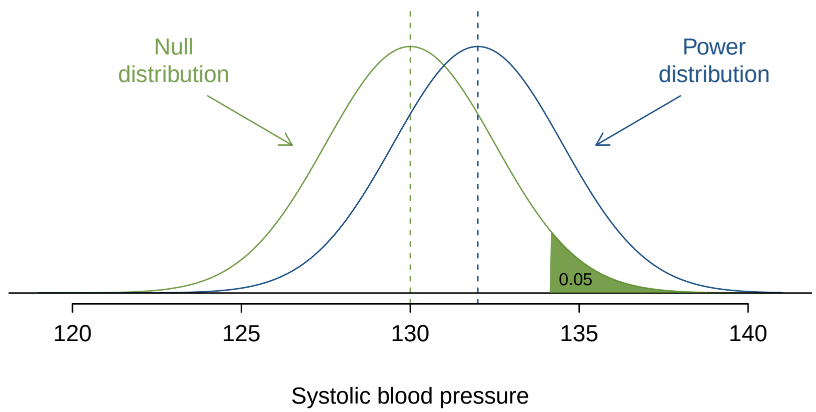
Putting it all together



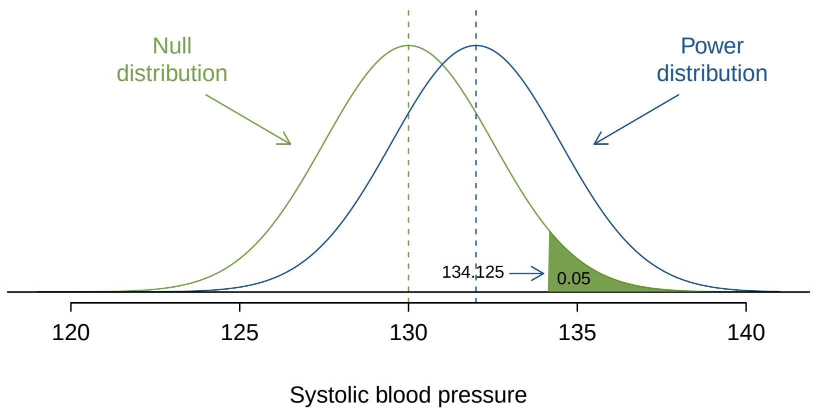
Putting it all together



Putting it all together

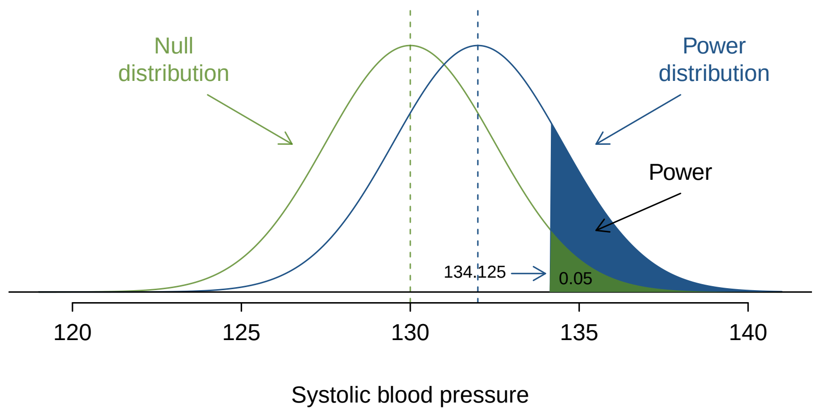


Putting it all together



Power

Putting it all together



Achieving desired power

There are several ways to increase power (and hence decrease type 2 error rate):

1. Increase the sample size.
2. Decrease the standard deviation of the sample, which essentially has the same effect as increasing the sample size (it will decrease the standard error). With a smaller s we have a better chance of distinguishing the null value from the observed point estimate. This is difficult to ensure but cautious measurement process and limiting the population so that it is more homogenous may help.
3. Increase α, which will make it more likely to reject H_0 (but note that this has the side effect of increasing the Type 1 error rate).
4. Consider a larger effect size. If the true mean of the population is in the alternative hypothesis but close to the null value, it will be harder to detect a difference.

Recap - Calculating Power

- Begin by picking a meaningful effect size δ and a significance level α
- Calculate the range of values for the point estimate beyond which you would reject H_0 at the chosen α level.
- Calculate the probability of observing a value from preceding step if the sample was derived from a population where $\bar{x} \sim N(\mu_{H_0} + \delta, SE)$

Example - Calculating power for a two sided hypothesis test

Going back to the blood pressure example, what would the power be to detect a 4 mmHg increase in average blood pressure for the hypothesis that the population average is different from 130 mmHg at a 95% significance level for a sample of 625 patients?

Step 0:

$$H_0: \mu = 130, \quad H_A: \mu \neq 130, \quad \alpha = 0.05, \quad n = 625, \quad \sigma = 25, \quad \delta = 4$$

Step 1:

$$P(Z > z \text{ or } Z < -z) < 0.05 \quad \Rightarrow \quad z > 1.96$$
$$\bar{x} > 130 + 1.96\frac{25}{\sqrt{625}} \text{ or } \bar{x} < 130 - 1.96\frac{25}{\sqrt{625}}$$
$$\bar{x} > 131.96 \text{ or } \bar{x} < 128.04$$

Step 2:

$$\bar{x} \sim N(\mu + \delta, SE) = N(134, 1)$$

$$\begin{aligned} P(\bar{x} > 131.96 \text{ or } \bar{x} < 128.04) &= P(Z > [131.96 - 134]/1) + P(Z < [128.04 - 134]/1) \\ &= P(Z > -2.04) + P(Z < -5.96) = 0.979 + 0 = 0.979 \end{aligned}$$

Example - Using power to determine sample size

Going back to the blood pressure example, how large a sample would you need if you wanted 90% power to detect a 4 mmHg increase in average blood pressure for the hypothesis that the population average is different from 130 mmHg at a 95% significance level?

Step 0:

$$H_0: \mu = 130, \quad H_A: \mu \neq 130, \quad \alpha = 0.05, \quad \beta = 0.10, \quad \sigma = 25, \quad \delta = 4$$

Step 1:

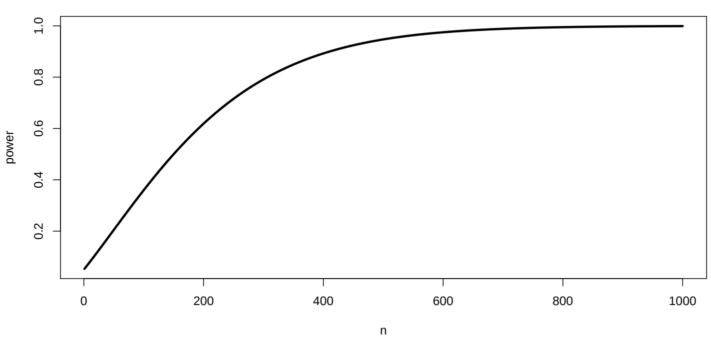
$$P(Z > z \text{ or } Z < -z) < 0.05 \quad \Rightarrow \quad z > 1.96$$
$$\bar{x} > 130 + 1.96\frac{25}{\sqrt{n}} \text{ or } \bar{x} < 130 - 1.96\frac{25}{\sqrt{n}}$$

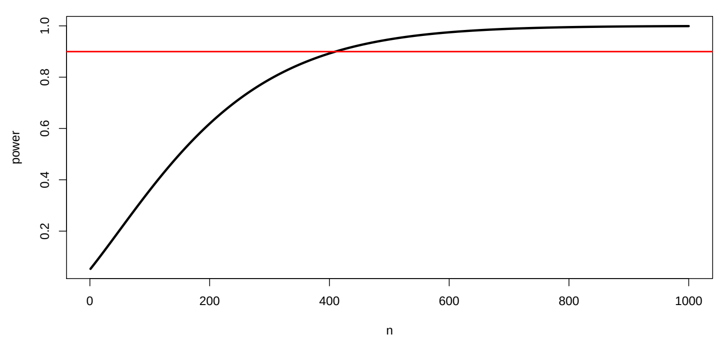
Step 2:

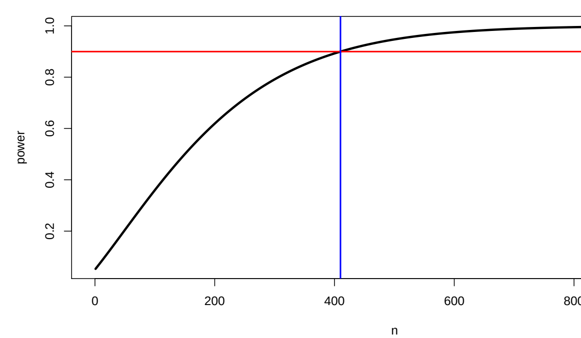
$$\bar{x} \sim N(\mu + \delta, SE) = N(134, 25/\sqrt{n})$$
$$P\left(\bar{x} > 130 + 1.96\frac{25}{\sqrt{n}} \text{ or } \bar{x} < 130 - 1.96\frac{25}{\sqrt{n}}\right) = 0.9$$
$$P\left(Z > 1.96 - 4\frac{\sqrt{n}}{25} \text{ or } Z < -1.96 - 4\frac{\sqrt{n}}{25}\right) = 0.9$$

Example - Using power to determine sample size (cont.)

So we are left with an equation we cannot solve directly, how do we evaluate it?







For $n = 410$ the power $= 0.8996$, therefore we need 411 subjects in our sample to achieve the desired level of power for the given circumstance.