### ACS Transformation Example

## From last lab

Just like in lab we load data, and subset for those who were employed.

load("acs.RData") acs\_sub = subset(acs, acs\$employment == "employed")

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ACS Transformation Example

## Predicting income

l = lm(income ~ hrs\_work + race + age + gender + edu + disability, data = acs\_sub);summary(1) ##

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```
## Call:
## lm(formula = income ~ hrs_work + race + age + gender + edu +
## disability, data = acs_sub)
##
## Residuals:
## Min 1Q Median 3Q Max
## -122650 -20503 -4597 10945 321681
##
## Coefficients:
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) -21737.3 7719.3 -2.816 0.004978 **
## hrs work 1000.1 135.5 7.379 3.86e-13 ***
## raceblack -6015.5 5877.3 -1.024 0.306359

        ## raceasian
        29595.6
        8030.0
        3.686
        0.000243
        ***

        ## raceother
        -8599.2
        6648.6
        -1.293
        0.196238

        ## age
        561.6
        118.9
        4.724
        2.71e-06
        ***

        ## gederfemale
        -18120.6
        3495.9
        -5.183
        2.74e-07
        ***

        ## educollege
        17273.8
        3827.5
        4.513
        7.31e-06
        ***

        ## edugrad
        58551.9
        5418.8
        10.805
        < 2e-16</td>
        ***

## disabilityyes -15852.0 6209.5 -2.553 0.010861 *
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 48270 on 833 degrees of freedom
## Multiple R-squared: 0.2935, Adjusted R-squared: 0.2859
## F-statistic: 38.46 on 9 and 833 DF, p-value: < 2.2e-16
                                                                                 ACS
```

## ACS Transformation Example

## Categorical variables with multiple levels

In model selection based on  $R_{adi}^2$ :

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Leave all levels in or drop the entire variable (even if one level is significant).

For example, the race variable in our model:

	Estimate	Std. Error	t value	$\Pr(> t )$
raceblack	-6015.53	5877.30	-1.02	0.31
raceasian	29595.59	8029.98	3.69	0.00
raceother	-8599.21	6648.63	-1.29	0.20

How do we interpret the slopes associated with the race variable?

#### ACS Transformation Example Diagnostics

# (1) Nearly normal residuals

## par(mfrow=c(1,2))

qqnorm(l\$residuals, main = "Normal prob. plot\nof residuals")
qqline(l\$residuals)
hist(l\$residuals, main = "Histogram of residuals")



#### ACS Transformation Example Diagnostics

## (2) Constant variability of residuals

# par(mfrow=c(1,2)) plot(l\$residuals ~ 1\$fitted, main = "Residuals vs. fitted") abline(h = 0, lty = 3) plot(abs(1\$residuals) ~ 1\$fitted, main = "Absolute value of\nresiduals vs. fitted") abline(h = 0, lty = 3)

Absolute value of Residuals vs. fitted residuals vs. fitted 3e+05  $\cap$ 8 250000 q 0 0 0 0 Ø  $\cap$ °° ©° © abs(I\$residuals) Ø  $\sim$ ୖୄୢ \$residuals 1e+05 100000 -1e+05 Q 0 0 0e+00 5e+04 1e+05 0e+00 5e+04 1e+05 I\$fitted I\$fitted Sta102 / BME102 (Colin Rundel) ACS April 10, 2015 Absolute value of Residuals vs. fitted residuals vs fitted ACS Transformation Example

• We saw that residuals have a right-skewed distribution, and the

relationship between hours worked per week and income is non-linear

• In these situations a transformation applied to the response variable

• In order to decide which transformation to use, we should examine

the distribution of the response variable.

CS Transformation Example Diagnostics

# (3) Independence

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plot(l\$residuals, main = "Residuals vs. order of data collection")

## Residuals vs. order of data collection



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Transformations

(exponential).

may be useful.

min = 0 Q1 = 12000

mean = 44098 median = 30000 Q3 = 55000max = 450000 be useful.

• The distribution is right skewed  $\rightarrow$ 

suggests that a log transformation may

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# Log of 0

## summary(acs\_sub\$income)

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##	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
##	0	12000	30000	44100	55000	450000

log(0)

## [1] -Inf

- Since there are some individuals who had 0 income (from salaries and wages) last year, we cannot take the log of their income, since log(0) = -∞.
- A commonly used trick is to add a very small number to all values before taking the log.

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## ACS Transformation Example Variance stabilizing transformations

## Logged income distribution

acs\_sub\$income\_log = log(acs\_sub\$income + 0.01)
par(mfrow=c(1,2))
hist(acs\_sub\$income)
hist(acs\_sub\$income\_log)



ACS Transformation Example Variance stabilizing transformation

## Logged income relationships

par(mfrow=c(1,2),mar=c(5, 4, 1, 2) + 0.1)

plot(acs\_sub\$income\_log ~ acs\_sub\$hrs\_work)



We still might want to do something about those 0 incomes, it doesn't make sense to model them with the rest of the data.

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ACS Transformation Example Variance stabilizing transformations

## Further subsetting the data

People who work more than 0 hours per week but make 0 income in salaries and wages are different than others whose income is proportional to number of hours they work. So we have reason to omit these people from the analysis (and model their income differently based on other variables).

acs\_sub2 = subset(acs\_sub, acs\_sub\$income > 0)
acs\_sub2\$income\_log = log(acs\_sub2\$income)

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## Logged relationships - for those with any income

## par(mfrow=c(1,2))

plot(acs\_sub2\$income\_log ~ acs\_sub2\$hrs\_work)
plot(acs\_sub2\$income\_log ~ acs\_sub2\$age)



## Predicting log of income

l\_log = lm(income\_log ~ hrs\_work + race + age + gender + edu + disability, data = acs\_sub2);summary(l\_log)

```
##
## Call:
## lm(formula = income_log ~ hrs_work + race + age + gender + edu +
##
      disability, data = acs_sub2)
##
## Residuals:
##
              10 Median
                             30
    Min
                                  Max
## -4.5725 -0.3936 0.0880 0.4993 3.1652
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                7.313684 0.140742 51.965 < 2e-16 ***
## hrs_work
                0.048793 0.002526 19.317 < 2e-16 ***
## raceblack
               -0.147579 0.104363 -1.414 0.158
## raceasian
                0.136877 0.141616 0.967
                                             0.334
## raceother
               -0.192193 0.121193 -1.586
                                             0.113
## age
                0.022229 0.002175 10.222 < 2e-16 ***
## genderfemale -0.276076 0.063702 -4.334 1.66e-05 ***
## educollege
               0.399230 0.069932
                                   5.709 1.62e-08 ***
## edugrad
                0.833686 0.098711 8.446 < 2e-16 ***
## disabilityyes -0.624479 0.115492 -5.407 8.53e-08 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.849 on 777 degrees of freedom
## Multiple R-squared: 0.5202, Adjusted R-squared: 0.5146
## F-statistic: 93.59 on 9 and 777 DF, p-value: < 2.2e-16
```

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ACS Transformation Example New model: log of income

ACS Transformation Example Diagnostics for model for logged income model

# (1) Nearly normal residuals

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# Final model for log of income

l\_log\_final = lm(income\_log ~ hrs\_work + age + gender + edu + disability, data = acs\_sub2);summary(l\_log\_final)

```
##
## Call:
## lm(formula = income_log ~ hrs_work + age + gender + edu + disability,
##
      data = acs_sub2)
##
## Residuals:
##
   Min 1Q Median 3Q Max
## -4.4098 -0.3936 0.0987 0.5124 3.1883
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                7.281258 0.139082 52.352 < 2e-16 ***
## hrs_work
                0.049017 0.002527 19.395 < 2e-16 ***
## age
                0.022309 0.002166 10.299 < 2e-16 ***
## genderfemale -0.287365 0.063569 -4.521 7.13e-06 ***
## educollege
                0.413555 0.069741 5.930 4.55e-09 ***
## edugrad
                0.844909 0.098323 8.593 < 2e-16 ***
## disabilityyes -0.632040 0.115558 -5.469 6.08e-08 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8503 on 780 degrees of freedom
## Multiple R-squared: 0.5168, Adjusted R-squared: 0.5131
## F-statistic: 139 on 6 and 780 DF, p-value: < 2.2e-16
```

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#### Constant variability of residuals (2)







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ACS Transformation Example Interpretations for model for logged income

# Interpretation

Which of the following is the correct interpretation of the slope of age hours worked per week?

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	7.28	0.14	52.35	0.00
hrs_work	0.05	0.00	19.39	0.00
age	0.02	0.00	10.30	0.00
genderfemale	-0.29	0.06	-4.52	0.00
educollege	0.41	0.07	5.93	0.00
edugrad	0.84	0.10	8.59	0.00
disabilityyes	-0.63	0.12	-5.47	0.00

ACS Transformation Example Interpretations for model for logged income

## Interpretation (cont.)

Which of the following is the correct interpretation of the slope of edu:college?

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	7.28	0.14	52.35	0.00
hrs_work	0.05	0.00	19.39	0.00
age	0.02	0.00	10.30	0.00
gender:female	-0.29	0.06	-4.52	0.00
edu:college	0.41	0.07	5.93	0.00
edu:grad	0.84	0.10	8.59	0.00
disability:yes	-0.63	0.12	-5.47	0.00

# (3) Independence

## Residuals vs. order of data collection