Lecture 14 - Tests of Proportions

Sta102 / BME 102

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Inference

Testing in Context

		Independent Variable		
	None	Categorical (2 levels)	Categorical (>2 levels)	Numerical
Numerical	Test of One Mean	Test of Two Means	ANOVA	Regression
Categorical (2 levels)	Test of One Proportion	Test of Two Proportions	χ^2 - Test of Independence	Logistic Regression
Categorical (>2 levels)	χ^2 - GoF	χ^2 - Test of Independence	χ^2 - Test of Independence	~~Multinomial Regression~~

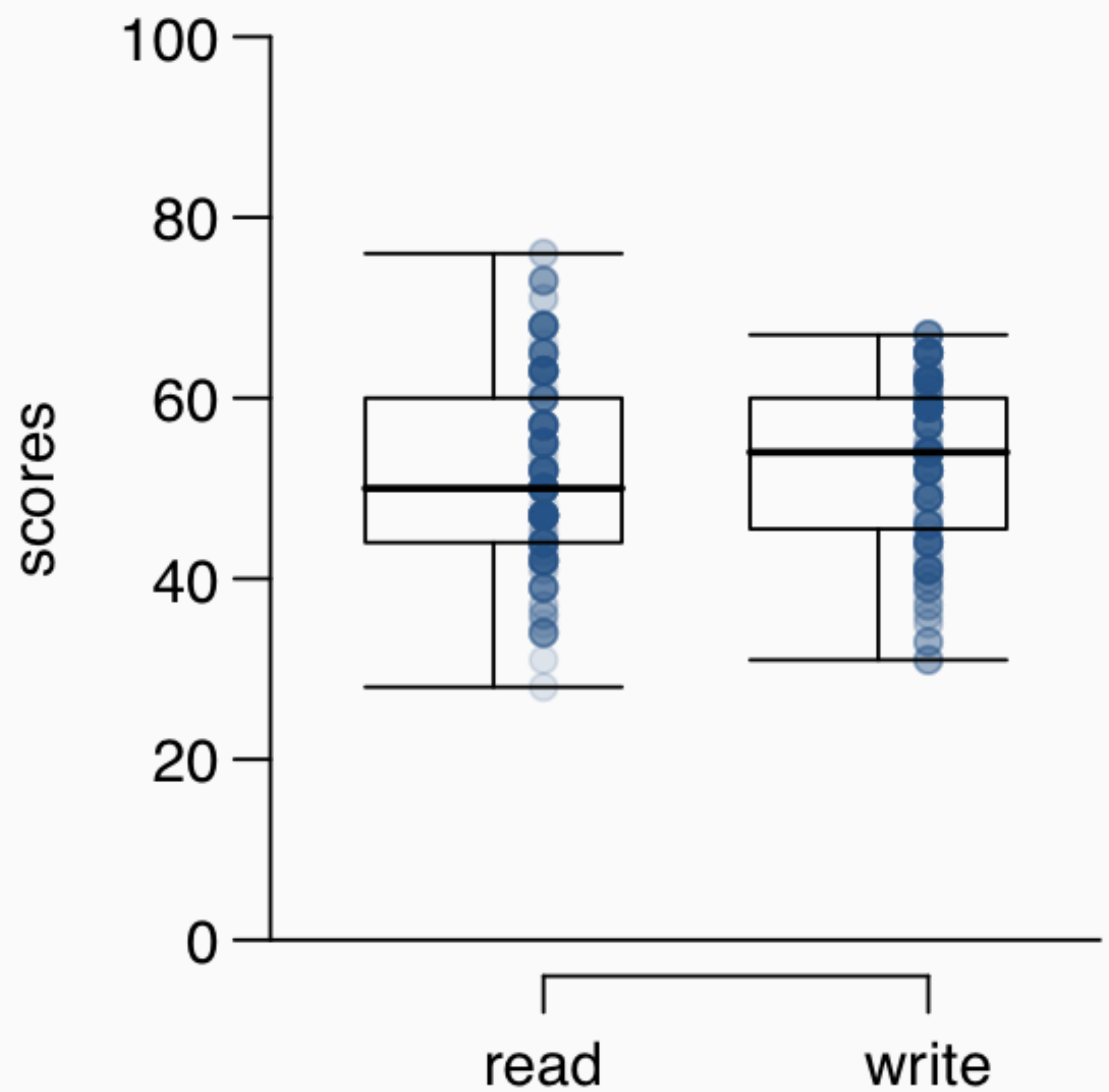
Dependent Variable

Paired Tests of Two Means

Example - Reading and Writing

200 randomly selected high school students took a reading and writing test and their scores are shown below. At a first glance, does there appear to be a difference between the average reading and writing test score?

	id	read	write
1	70	57	52
2	86	44	33
3	141	63	44
4	172	47	52
⋮	⋮	⋮	⋮
200	137	63	65



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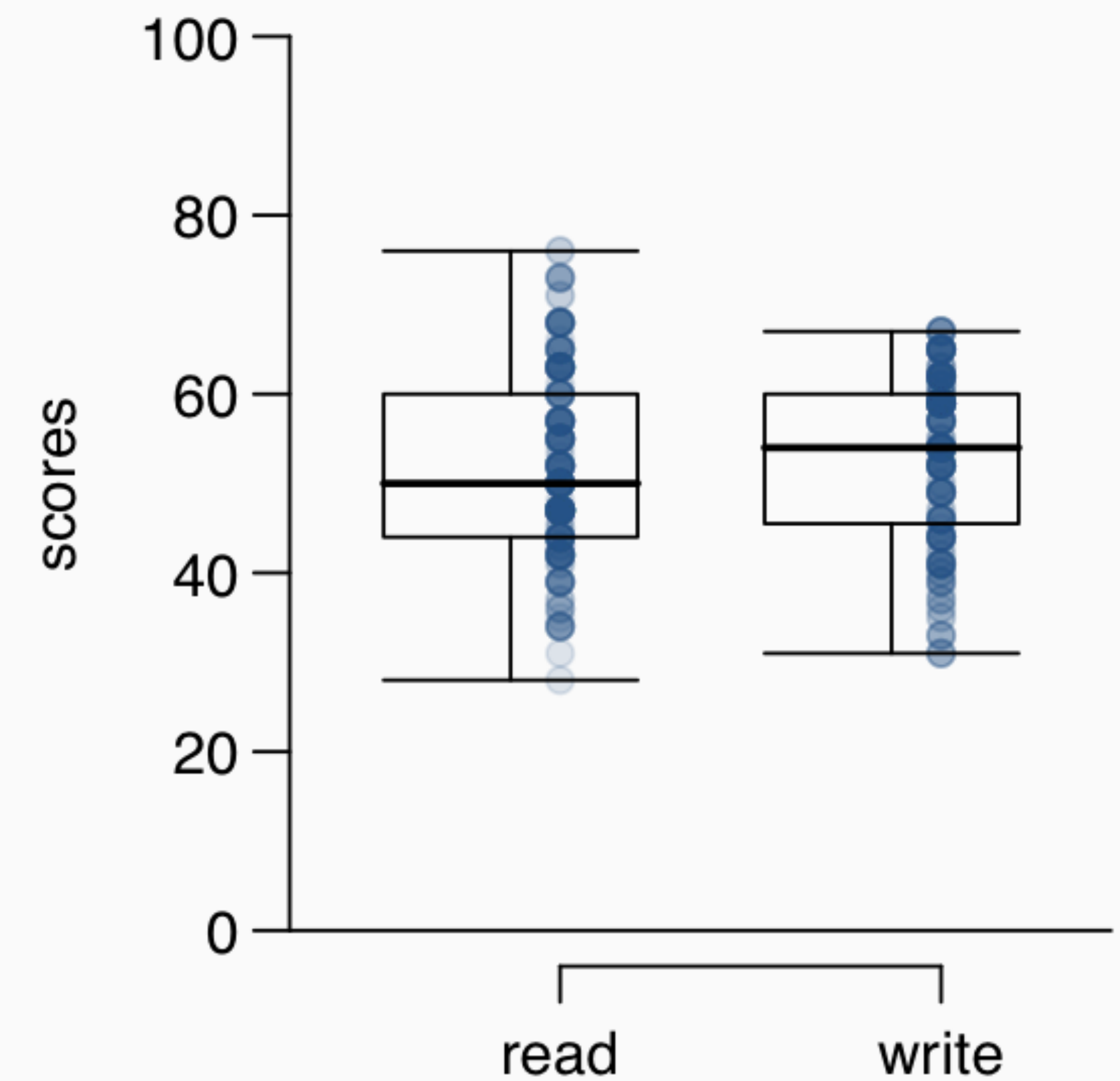

Do you think reading and writing scores are independent?

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$$\text{diff} = \text{read} - \text{write}$$

	id	read	write	diff
1	70	57	52	5
2	86	44	33	11
3	141	63	44	19
4	172	47	52	-5
⋮	⋮	⋮	⋮	⋮
200	137	63	65	-2

Parameter and point estimate

Parameter of interest: Average difference between the reading and writing scores of *all* high school students.

$$\mu_{diff}$$

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Point estimate: Average difference between the reading and writing scores of *sampled* high school students.

$$\bar{x}_{diff}$$

Setting the hypotheses

What are the hypotheses for testing if there is a difference between the average reading and writing scores?

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H_0: There is no difference between the average reading and writing score.

$$\mu_{diff} = 0$$

H_A: There is a difference between the average reading and writing score.

$$\mu_{diff} \neq 0$$

Nothing new here

We have already done this kind of analysis previously.

- We have data from *one* numeric variable - the difference.
- We are testing to see if this variable is or is not equal to 0.

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$\bar{x}$	-0.545
s	8.89
n	200

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$$H_0: \ \mu_{diff} = 0$$
$$H_A: \ \mu_{diff} \neq 0$$

$$T = \frac{\bar{X} - \mu}{SE} = \frac{-0.545 - 0}{8.89/\sqrt{200}} = -0.877$$

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$$T = \frac{\bar{X} - \mu}{SE} = \frac{-0.545 - 0}{8.89/\sqrt{200}} = -0.877$$

$\hookrightarrow df = 199$

$$\text{p-value} = P(T < -0.877 \text{ or } T > 0.877)$$

$$= 2 \times P(T < -0.877) = 2 \times 0.19 = 0.38$$

Example - Zinc

Trace metals in drinking water affect the flavor and unusually high concentrations can pose a health hazard. Data were collected by measuring zinc concentration at the bottom and at the surface of 10 randomly sampled wells in Wake country.

We would like to evaluate whether the true average concentration of zinc at the bottom of the well water exceeds that of the surface water. Data are given below.

well	zinc	location
1	0.43	bottom
2	0.266	bottom
3	0.567	bottom
4	0.531	bottom
5	0.707	bottom
6	0.716	bottom
7	0.651	bottom

well	zinc	location
8	0.589	bottom
9	0.469	bottom
10	0.723	bottom
1	0.415	surface
2	0.238	surface
3	0.39	surface
4	0.41	surface

well	zinc	location
5	0.605	surface
6	0.609	surface
7	0.632	surface
8	0.523	surface
9	0.411	surface
10	0.612	surface

Tidying the data

We prefer data where each row represents a *unit of observation* - in this case a well. What does that look like?

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well	zinc bottom	zinc top
1	0.43	0.415
2	0.266	0.238
3	0.567	0.39
4	0.531	0.41
5	0.707	0.605
6	0.716	0.609
7	0.651	0.632
8	0.589	0.523
9	0.469	0.411
10	0.723	0.612

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well	zinc bottom	zinc top	diff
1	0.43	0.415	0.015
2	0.266	0.238	0.028
3	0.567	0.39	0.177
4	0.531	0.41	0.121
5	0.707	0.605	0.102
6	0.716	0.609	0.107
7	0.651	0.632	0.019
8	0.589	0.523	0.066
9	0.469	0.411	0.058
10	0.723	0.612	0.111

Inference

Lets use a confidence interval to evaluate the difference in zinc concentration between the bottom and top of a well.

$$\bar{x}_{diff} = 0.08, \quad s = 0.052, \quad n = 10$$

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95% Confidence Interval:

$$PE \pm CV \times SE$$
$$\bar{x}_{diff} \pm t^{\star}_{df=9} \times \frac{s}{\sqrt{n}}$$
$$0.08 \pm 2.26 \times \frac{0.052}{\sqrt{10}}$$
$$(0.043, 0.118)$$

Calculating power - Step 0 and 1

If we were to conduct a hypothesis test to evaluate if the difference is between the bottom and top is statistically significant, what is the power of these hypotheses to detect a difference of 0.08? $= \delta$

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Step 0: What do we know?

$H_0 : \mu_{diff} = 0,\ H_A : \mu_{diff} \neq 0,\ \alpha = 0.05,\ n = 10,\ SE = 0.0164,\ \delta = 0.08,\ 1-\beta = ?$

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$$P(T > t \text{ or } T < -t) < 0.05 \quad \Rightarrow \quad t_{df=9} > 2.26$$

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$$P\left(\frac{\bar{x} - 0}{0.0164} > 2.26 \text{ or } \frac{\bar{x} - 0}{0.0164} < -2.26\right) = 0.05$$

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$$\bar{x} > 0.0164 \times 2.26 \text{ or } \bar{x} > 0.0164 \times -2.26$$

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$$\bar{x} > 0.0164 \times 2.26 \text{ or } \bar{x} > 0.0164 \times -2.26$$

$$\bar{x} > 0.037 \text{ or } \bar{x} < -0.037$$

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$$\bar{x} > 0.037 \text{ or } \bar{x} < -0.037$$

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Step 1: What values of $\hat{p}$ would let us reject H_0?

$$\bar{x} > 0.037 \text{ or } \bar{x} < -0.037$$

Step 2: Assume $p = 0 + \delta = 0.08$, what is the probability we reject H_0?

$$\begin{aligned} &P(\bar{x} > 0.037 \text{ or } \bar{x} < -0.037 | \mu_{diff} = 0.08) \\ &= P\left(T > \frac{0.037 - 0.08}{0.0168}\right) + P\left(T < \frac{-0.037 - 0.08}{0.0168}\right) + \\ &= P(T > -2.56) + P(T < -6.96) \\ &= 0.985 \end{aligned}$$

Inference for proportions

Example - Experimental Design

Two scientists want to know if a certain drug is effective against high blood pressure. The first scientist wants to give the drug to 1000 people with high blood pressure and see how many of them experience lower blood pressure levels. The second scientist wants to give the drug to 500 people with high blood pressure, and not give the drug to another 500 people with high blood pressure, and see how many in both groups experience lower blood pressure levels. Which is the better way to test this drug?

(a) All 1000 get the drug

(b) 500 get the drug, 500 don't

Results from the GSS

The GSS asks the same question, below is the distribution of responses from the 2010 survey:

All 1000 get the drug	99
500 get the drug 500 don't	571
Total	670

Parameter and point estimate

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$$p \left(\text{a population proportion}\right)$$

- *Point estimate:* Proportion of *sampled* Americans who have good intuition about experimental design.

$$\hat{p} \left(\text{a sample proportion}\right)$$

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- What we need to know then is

$$SE_{\hat{p}} = ? \qquad CV = ?$$

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It may be useful to instead think about $n\hat{p}$, what distribution will that have?

$$n\hat{p} \sim Binom(n=n, p=p)$$

$$n\hat{p} \approx x' \sim N(\mu = np, \sigma^2 = np(1-p))$$

$$\hat{p} \approx \frac{x'}{n} \sim N\left(\mu = p, \sigma^2 = \frac{p(1-p)}{n}\right)$$

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$$n\hat{p} \approx X' \sim N(\mu = np, \sigma^2 = np(1-p))$$

We can then find the distribution of $\hat{p}$ by dividing by n,

$$\hat{p} \approx X'/n \sim N(\mu = p, \sigma^2 = p(1-p)/n)$$

Central limit theorem (as applied to proportions)

A sample proportion will have a sampling distribution that is approximately normal with,

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Assumptions/conditions:

1. *Independence*:
   - *Random sample*
   - *10% condition*: If sampling without replacement, $n <$ 10% of the population.
2. *Normality*: At least 10 successes ($np \geq 10$) and 10 failures ($n(1-p) \geq 10$).

Back to experimental design...

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Given: $n = 670$, $\hat{p} = \frac{571}{670} = 0.85$.

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Are CLT conditions met?

1. *Independence*: The sample is random, and $670 < 10\%$ of all Americans, therefore we can assume that one respondent's response is independent of another.
2. *Success-failure*: 571 people answered correctly (successes) and 99 answered incorrectly (failures), both are greater than 10.

$n\hat{p}$

$n(1-\hat{p})$

Calculating the Confidence Interval

We are given that $n = 670$, $\hat{p} = 0.85$, we also just learned that the standard error of the sample proportion is $SE = \sqrt{\frac{p(1-p)}{n}}$. What is the 95% confidence interval for this proportion?

$$CI = \text{point estimate} \pm \text{margin of error}$$

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CI &= \text{point estimate} \pm \text{margin of error} \\
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&= \hat{p} \pm z^* \times SE
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$$\begin{aligned} CI &= \text{point estimate} \pm \text{margin of error} \\ &= \text{point estimate} \pm \text{critical value} \times SE \\ &= \hat{p} \pm z^* \times SE \\ &= 0.85 \pm 1.96 \times \sqrt{\frac{0.85 \times 0.15}{670}} = (0.82, 0.88) \end{aligned}$$

Choosing a sample size

How many people should you sample in order to reduce the margin of error of a 95% confidence interval down to 1%.

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$$0.01 \quad \geq \quad 1.96 \times \sqrt{\frac{p \times (1 - p)}{n}}$$

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$$0.01 \geq 1.96 \times \sqrt{\frac{p \times (1-p)}{n}}$$

$$0.01 \geq 1.96 \times \sqrt{\frac{0.85 \times 0.15}{n}} \rightarrow \textit{Using } \hat{p} \textit{ from previous study}$$

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0.01^2 &\geq 1.96^2 \times \frac{0.85 \times 0.15}{n}
\end{aligned}$$

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0.01 &\geq 1.96 \times \sqrt{\frac{0.85 \times 0.15}{n}} \rightarrow \textit{Using } \hat{p} \textit{ from previous study} \\
0.01^2 &\geq 1.96^2 \times \frac{0.85 \times 0.15}{n} \\
n &\geq \frac{1.96^2 \times 0.85 \times 0.15}{0.01^2}
\end{aligned}$$

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0.01 &\geq 1.96 \times \sqrt{\frac{0.85 \times 0.15}{n}} \rightarrow \textit{Using } \hat{p} \textit{ from previous study} \\
0.01^2 &\geq 1.96^2 \times \frac{0.85 \times 0.15}{n} \\
n &\geq \frac{1.96^2 \times 0.85 \times 0.15}{0.01^2} \\
n &\geq 4898.04
\end{aligned}
$$

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$$ME = z^{\star} \times SE$$

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0.01 &\geq 1.96 \times \sqrt{\frac{p \times (1-p)}{n}} \\
0.01 &\geq 1.96 \times \sqrt{\frac{0.85 \times 0.15}{n}} \rightarrow \textit{Using } \hat{p} \textit{ from previous study} \\
0.01^2 &\geq 1.96^2 \times \frac{0.85 \times 0.15}{n} \\
n &\geq \frac{1.96^2 \times 0.85 \times 0.15}{0.01^2} \\
n &\geq 4898.04 \rightarrow n \;\; \textit{should be at least 4,899}
\end{aligned}$$

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... use $\hat{p} = 0.5$. Why?

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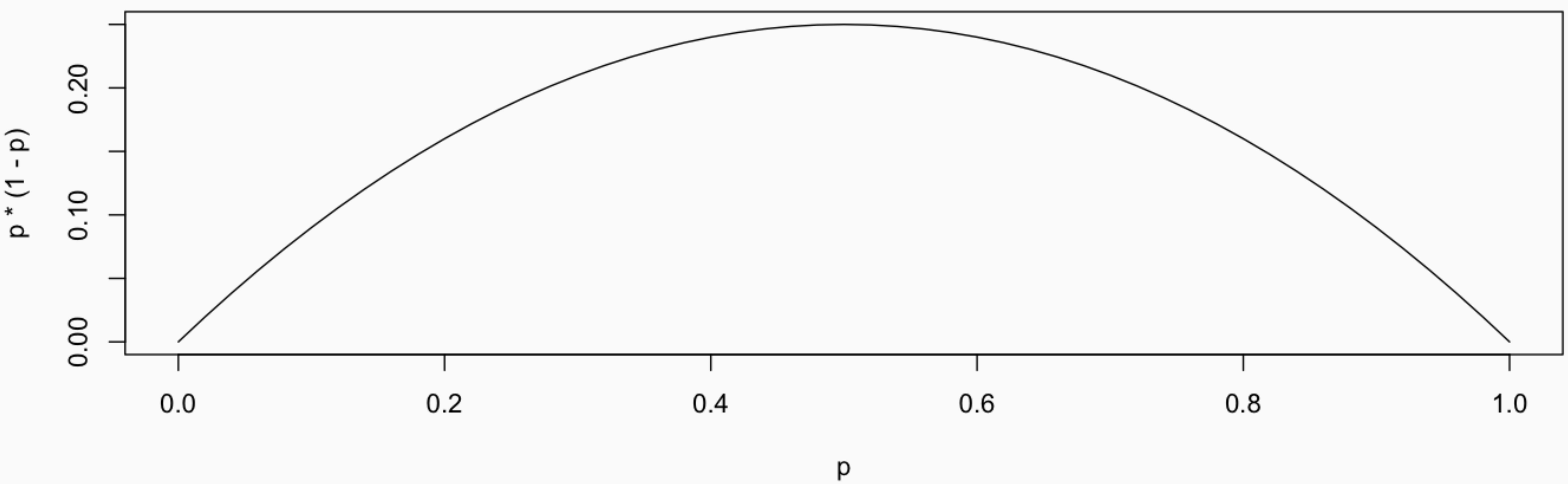
$$SE = \sqrt{\frac{p(1-p)}{n}} \quad \uparrow$$

What if there isn't a previous study?

... use $\hat{p} = 0.5$. Why?

- if you don't know any better, 50-50 is a good guess
- $\hat{p} = 0.5$ gives the most conservative estimate – largest standard error and thus the largest possible sample size.



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Hypotheses:

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CI vs. HT for proportions

For a test of one proportion our null and alternative hypotheses are about p, therefore when we assume H_0 is true we fix $p = p_0$. Hence,

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- Success-failure condition:
  - CI: At least 10 *observed* successes and failures, calculated using the sample proportion, $\hat{p}$
  - HT: At least 10 *expected* successes and failures, calculated using the null value, p_0

Back to the GSS

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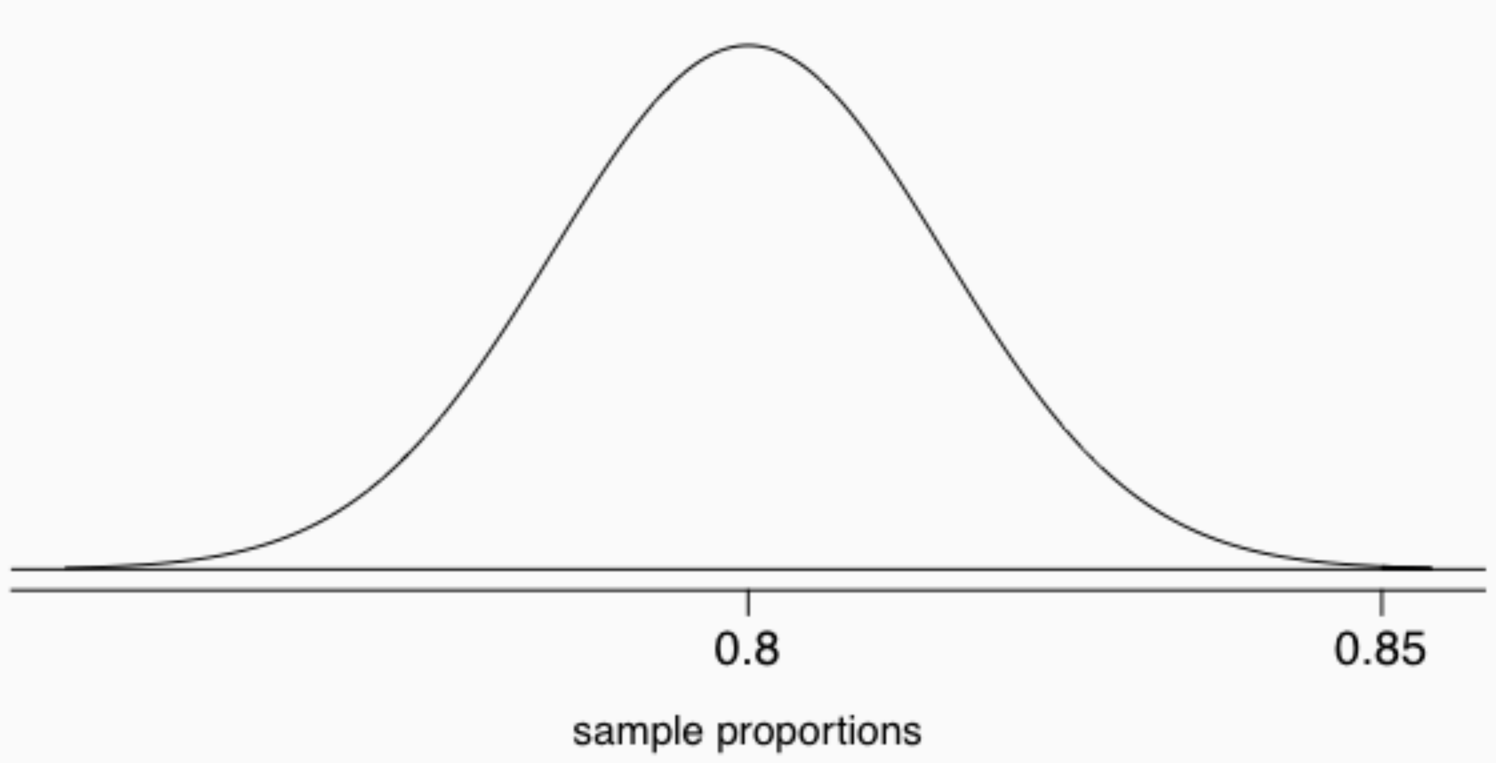

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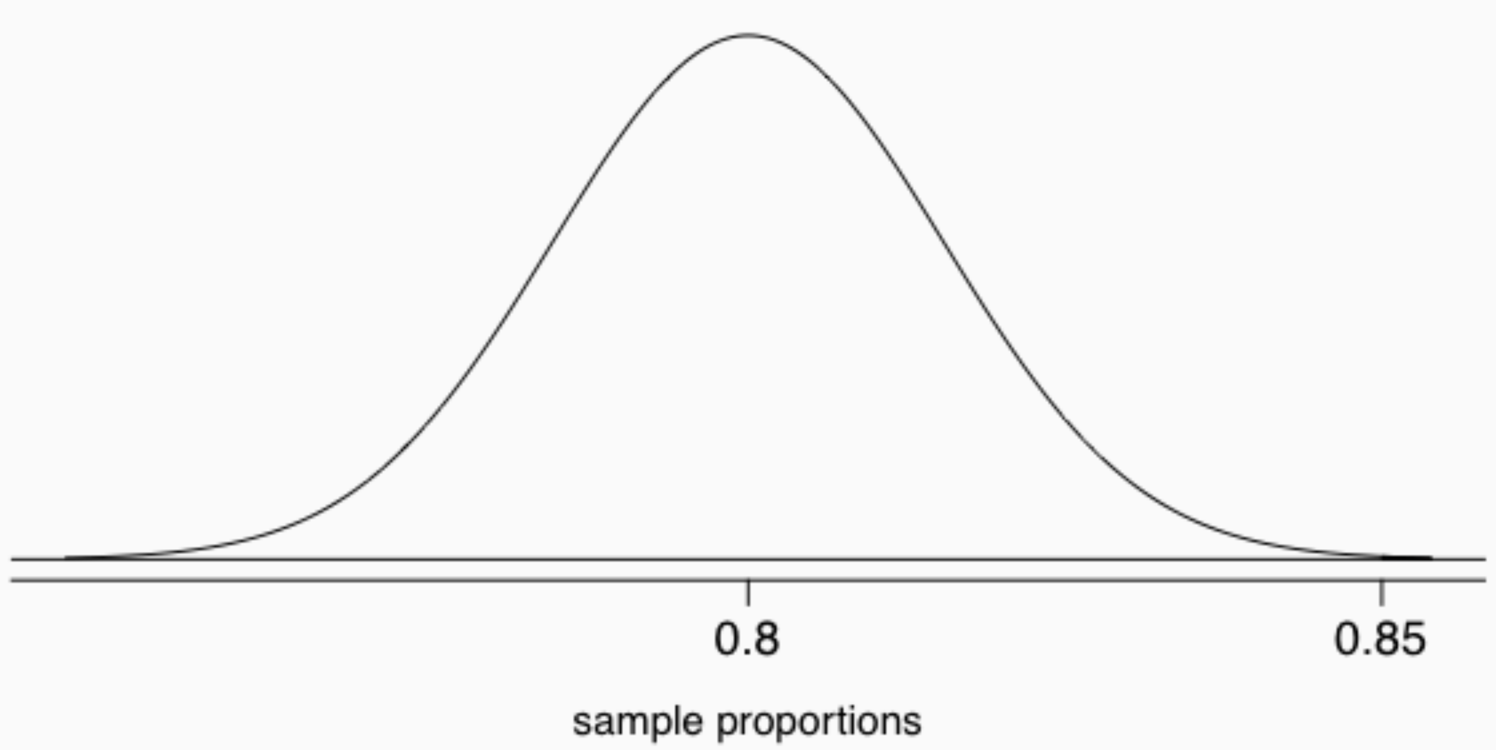

Since p-value is small we reject H_0.

Calculating power - Step 0 and 1

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Step 0: What do we know?

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Assume $p = 0.08$

$$P(Z > z) < 0.05 \quad \Rightarrow \quad z > 1.645$$

$H_0: p = 0.8$

$$P\left(\frac{\hat{p} - 0.8}{0.0154} > 1.645\right) = 0.05$$

$\hookrightarrow SE = \sqrt{\frac{0.8(1-0.8)}{670}}$

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Step 1: What values of $\hat{p}$ would let us reject H_0?

$$\hat{p} > 0.825$$

Step2: Find Prob of region assuming H_A true

$$P(\hat{p} > 0.825 \mid p = p_0 + \delta = 0.8 + 0.05 = 0.85)$$

$$SE = \sqrt{\frac{0.85(1-0.85)}{670}} \neq 0.0154$$

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Step 0: What do we know?

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Step 1: What values of $\hat{p}$ would let us reject H_0?

$$\hat{p} > 0.825$$

Step 2: Assume $p = 0.8 + \delta = 0.85$, what is the probability we reject H_0?
Since p changed, so does $SE = \sqrt{0.85(1 - 0.85)/670} = 0.0138$.

$$\begin{aligned} & P(\hat{p} > 0.825 | p = 0.85) \\ &= P\left(Z > \frac{0.825 - 0.85}{0.0138}\right) \\ &= P\left(Z > -1.811\right) \\ &= 0.965 \end{aligned}$$

Common Misinterpretations

11% of 1,001 Americans responding to a 2006 Gallup survey stated that they have objections to celebrating Halloween on religious grounds. At 95% confidence level, the margin of error for this survey a is $\pm 3\%$. A news piece on this study's findings states: "More than 10% of all Americans have objections on religious grounds to celebrating Halloween."

Is this statement justified?