Lecture 15 - χ^2 Tests

Sta 102

June 8th, 2016

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χ^2 test of GOF

Imazethapyr

Shivrain et al. (2006) crossed clearfield rice, which are resistant to the herbicide imazethapyr, with red rice, which are susceptible to imazethapyr. They then crossed the hybrid offspring and examined the F_2 generation, where they found 772 resistant plants, 1611 moderately resistant plants, and 737 susceptible plants. If resistance is controlled by a single gene with two co-dominant alleles, you would expect a 1:2:1 ratio. Do these data provide convincing evidence of divergence from the expected ratios?

Shivrain, V.K., N.R. Burgos, K.A.K. Moldenhauer, R.W. McNew, and T.L. Baldwin. 2006. Characterization of spontaneous crosses between Clearfield rice (Oryza sativa) and red rice (Oryza sativa). Weed Technology 20: 576-584.

Summarizing the study results

We saw that the researchers observed "772 resistant plants, 1611 moderately resistant plants, and 737 susceptible plants" and we also know that the expected ratio is 1:2:1.

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Outcome	Observed	Expected
Resistant	772	780
Moderately resistant	1611	1560
Susceptible	737	780
Total	3120	3120

Setting the hypotheses

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H_0: There is no inconsistency between the observed and the expected counts. *The observed counts follow the same distribution as the expected counts.*

H_A: There is an inconsistency between the observed and the expected counts. *The observed counts do not follow the same distribution as the expected counts.*

Evaluating the hypotheses

To evaluate these hypotheses, we quantify how different the observed counts are from the expected counts.

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Large deviations from what would be expected based on sampling variation (chance) alone provide strong evidence against the null hypothesis.

This is called a *goodness of fit* test since we're evaluating how well the observed data fit the expected distribution.

Anatomy of a test statistic

The general form of the test statistics we've seen this far is

$$\frac{\text{point estimate} - \text{null value}}{\text{SE of point estimate}}$$

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2. standardizing that difference using the standard error of the point estimate.

These two ideas will help in the construction of an appropriate test statistic for count data.

χ^2 statistic

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The χ^2 statistic is defined to be

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

where $k =$ total number of categories

Calculating the χ^2 statistic

Outcome	Observed	Expected	$\frac{(O-E)^2}{E}$
Resistant	772	780	$\frac{(772-780)^2}{780} = 0.0821$

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Total	3120	3120	4.1199

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Where have we seen this before?

Conditions for the χ^2 test

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Failing to check conditions may unintentionally effect the test's error rates.

The χ^2 distribution

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So far we've seen three other continuous distributions:

- Normal - unimodal and symmetric with two parameters: μ (center) and σ^2 (spread)
- T - unimodal and symmetric with one parameter: *df* (spead, kurtosis)
- T - unimodal and non-symmetric with two parameters: df_1, df_2

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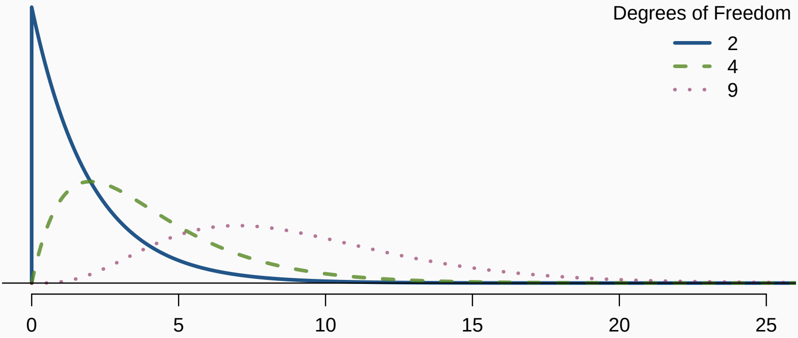
- If we define $Z \sim N(0,1)$ then the quantity,

$$Z^2 \sim \chi^2_{df=1}$$

- If we have $Z_1, \ldots, Z_n \sim N(0,1)$ then the quantity,

$$Z_1{}^2 + Z_2{}^2 + \ldots + Z_n{}^2 \overset{iid}{\sim} \chi^2_{df=n}$$

The χ^2 distribution (cont.)



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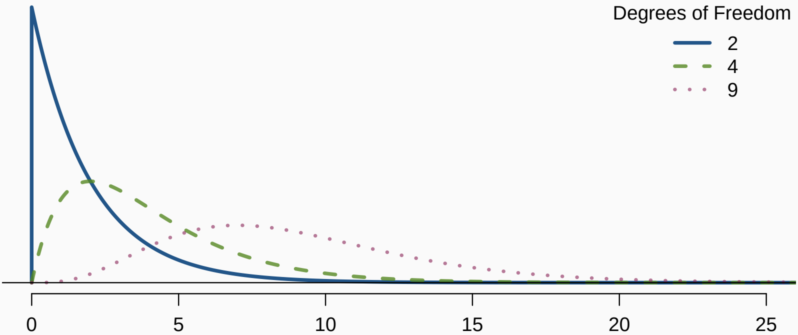
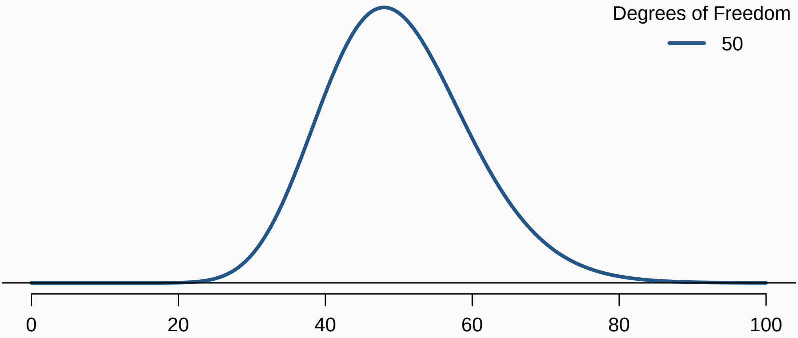


As the df increases:

- the center of the χ^2 distribution increases
- the variability of the χ^2 distribution increases

The χ^2 distribution (cont.)



Also, for large *df* the χ^2 distribution converges to the normal distribution with

$$\mathcal{N}(\mu = df,\ \sigma^2 = 2\,df).$$

Finding areas under the χ^2 curve

- p-value = tail area under the χ^2 distribution (as usual)

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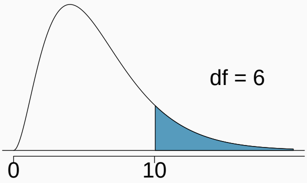
- This table is similar to the t table, it provides upper tail probabilities.

Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
	2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
	3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52
	6	7.23	8.56	10.64	12.59	15.03	16.81	18.55	22.46
	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
	...								

Finding areas under the χ^2 curve (cont.)

Estimate the shaded area under the χ^2 curve with $df = 6$.



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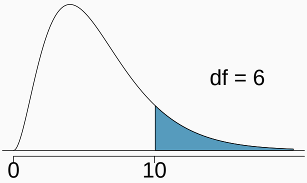

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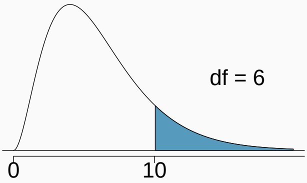

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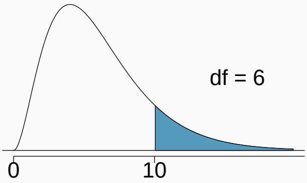

$P(\chi^2_{df=6} > 10)$
is between 0.1 and 0.2

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df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
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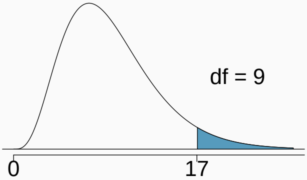
Finding areas under the χ^2 curve (cont.)

Estimate the shaded area (above 17) under the χ^2 curve with $df = 9$.



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	9	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88
	10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59
	11	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26

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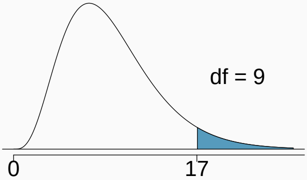

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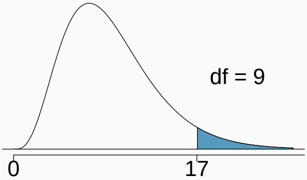

$P(\chi^2_{df=9} > 17)$
is between 0.02 and 0.05

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df	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
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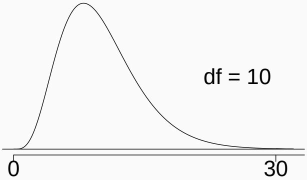
Finding areas under the χ^2 curve (one more)

Estimate the shaded area (above 30) under the χ^2 curve with $df = 10$.



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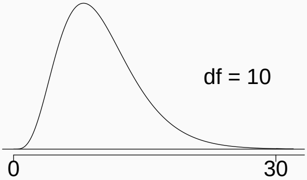

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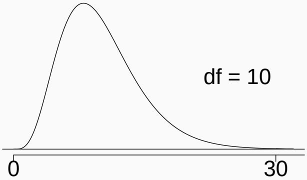

$P(\chi^2_{df=10} > 30)$
is less than 0.001

Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	*0.001*	→
df	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32	
	8	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12	
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	10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	*29.59*	→
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## [1] 0.0008566412
```

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pchisq(q = 30, df = 10, lower.tail = FALSE)

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- Using a web app - https://gallery.shinyapps.io/dist_calc/

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H_0: There is no inconsistency between the observed and the expected counts. *The observed counts follow the same distribution as the expected counts.*

H_A: There is an inconsistency between the observed and the expected counts. *The observed counts **do not** follow the same distribution as the expected counts.*

Our test statistic was $\chi^2 = 4.1199$.

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Our test statistic was $\chi^2 = 4.1199$.

All we need is the *df* and we can calculate the tail area (the p-value) and make a decision on the hypotheses.

Degrees of freedom for a goodness of fit test

When conducting a goodness of fit test to evaluate how well the observed data follow an expected distribution, the degrees of freedom are calculated as the number of cells (k) minus 1.

$$df = k - 1$$

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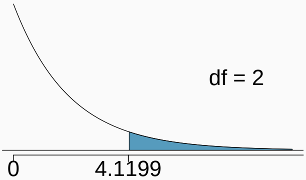
When conducting a goodness of fit test to evaluate how well the observed data follow an expected distribution, the degrees of freedom are calculated as the number of cells (k) minus 1.

$$df = k - 1$$

For these data, $k = 3$ therefore

$$df = 3 - 1 = 2$$

Finding a p-value for a χ^2 test



$$p\text{-value} = P(\chi^2_{df=2} > 4.1199)$$

Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
	2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
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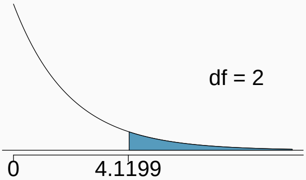
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52

Finding a p-value for a χ^2 test



$$p\text{-}value = P(\chi^2_{df=2} > 4.1199)$$

Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
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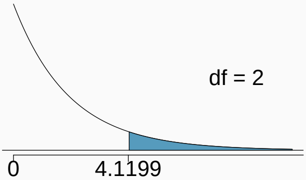

$$p\text{-}value = P(\chi^2_{df=2} > 4.1199)$$
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Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
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Conclusion of the hypothesis test

We calculated a p-value of 0.1275. At the 5% significance level, what is the conclusion of the hypothesis test?

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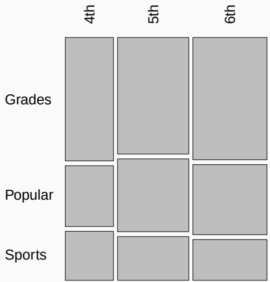
Fail to reject H_0, the data do not provide convincing evidence that the observed counts diverge from the expected counts.

χ^2 test of independence

Popular kids

Students in grades 4-6 were asked whether good grades, athletic ability, or popularity was most important to them. A two-way table separating the students by grade and by choice of most important factor is shown below. Do these data provide evidence to suggest that goals vary by grade?

	Grades	Popular	Sports
4^{th}	63	31	25
5^{th}	88	55	33
6^{th}	96	55	32



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Our hypotheses are:

H_0: Grade and goals are independent. Goals do not vary by grade.

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Conditions for the χ^2 test of independence

- *Independence:* Each case that contributes a count to the table must be independent of all the other cases in the table.
- *Sample size:* Each cell must have at least 5 *expected* counts.

χ^2 test of independence

The test statistic is calculated using

$$\chi^2_{df} = \sum_{i=1}^{k} \frac{(O - E)^2}{E}$$

where k is the number of cells and $df = (R - 1) \times (C - 1)$.

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The p-value is the area under the χ^2 distribution curve to the right of the calculated test statistic,

$$\text{p-value} = P\left(\chi^2_{df} > \sum_{i=1}^{k} \frac{(O - E)^2}{E}\right).$$

χ^2 test of independence (cont.)

Expected counts in two-way tables:

$$\text{Expected Counts} = \frac{(\text{row total}) \times (\text{column total})}{\text{table total}}$$

χ^2 test of independence (cont.)

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	Grades	Popular	Sports	Total
4^{th}	*63*	*31*	25	119
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Total	247	141	90	478

χ^2 test of independence (cont.)

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Expected counts in two-way tables

What is the expected count for the highlighted cell?

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$$E_{5,Pop} = \frac{176 \times 141}{478} = 52$$

more 5th graders than expected have a goal of being popular

Calculating the test statistic in two-way tables

Expected counts are shown in (blue) next to the observed counts.

	Grades	Popular	Sports	Total
4^{th}	63 (61)	31 (35)	25 (23)	119
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$$df = (R-1) \times (C-1) = (3-1) \times (3-1) = 2 \times 2 = 4$$

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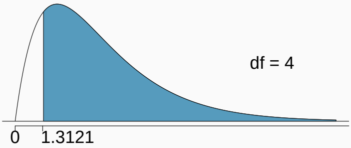

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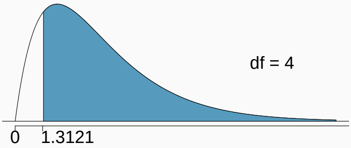

$P(\chi^2_{df=4} > 1.3121)$ *is more than 0.3*

P-value > 0.3

Conclusion

Do these data provide evidence to suggest that goals vary by grade?

H_0: Grade and goals are independent. Goals do not vary by grade.

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Conclusion

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H_0: Grade and goals are independent. Goals do not vary by grade.
H_A: Grade and goals are dependent. Goals vary by grade.

Since p-value is large, we fail to reject H_0. The data do not provide convincing evidence that grade and goals are dependent. It doesn't appear that goals vary by grade.

Summary

Summary - χ^2 test of goodness of fit

- Data:

  y - categorical variable w/ 3 or more levels,

  x - none.

- Hypotheses:

  H_0 - data follow the given distribution,

  H_A - data do not follow the given distribution.

- Conditions:

  Independent observations, all $E_i \geq 5$.

- Test statistic:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}, \quad df = k - 1$$

Summary - χ^2 test of independence

- Data:

  y - categorical variable w/ 2 or more levels,

  x - categorical variable w/ 2 or more levels.

- Hypotheses:

  H_0 - x and y are independent,

  H_A - x and y are dependent.

- Conditions:

  Independent observations, all $E_{i,j} \geq 5$.

- Test statistic:

$$\chi^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{(O_{i,j} - E_{i,j})^2}{E_{i,j}}, \quad df = (m-1)(n-1)$$

$$E_{i,j} = \frac{N_{i,\cdot} \times N_{\cdot,j}}{N_{\cdot,\cdot}}$$