Lecture 15: Joint Distributions

Sta230 / Mth230

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Example 4

Let Y be the rate of calls at a help desk, and X the number of calls in a 2 hour window to that help desk; Let's say that:

$$f(x,y) = \frac{(2y)^x}{x!} e^{-3y}$$

for $y > 0$, $x = 0, 1, 2, \ldots$.

Find:

a) $P(X = 0)$

b) $P(Y > 2)$

c) What is the marginal distirbution of Y?

Example 4.a

Find $P(X = 0)$

Example 4.b

Find $P(Y > 2)$

Example 4.c

Example 5

Let $\theta \sim \text{Unif}(0, 2\pi)$ and $X = \cos(\theta),\ Y = \sin(\theta)$.

Find:

a) $P[X + Y > 1]$

b) $P[Y > 1/2]$

c) Let $\theta \sim \text{Ex}(1)$ and answer a) and b)

Example 5.a

Let $\theta \sim \text{Unif}(0, 2\pi)$ and $X = \cos(\theta),\ Y = \sin(\theta)$.

Example 5.b

Let $\theta \sim \text{Unif}(0, 2\pi)$ and $X = \cos(\theta),\ Y = \sin(\theta)$.

Example 5.c

Let $\theta \sim \text{Exp}(1)$ and $X = \cos(\theta)$, $Y = \sin(\theta)$.

Example 6 - 5.2.9

Consider a fishing experiment where we catch λ fish per hour.

Let X, Y be the times of first and second fish-catch events respectively.

Find:

a) $f(x, y)$

b) $f_X(x)$, $f_Y(y)$

c) Are X and Y independent?

d) $P[Y > X + 2]$

Example 6.a

Let X, Y be the times of first and second fish-catch events respectively ($\lambda = 1$).

We can also think of this as $M, N \sim \text{Exp}(1)$ where $X = \min(M, N)$, $Y = \max(M, N)$.

Example 6.a, cont.

As always, it is good to check that this is a proper joint density:

Example 6.b

Example 6.c

If X and Y are independent then $f(x, y) = f_X(x) f_Y(y)$.

Example 6.d

Example 7

Suppose that X and Y have a continuous joint distribution for which the joint p.d.f. is defined as follows:

$$f(x, y) = \begin{cases} 3/2y^2 & \text{for } 0 \le x \le 2 \text{ and } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Determine the marginal pdf's of X and Y.

b) Are X and Y independent?

c) Are the event $X < 1$ and the event $Y \le 1/2$ independent?

Example 7.a

Example 7.b and 7.c

Example 8

Suppose that two persons make an appointment to meet between 5 p.m. and 6 p.m. at a certain location, and they agree that neither person will wait more than 10 minutes for the other person. If they arrive independently at random times between 5 p.m. and 6 p.m., what is the probability that they will meet?

a) Write down the joint pdf for the two people's arrival times

b) Find a formula to represent the event the two people meetup

c) Find the probability of the event in b)