

# Lecture 10

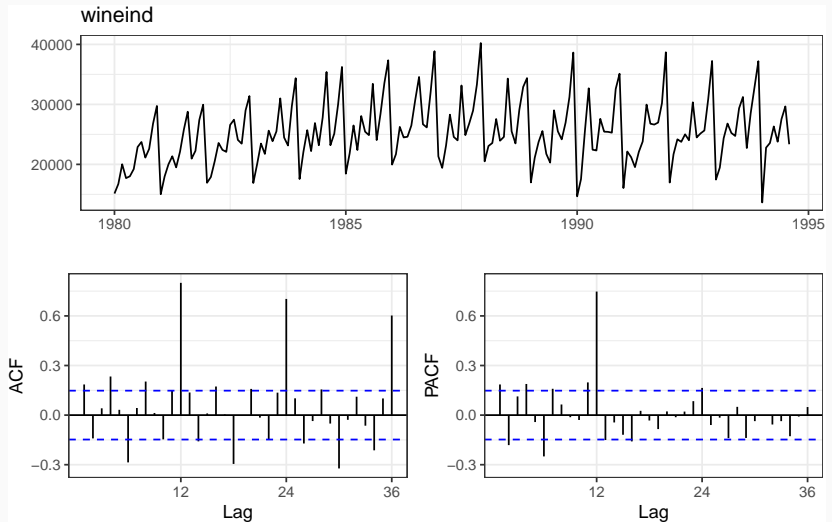
## Seasonal Arima

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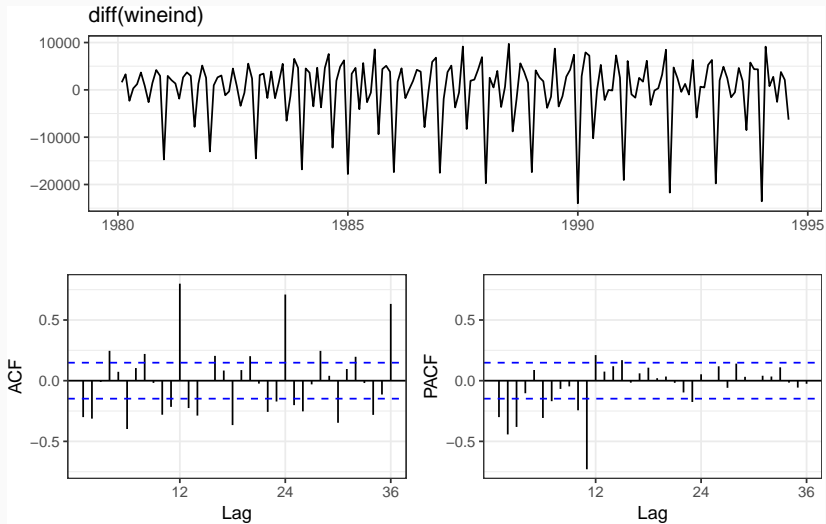
10/05/2018

# Australian Wine Sales Example (Lecture 6)

Australian total wine sales by wine makers in bottles  $\leq 1$  litre. Jan 1980 – Aug 1994.



# Differencing



We can extend the existing Arima model to handle these higher order lags (without having to include all of the intervening lags).

Seasonal ARIMA  $(p, d, q) \times (P, D, Q)_s$ :

$$\Phi_P(L^s) \phi_p(L) \Delta_s^D \Delta^d y_t = \delta + \Theta_Q(L^s) \theta_q(L) w_t$$

We can extend the existing Arima model to handle these higher order lags (without having to include all of the intervening lags).

Seasonal ARIMA  $(p, d, q) \times (P, D, Q)_s$ :

$$\Phi_P(L^s) \phi_p(L) \Delta_s^D \Delta^d y_t = \delta + \Theta_Q(L^s) \theta_q(L) w_t$$

where

$$\phi_p(L) = 1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p$$

$$\theta_q(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_p L^q$$

$$\Delta^d = (1 - L)^d$$

$$\Phi_P(L^s) = 1 - \Phi_1 L^s - \Phi_2 L^{2s} - \dots - \Phi_P L^{Ps}$$

$$\Theta_Q(L^s) = 1 + \Theta_1 L^s + \Theta_2 L^{2s} + \dots + \Theta_Q L^{Qs}$$

$$\Delta_s^D = (1 - L^s)^D$$

## Seasonal Arima for wineind - AR

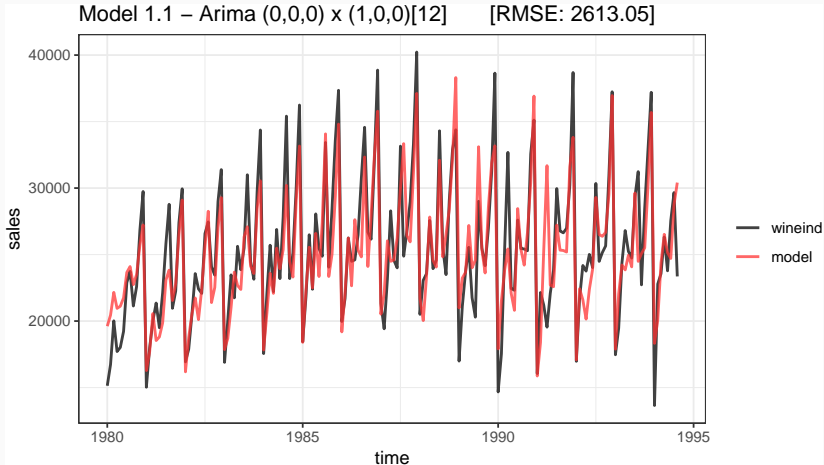
Lets consider an ARIMA(0,0,0)  $\times$  (1,0,0)<sub>12</sub>:

$$(1 - \Phi_1 L^{12}) y_t = \delta + w_t$$

$$y_t = \Phi_1 y_{t-12} + \delta + w_t$$

```
(m1.1 = forecast::Arima(wineind, seasonal=list(order=c(1,0,0), period=12)))  
## Series: wineind  
## ARIMA(0,0,0)(1,0,0)[12] with non-zero mean  
##  
## Coefficients:  
##          sar1          mean  
##      0.8780    24489.24  
## s.e.  0.0314    1154.48  
##  
## sigma^2 estimated as 6906536:  log likelihood=-1643.39  
## AIC=3292.78   AICc=3292.92   BIC=3302.29
```

# Fitted model



## Seasonal Arima for wineind - Diff

Lets consider an  $ARIMA(0, 0, 0) \times (0, 1, 0)_{12}$ :

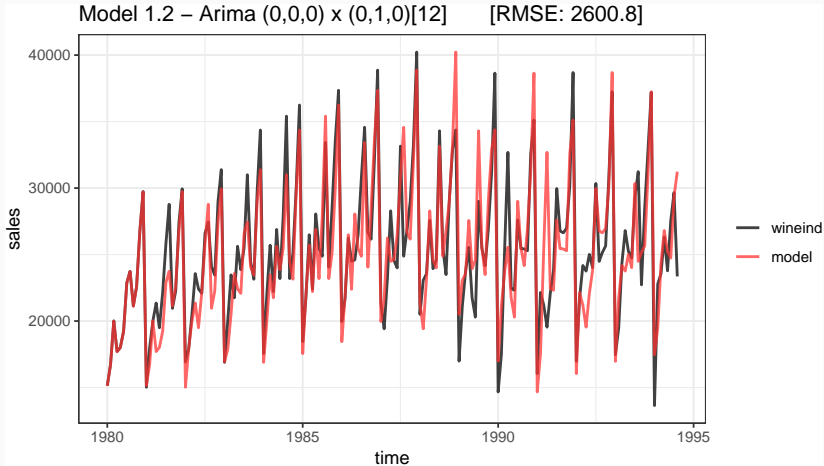
$$(1 - L^{12})y_t = \delta + w_t$$

$$y_t = y_{t-12} + \delta + w_t$$

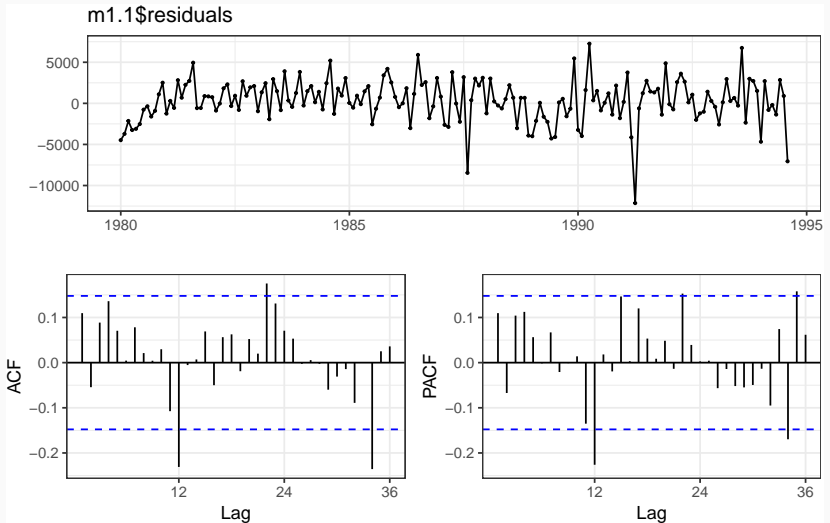
```
(m1.2 = forecast::Arima(wineind, seasonal=list(order=c(0,1,0), period=12)))  
## Series: wineind  
## ARIMA(0,0,0)(0,1,0)[12]  
##  
## sigma^2 estimated as 7259076: log likelihood=-1528.12  
## AIC=3058.24 AICc=3058.27 BIC=3061.34
```



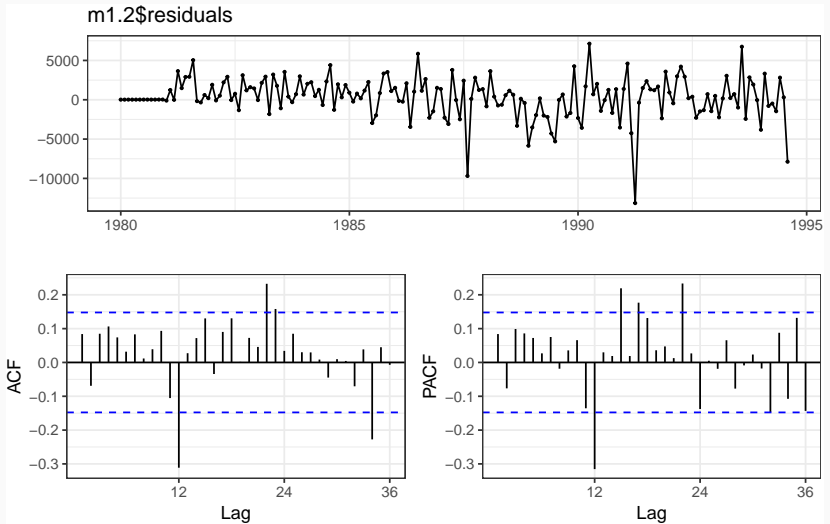
# Fitted model



# Residuals - Model 1.1



# Residuals - Model 1.2



ARIMA(0, 0, 0)  $\times$  (0, 1, 1)<sub>12</sub>:

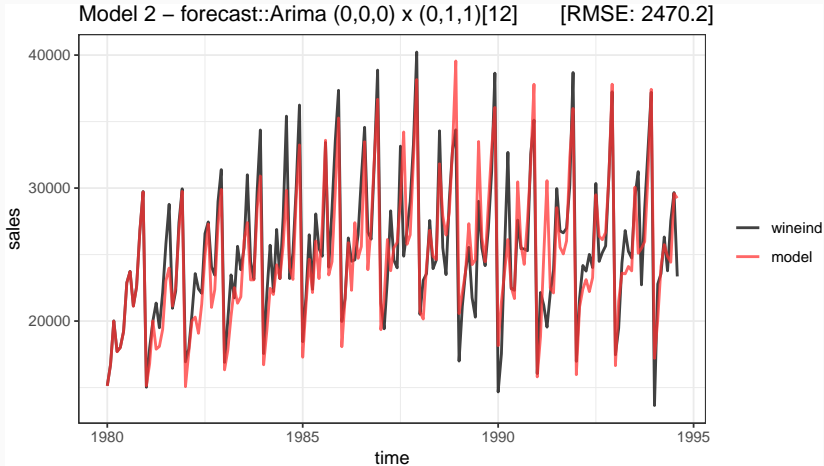
$$(1 - L^{12})y_t = \delta + (1 + \Theta_1 L^{12})w_t$$

$$y_t - y_{t-12} = \delta + w_t + \Theta_1 w_{t-12}$$

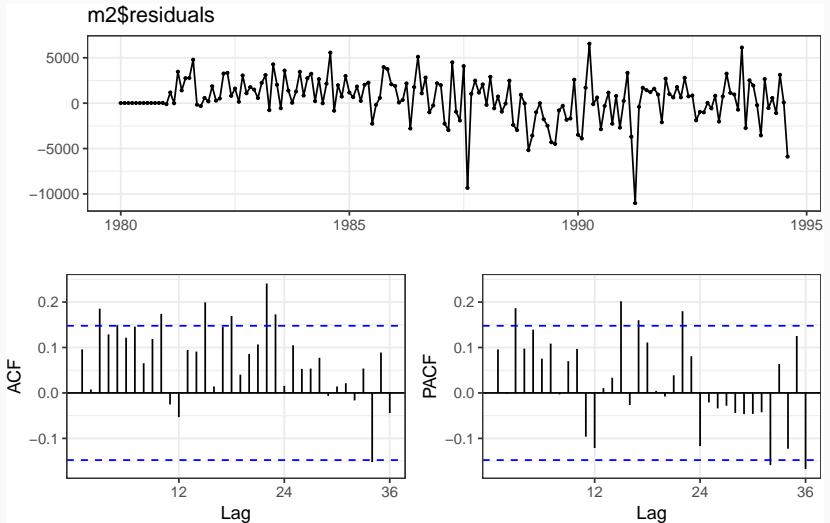
$$y_t = \delta + y_{t-12} + w_t + \Theta_1 w_{t-12}$$

```
(m2 = forecast::Arima(wineind, order=c(0,0,0),
                      seasonal=list(order=c(0,1,1), period=12)))
## Series: wineind
## ARIMA(0,0,0)(0,1,1)[12]
##
## Coefficients:
##          sma1
##         -0.3246
## s.e.      0.0807
##
## sigma^2 estimated as 6588531:  log likelihood=-1520.34
## AIC=3044.68   AICc=3044.76   BIC=3050.88
```

# Fitted model



# Residuals



## Model 3

ARIMA(3, 0, 0)  $\times$  (0, 1, 1)<sub>12</sub>

$$(1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3)(1 - L^{12})y_t = \delta + (1 + \Theta_1 L)w_t$$

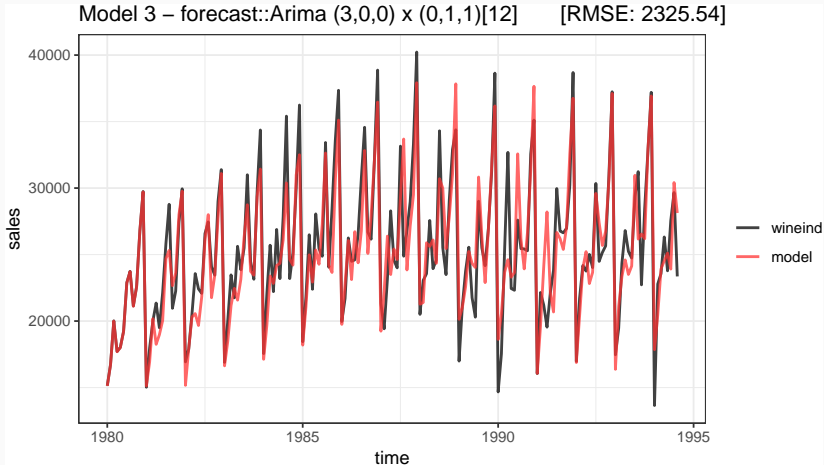
$$(1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3)(y_t - y_{t-12}) = \delta + w_t + w_{t-12}$$

$$y_t = \delta + \sum_{i=1}^3 \phi_i y_{t-1} + y_{t-12} - \sum_{i=1}^3 \phi_i y_{t-12-i} + w_t + w_{t-12}$$

```
(m3 = forecast::Arima(wineind, order=c(3,0,0),  
                      seasonal=list(order=c(0,1,1), period=12)))
```

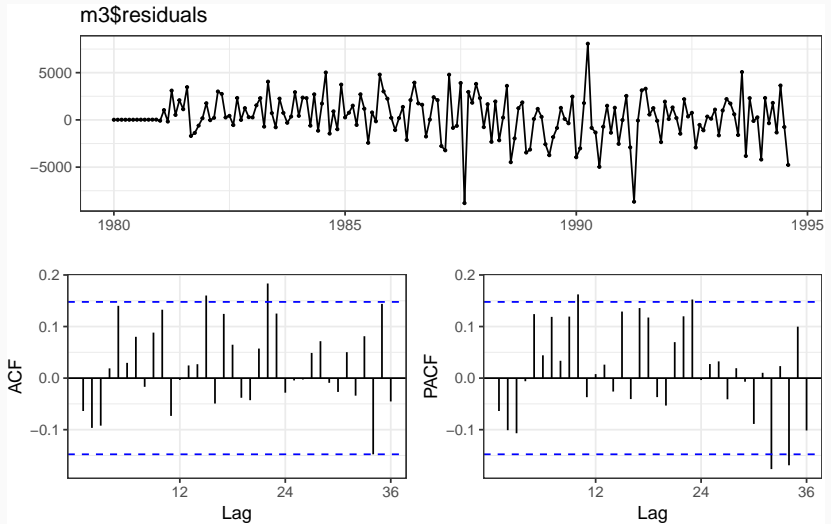
```
## Series: wineind  
## ARIMA(3,0,0)(0,1,1)[12]  
##  
## Coefficients:  
##          ar1      ar2      ar3      sma1  
##      0.1402  0.0806  0.3040  -0.5790  
## s.e.  0.0755  0.0813  0.0823   0.1023  
##  
## sigma^2 estimated as 5948935:  log likelihood=-1512.38  
## AIC=3034.77   AICc=3035.15   BIC=3050.27
```

# Fitted model





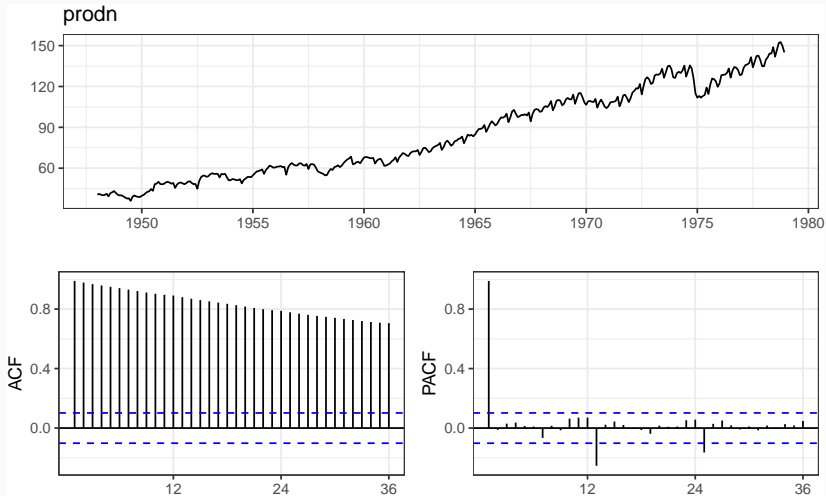
# Model - Residuals



## prodn from the astsa package

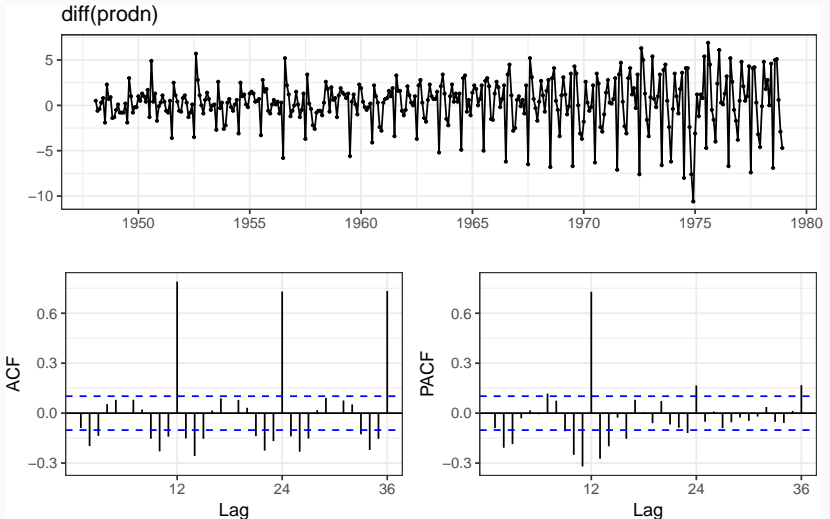
Monthly Federal Reserve Board Production Index (1948-1978)

```
data(prodn, package="astsa"); forecast::ggsdisplay(prodn, points = FALSE)
```



# Differencing

Based on the ACF it seems like standard differencing may be required



Additional seasonal differencing also seems warranted

```
(fr_m1 = forecast::Arima(prodn, order = c(0,1,0),  
                          seasonal = list(order=c(0,0,0), period=12)))
```

```
## Series: prodn
```

```
## ARIMA(0,1,0)
```

```
##
```

```
## sigma^2 estimated as 7.147: log likelihood=-891.26
```

```
## AIC=1784.51 AICc=1784.52 BIC=1788.43
```

```
(fr_m2 = forecast::Arima(prodn, order = c(0,1,0),  
                          seasonal = list(order=c(0,1,0), period=12)))
```

```
## Series: prodn
```

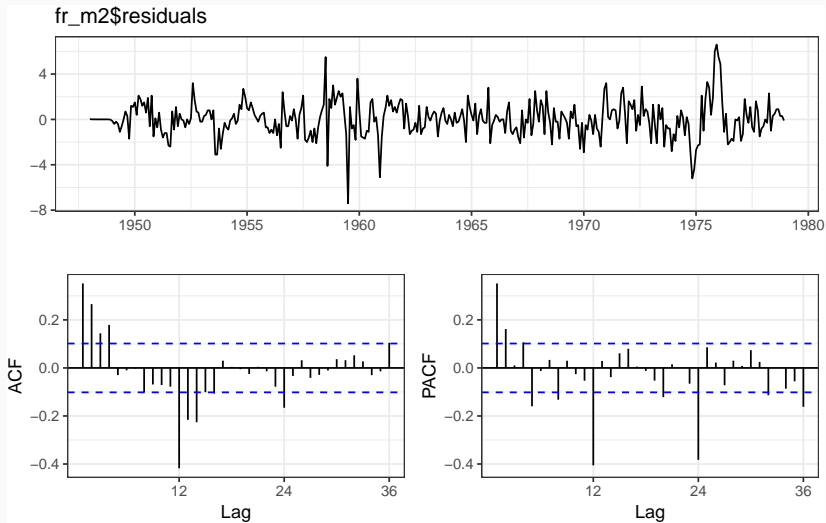
```
## ARIMA(0,1,0)(0,1,0)[12]
```

```
##
```

```
## sigma^2 estimated as 2.52: log likelihood=-675.29
```

```
## AIC=1352.58 AICc=1352.59 BIC=1356.46
```

# Residuals



## Adding Seasonal MA

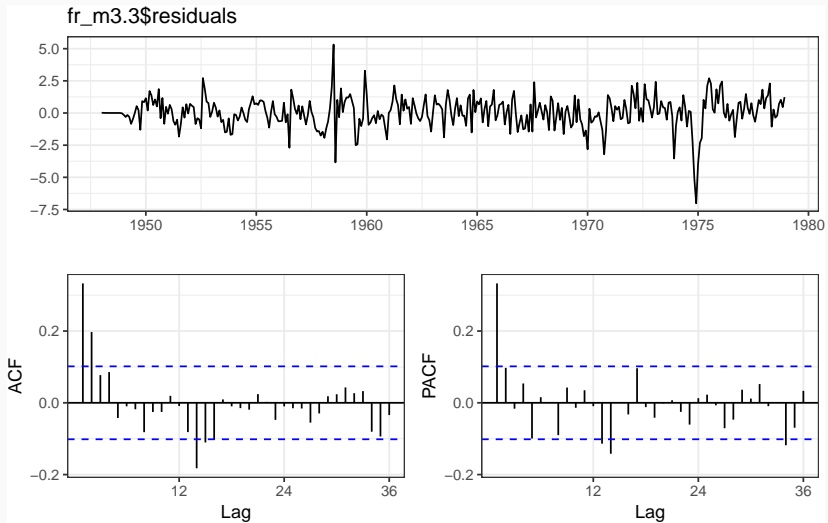
```
(fr_m3.1 = forecast::Arima(prodn, order = c(0,1,0),
                           seasonal = list(order=c(0,1,1), period=12)))
## Series: prodn
## ARIMA(0,1,0)(0,1,1)[12]
##
## Coefficients:
##          sma1
##        -0.7151
## s.e.    0.0317
##
## sigma^2 estimated as 1.616:  log likelihood=-599.29
## AIC=1202.57  AICc=1202.61  BIC=1210.34
```

```
(fr_m3.2 = forecast::Arima(prodn, order = c(0,1,0),
                           seasonal = list(order=c(0,1,2), period=12)))
## Series: prodn
## ARIMA(0,1,0)(0,1,2)[12]
##
## Coefficients:
##          sma1    sma2
##        -0.7624  0.0520
## s.e.    0.0689  0.0666
##
## sigma^2 estimated as 1.615:  log likelihood=-598.98
## AIC=1203.96  AICc=1204.02  BIC=1215.61
```

## Adding Seasonal MA (cont.)

```
(fr_m3.3 = forecast::Arima(prodn, order = c(0,1,0),
                           seasonal = list(order=c(0,1,3), period=12)))
## Series: prodn
## ARIMA(0,1,0)(0,1,3)[12]
##
## Coefficients:
##          sma1      sma2      sma3
##      -0.7853  -0.1205   0.2624
## s.e.   0.0529   0.0644   0.0529
##
## sigma^2 estimated as 1.506:  log likelihood=-587.58
## AIC=1183.15   AICc=1183.27   BIC=1198.69
```

## Residuals - Model 3.3



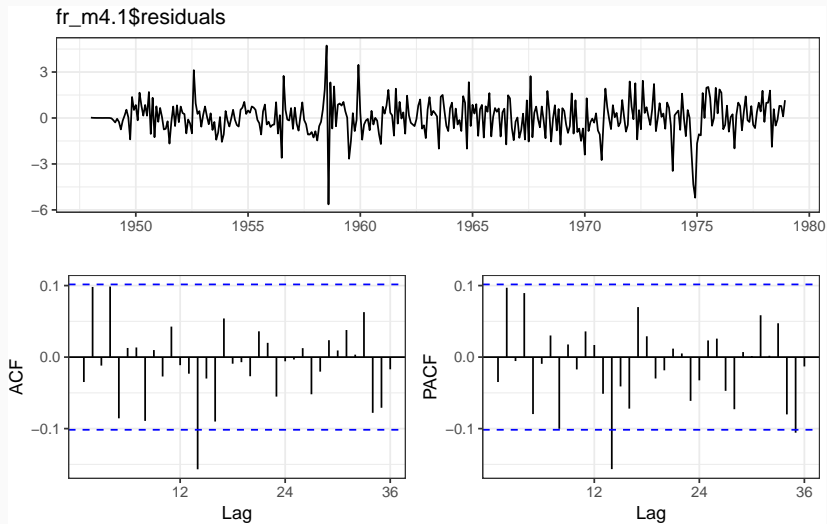


## Adding AR

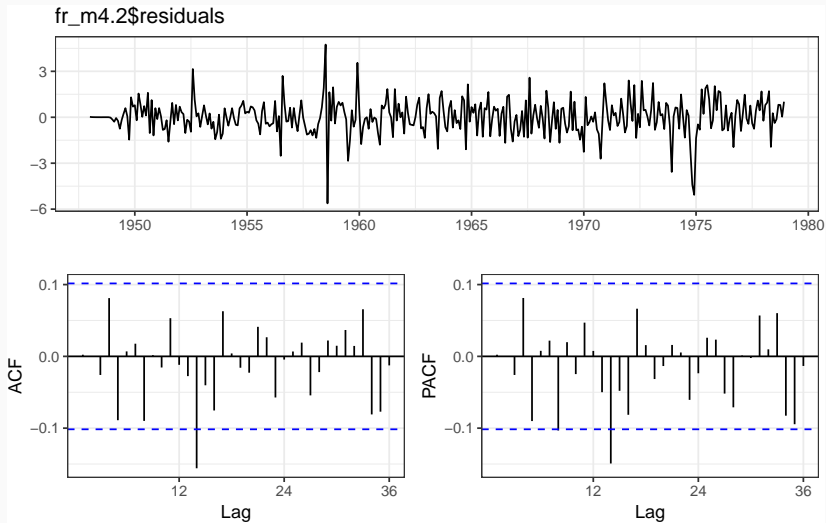
```
(fr_m4.1 = forecast::Arima(prodn, order = c(1,1,0),
                           seasonal = list(order=c(0,1,3), period=12)))
## Series: prodn
## ARIMA(1,1,0)(0,1,3)[12]
##
## Coefficients:
##          ar1      sma1      sma2      sma3
##          0.3393 -0.7619 -0.1222  0.2756
## s.e.    0.0500  0.0527  0.0646  0.0525
##
## sigma^2 estimated as 1.341: log likelihood=-565.98
## AIC=1141.95  AICc=1142.12  BIC=1161.37
```

```
(fr_m4.2 = forecast::Arima(prodn, order = c(2,1,0),
                           seasonal = list(order=c(0,1,3), period=12)))
## Series: prodn
## ARIMA(2,1,0)(0,1,3)[12]
##
## Coefficients:
##          ar1      ar2      sma1      sma2      sma3
##          0.3038  0.1077 -0.7393 -0.1445  0.2815
## s.e.    0.0526  0.0538  0.0539  0.0653  0.0526
##
## sigma^2 estimated as 1.331: log likelihood=-563.98
## AIC=1139.97  AICc=1140.2  BIC=1163.26
```

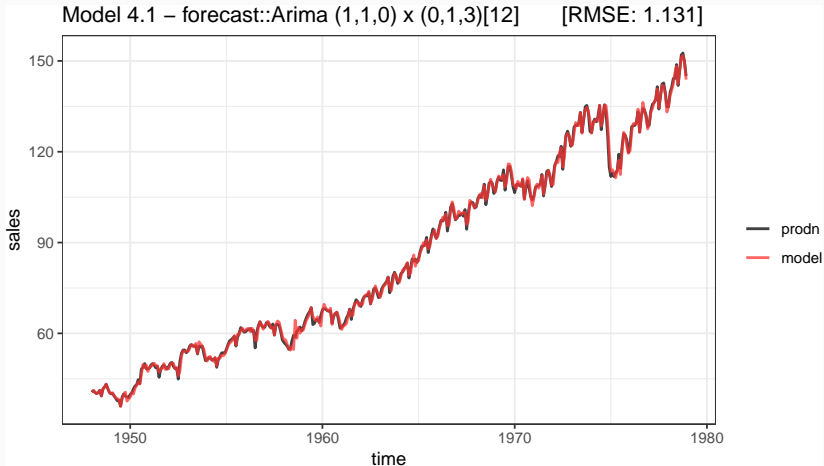
## Residuals - Model 4.1



## Residuals - Model 4.2



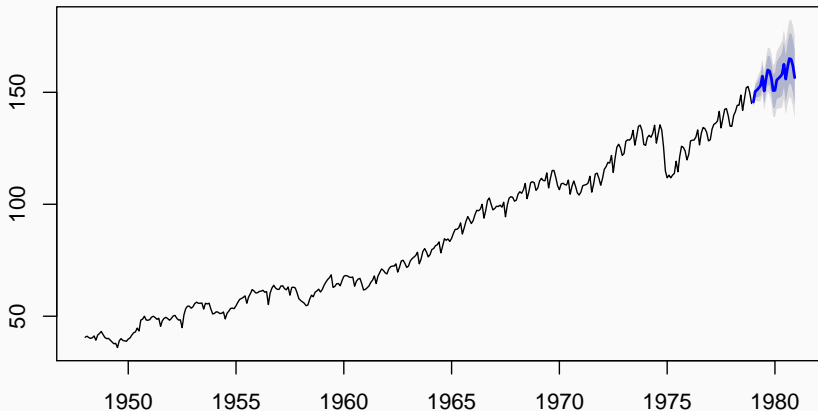
# Model Fit



# Model Forecast

```
forecast::forecast(fr_m4.1) %>% plot()
```

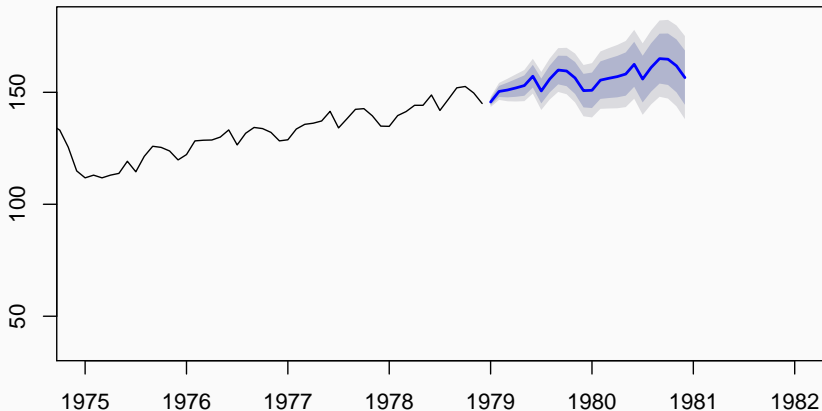
## Forecasts from ARIMA(1,1,0)(0,1,3)[12]



## Model Forecast (cont.)

```
forecast::forecast(fr_m4.1) %>% plot(xlim=c(1975,1982))
```

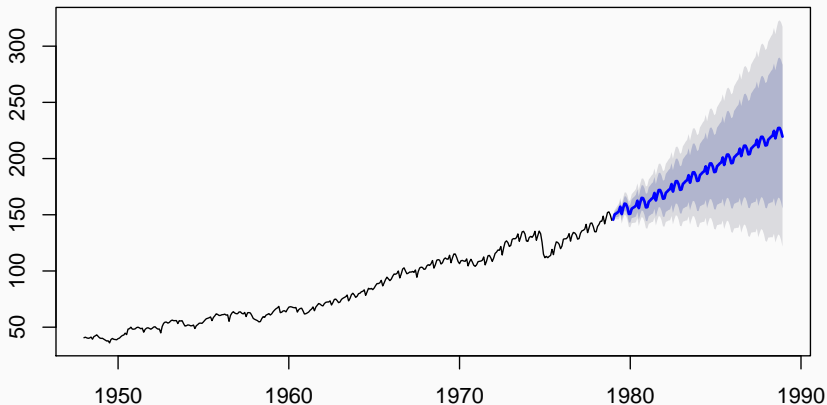
### Forecasts from ARIMA(1,1,0)(0,1,3)[12]



## Model Forecast (cont.)

```
forecast::forecast(fr_m4.1, 120) %>% plot()
```

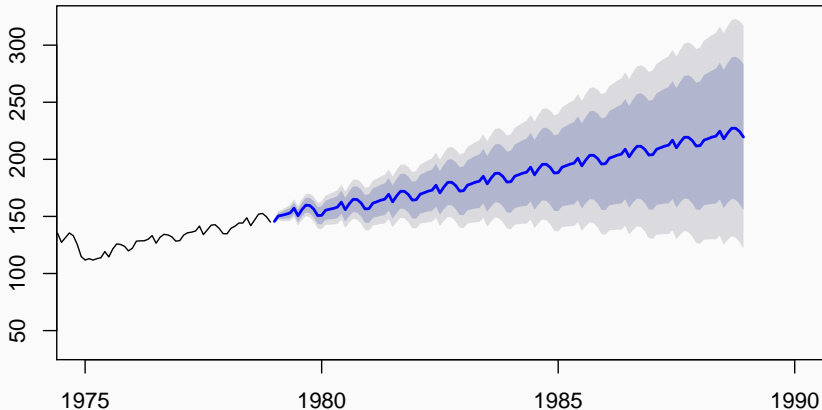
### Forecasts from ARIMA(1,1,0)(0,1,3)[12]



## Model Forecast (cont.)

```
forecast::forecast(fr_m4.1, 120) %>% plot(xlim=c(1975,1990))
```

### Forecasts from ARIMA(1,1,0)(0,1,3)[12]

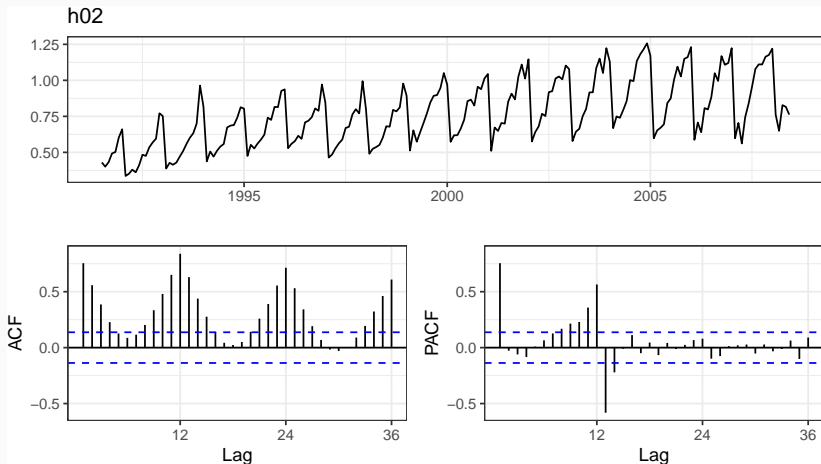




## Exercise - Cortecosteroid Drug Sales

Monthly cortecosteroid drug sales in Australia from 1992 to 2008.

```
data(h02, package="fpp")  
forecast::ggtsdisplay(h02, points=FALSE)
```





# Forecasting

---

Forecasts for stationary models necessarily revert to mean

- Remember,  $E(y_t) \neq \delta$  but rather  $\delta / (1 - \sum_{i=1}^p \phi_i)$ .
- Differenced models revert to trend (usually a line)
- Why? AR gradually damp out, MA terms disappear

Like any other model, accuracy decreases as we extrapolate and the prediction interval expands rapidly

## One step ahead forecasting

Take a fitted ARMA(1,1) process where we know  $\delta$ ,  $\phi$ , and  $\theta$  then

