Lecture 15

GPs for GLMs + Spatial Data

10/26/2018

GPs and GLMs

Logistic Regression

A typical logistic regression problem uses the following model,

$$\begin{aligned} y_i &\sim \text{Bern}(p_i) \\ \text{logit}(p_i) &= \mathbf{X}\,\boldsymbol{\beta} \\ &= \beta_0 + \beta_1\,x_{i1} + \cdots + \beta_k\,x_{ik} \end{aligned}$$

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$$\begin{aligned} y_i &\sim \text{Bern}(p_i) \\ \text{logit}(p_i) &= \mathbf{X}\,\boldsymbol{\beta} \\ &= \beta_0 + \beta_1\, x_{i1} + \cdots + \beta_k\, x_{ik} \end{aligned}$$

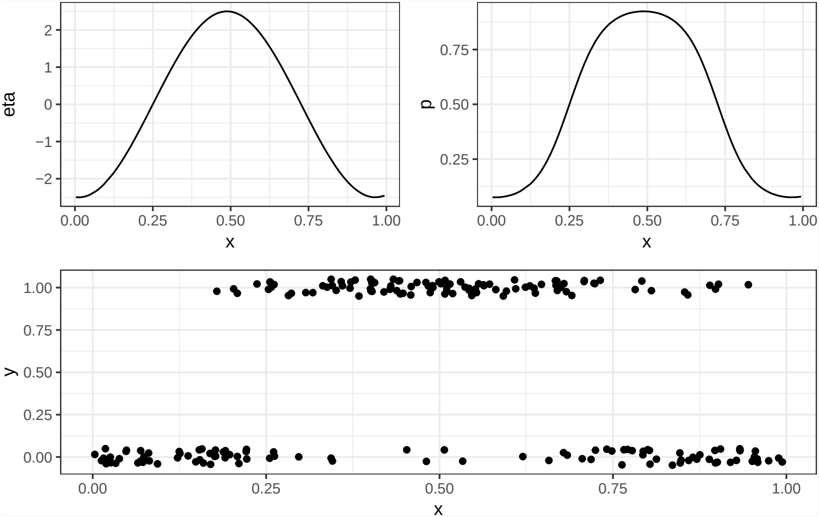
there is no reason that the linear equation above can't contain thing like random effects or GPs

$$\begin{aligned} y_i &\sim \text{Bern}(p_i) \\ \text{logit}(p_i) &= \mathbf{X}\,\boldsymbol{\beta} + w(\mathbf{x}) \end{aligned}$$

where

$$w(\mathbf{x}) \sim \mathcal{N}(0, \Sigma)$$

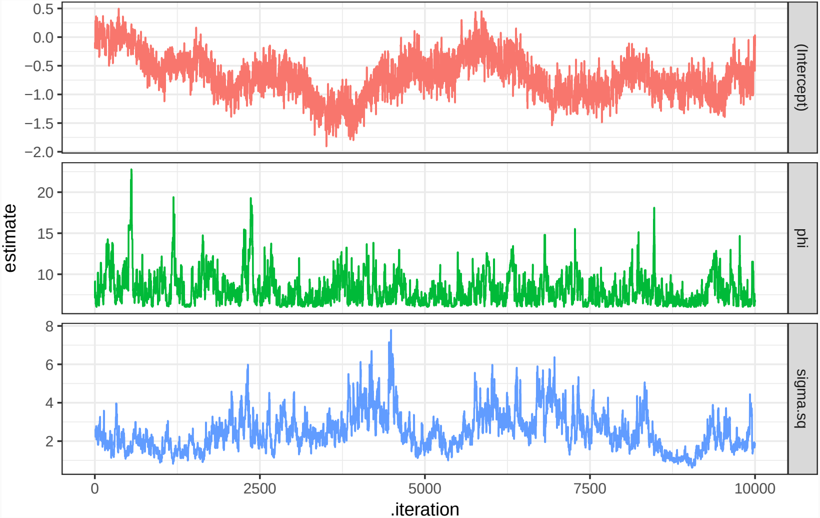
A toy example



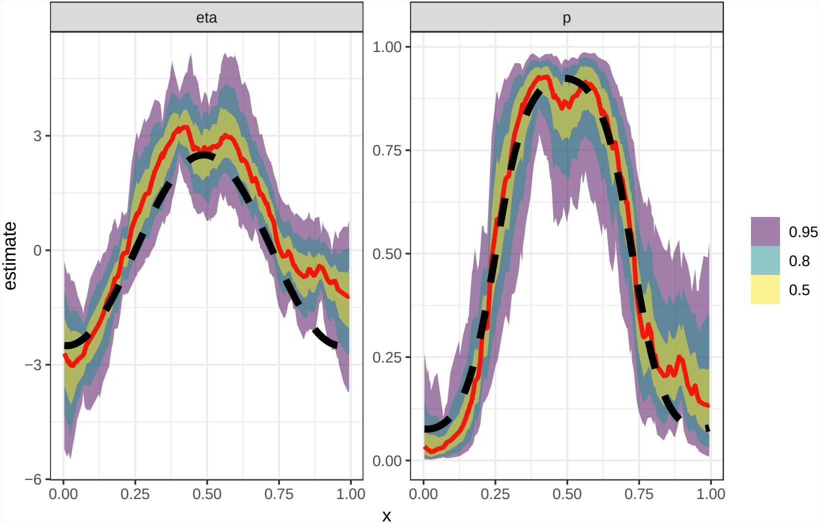
Jags Model*

```
logistic_model = "model{
  for(i in 1:N) {
    y[i] ~ dbern(p[i])
    logit(p[i]) = beta0 + eta[i]
  }

  eta ~ dmnorm(rep(0,N), inverse(Sigma))

  for (i in 1:(length(y)-1)) {
    for (j in (i+1):length(y)) {
      Sigma[i,j] = sigma2 * exp(- l * d[i,j]))
      Sigma[j,i] = Sigma[i,j]
    }
  }

  for (i in 1:length(y)) {
    Sigma[i,i] = sigma2
  }

  beta0 ~ dnorm(0, 1)
  sigma2 = 1/tau
  tau ~ dgamma(1, 2)
  l ~ dunif(3/0.5, 3/0.01)
}"
```

Model Results - Diagnostics

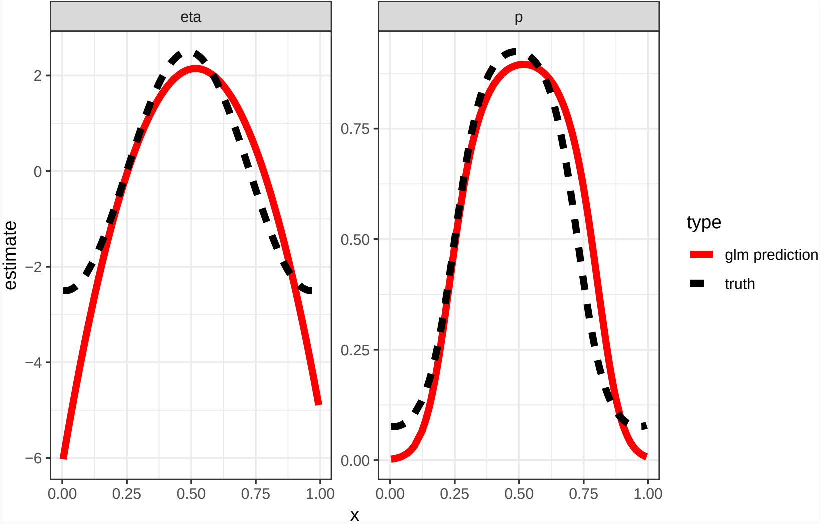


Model Results - Fit



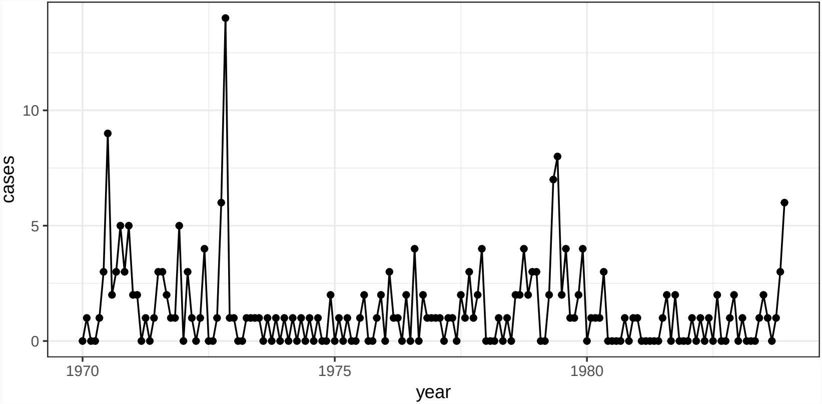
Model vs glm

```
g = glm(y~poly(x,2), data=d, family="binomial")
```



`Polio` from the glarma package.

> *This data set gives the monthly number of cases of poliomyelitis in the U.S. for the years 1970–1983 as reported by the Center for Disease Control.*



Polio Model

Model:

$$y_i \sim \text{Pois}(\lambda_i)$$
$$\log(\lambda_i) = \beta_0 + w(\mathbf{t})$$

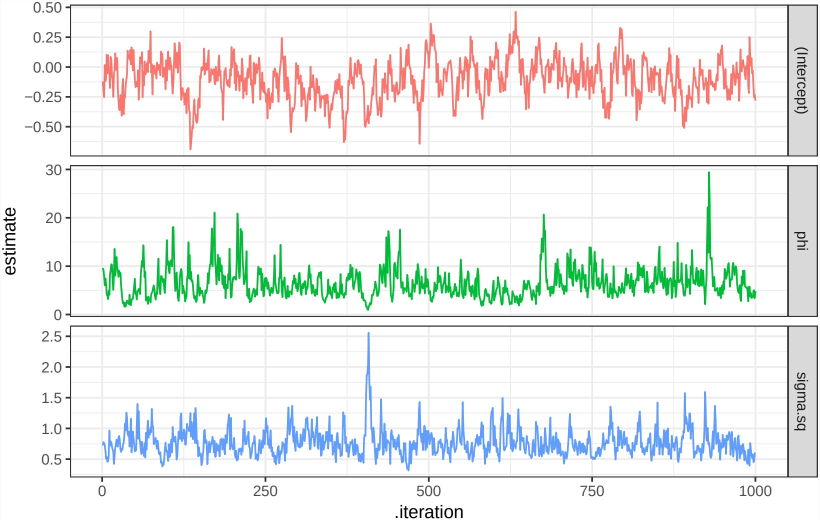
where

$$w(\mathbf{t}) \sim \mathcal{N}(0, \Sigma)$$
$$\{\boldsymbol{\Sigma}\}_{ij} = \sigma^2 \exp(-|\phi\, d_{ij}|)$$

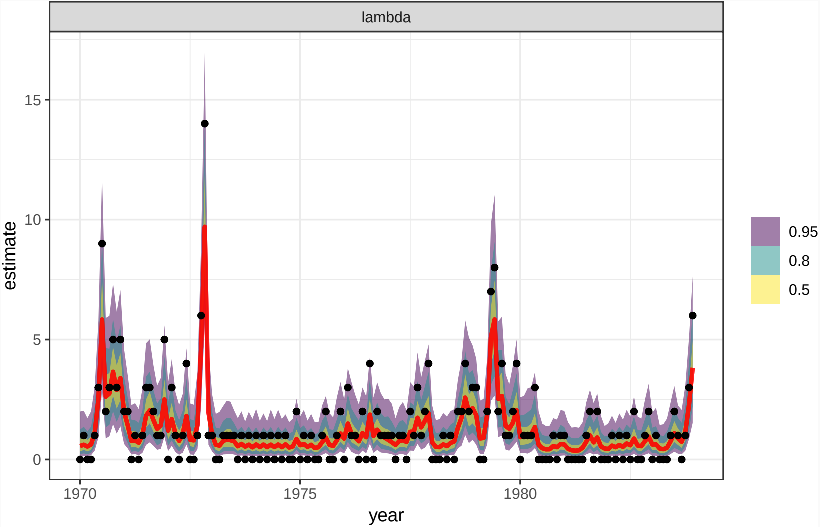
Priors:

$$\beta_0 \sim \mathcal{N}(0, 1)$$
$$\phi \sim \text{Unif}\left(\frac{3}{6}, \frac{3}{1/12}\right)$$
$$1/\sigma^2 \sim \text{Gamma}(2, 1)$$

Model Results - Diagnostics



Model Results - Fit



Spatial data in R

Analysis of geospatial data in R

R has a rich package ecosystem for read/writing, manipulating, and analyzing geospatial data.

Some core packages (CRAN - Spatial task view):

- `sp` - core classes for handling spatial data, additional utility functions.
- `rgdal` - R interface to `gdal` (Geospatial Data Abstraction Library) for reading and writing spatial data.
- `rgeos` - R interface to `geos` (Geometry Engine Open Source) library for querying and manipulating spatial data. Reading and writing WKT.
- `sf` - Combines the functionality of `sp`, `rgdal`, and `rgeos` into a single package based on tidy priciples.
- `lwgeom` - additional functionality for `sf` using PostGIS' liblwgeom.
- `raster` - classes and tools for handling raster data.

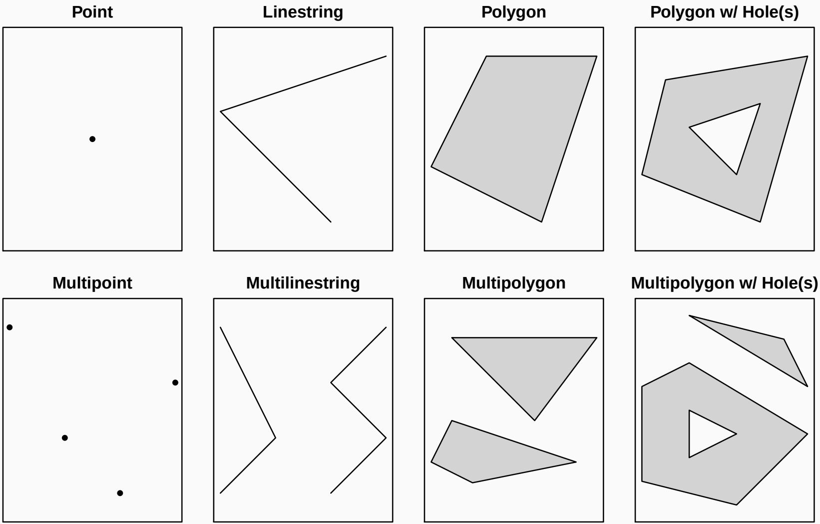
Installing `sf`

This is the hardest part of using the `sf` package, difficulty comes from is dependence on several external libraries (`geos`, `gdal`, and `proj`).

- *Windows* - installing from source works when Rtools is installed (system requirements are downloaded from rwinlib)
- *MacOS* - install dependencies via homebrew: `gdal2`, `geos`, `proj`.
- *Linux* - Install development pacakages for GDAL (>= 2.0.0), GEOS (>= 3.3.0) and Proj.4 (>= 4.8.0) from your package manager of choice.

More specific details are included in the repo readme on github.

Simple Features



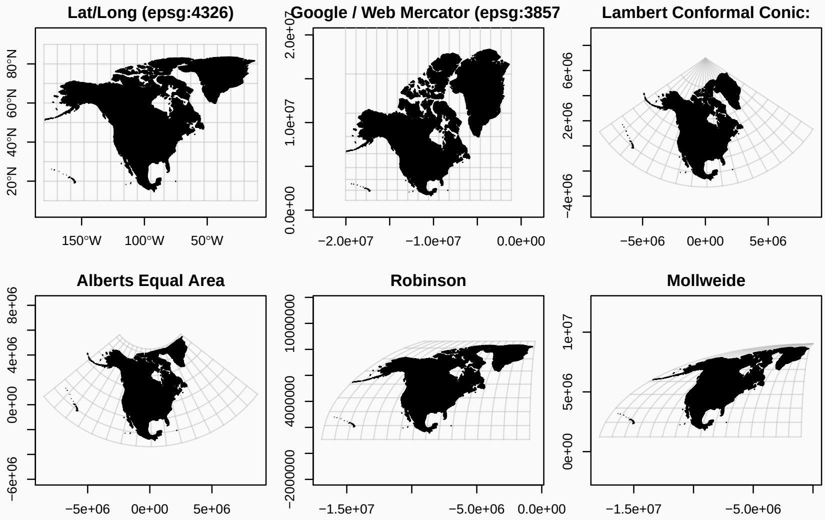
Reading, writing, and converting simple features

- ~~`maptools`~~
  - ~~`readShapePoints` / `writeShapePoints` - Shapefile w/ points~~
  - ~~`readShapeLines` / `writeShapeLines` - Shapefile w/ lines~~
  - ~~`readShapePoly` / `writeShapePoly` - Shapefile w/ polygons~~
  - ~~`readShapeSpatial` / `writeShapeSpatial` - Shapefile~~
- ~~`rgdal`~~
  - ~~`readOGR` / `writeOGR` - Shapefile, GeoJSON, KML, …~~
- `rgeos`
  - ~~`readWKT` / `writeWKT` - Well Known Text~~
- `sf`
  - `st_read` / `st_write` - Shapefile, GeoJSON, KML, …
  - `st_as_sfc` / `st_as_wkt` - WKT
  - `st_as_sfc` / `st_as_binary` - WKB
  - `st_as_sfc` / `as(x, ”Spatial”)` - sp

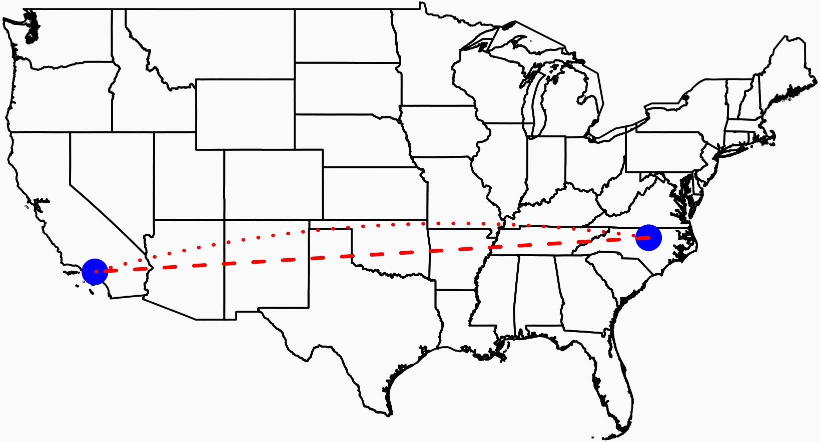
See sf vignette #2.

Geospatial data in the real world

Projections

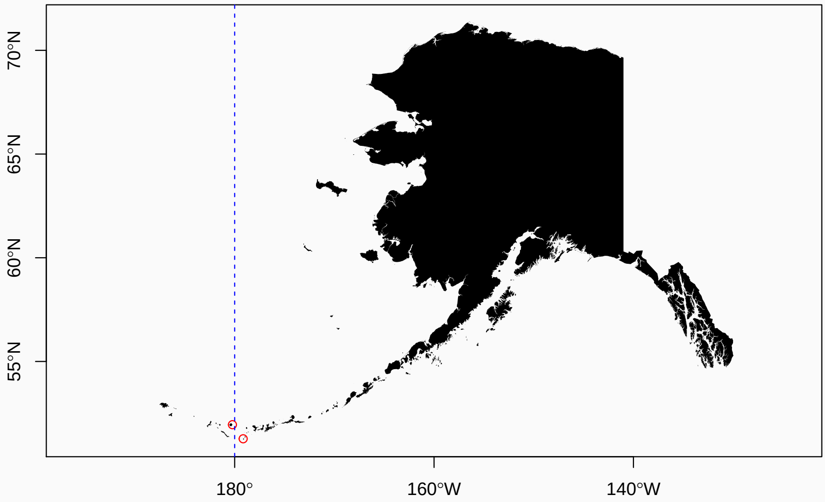


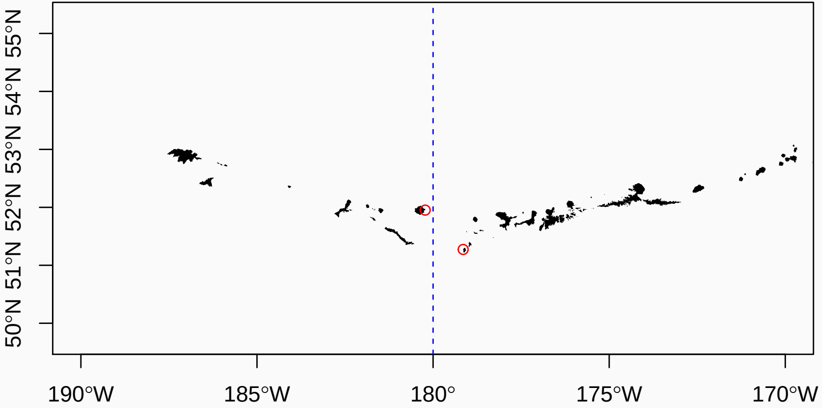
Distance on a Sphere

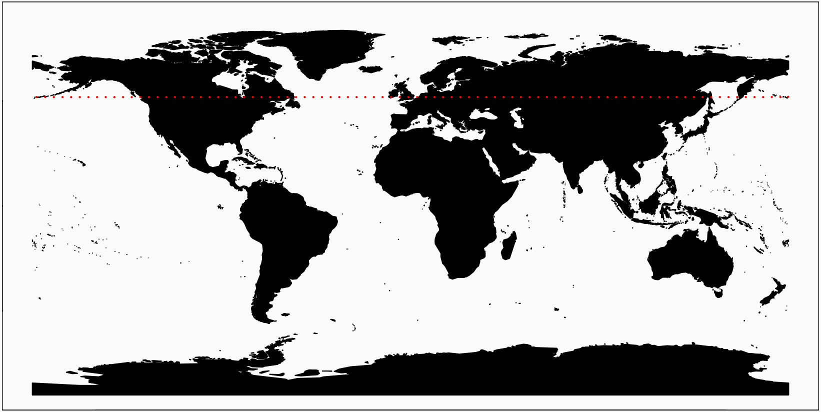


Dateline

Want to fly from the Western most point in the US to the Eastern most point?




55°N
54°N
53°N
52°N
51°N
50°N
190°W
185°W
180°
175°W
170°W



Using `sf`

Example data

```r
nc  = st_read("../data/gis/nc_counties/", quiet=TRUE, stringsAsFactors=FALSE)
air = st_read("../data/gis/airports/", quiet=TRUE, stringsAsFactors=FALSE)
hwy = st_read("../data/gis/us_interstates/", quiet=TRUE, stringsAsFactors=FALSE)

tbl_df(nc)
## # A tibble: 100 x 9
##      AREA PERIMETER COUNTYP010 STATE COUNTY FIPS  STATE_FIPS SQUARE_MIL
##     <dbl>     <dbl>      <dbl> <chr> <chr>  <chr> <chr>           <dbl>
##  1 0.112       1.61       1994 NC    Ashe ~ 37009 37               429.
##  2 0.0616      1.35       1996 NC    Alleg~ 37005 37               236.
##  3 0.140       1.77       1998 NC    Surry~ 37171 37               539.
##  4 0.0891      1.43       1999 NC    Gates~ 37073 37               342.
##  5 0.0687      4.43       2000 NC    Curri~ 37053 37               264.
##  6 0.119       1.40       2001 NC    Stoke~ 37169 37               456.
##  7 0.0626      2.11       2002 NC    Camde~ 37029 37               241.
##  8 0.115       1.46       2003 NC    Warre~ 37185 37               444.
##  9 0.143       2.40       2004 NC    North~ 37131 37               551.
## 10 0.0925      1.81       2005 NC    Hertf~ 37091 37               356.
## # ... with 90 more rows, and 1 more variable: geometry <MULTIPOLYGON [°]>
```

```
tbl_df(air)
## # A tibble: 940 x 17
##    AIRPRTX010 FEATURE ICAO  IATA  AIRPT_NAME CITY  STATE STATE_FIPS COUNTY
##         <dbl> <chr>   <chr> <chr> <chr>      <chr> <chr> <chr>      <chr>
##  1          0 AIRPORT KGON  GON   GROTON-NE~ GROT~ CT    09         NEW L~
##  2          3 AIRPORT K6S5  6S5   RAVALLI C~ HAMI~ MT    30         RAVAL~
##  3          4 AIRPORT KMHV  MHV   MOJAVE AI~ MOJA~ CA    06         KERN
##  4          6 AIRPORT KSEE  SEE   GILLESPIE~ SAN ~ CA    06         SAN D~
##  5          7 AIRPORT KFPR  FPR   ST LUCIE ~ FORT~ FL    12         ST LU~
##  6          8 AIRPORT KRYY  RYY   COBB COUN~ ATLA~ GA    13         COBB
##  7         10 AIRPORT KMKL  MKL   MC KELLAR~ JACK~ TN    47         MADIS~
##  8         11 AIRPORT KCCR  CCR   BUCHANAN ~ CONC~ CA    06         CONTR~
##  9         13 AIRPORT KJYO  JYO   LEESBURG ~ LEES~ VA    51         LOUDO~
## 10         15 AIRPORT KCAD  CAD   WEXFORD C~ CADI~ MI    26         WEXFO~
## # ... with 930 more rows, and 8 more variables: FIPS <chr>, TOT_ENP <dbl>,
## #   LATITUDE <dbl>, LONGITUDE <dbl>, ELEV <dbl>, ACT_DATE <chr>,
## #   CNTL_TWR <chr>, geometry <POINT [°]>
```

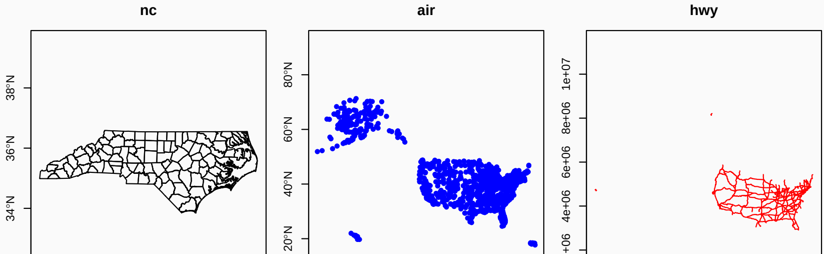
```r
tbl_df(hwy)
## # A tibble: 233 x 4
##    ROUTE_NUM DIST_MILES DIST_KM                                     geometry
##    <chr>          <dbl>   <dbl>                        <MULTILINESTRING [m]>
##  1 I10          2449.   3941.   ((-1881200 4072307, -1879922 4072943, -18~
##  2 I105           20.8    33.4  ((-1910156 5339585, -1910139 5339705, -19~
##  3 I110           41.4    66.6  ((1054139 3388879, 1054287 3385988, 10549~
##  4 I115            1.58    2.55 ((-1013796 5284243, -1013138 5283839, -10~
##  5 I12            85.3   137.   ((680741.7 3366581, 682709.8 3366521, 683~
##  6 I124            1.73    2.79 ((1201467 3906285, 1201643 3905927, 12016~
##  7 I126            3.56    5.72 ((1601502 3829718, 1602136 3829053, 16024~
##  8 I129            3.1     4.99 ((217446 4705389, 217835.1 4705377, 21924~
##  9 I135           96.3   155.   ((96922.97 4313125, 96561.85 4310056, 966~
## 10 I15          1436.   2311    ((-882875.7 5602902, -882998.3 5602422, -~
## # ... with 223 more rows
```

sf classes

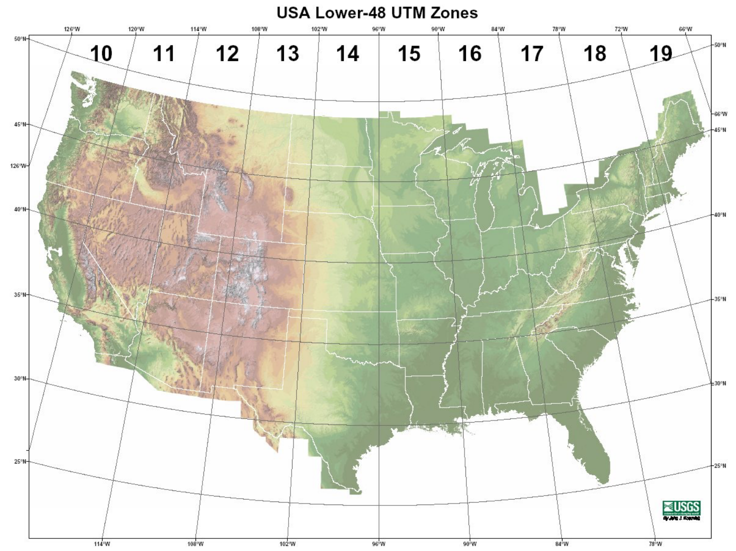
```r
str(nc)
## Classes 'sf' and 'data.frame':   100 obs. of  9 variables:
##  $ AREA      : num  0.1118 0.0616 0.1402 0.0891 0.0687 ...
##  $ PERIMETER : num  1.61 1.35 1.77 1.43 4.43 ...
##  $ COUNTYP010: num  1994 1996 1998 1999 2000 ...
##  $ STATE     : chr  "NC" "NC" "NC" "NC" ...
##  $ COUNTY    : chr  "Ashe County" "Alleghany County" "Surry County" "Gates County"
##  $ FIPS      : chr  "37009" "37005" "37171" "37073" ...
##  $ STATE_FIPS: chr  "37" "37" "37" "37" ...
##  $ SQUARE_MIL: num  429 236 539 342 264 ...
##  $ geometry  :sfc_MULTIPOLYGON of length 100; first list element: List of 1
##   ..$ :List of 1
##   .. ..$ : num [1:1030, 1:2] -81.7 -81.7 -81.7 -81.6 -81.6 ...
##   ..- attr(*, "class")= chr  "XY" "MULTIPOLYGON" "sfg"
##  - attr(*, "sf_column")= chr "geometry"
##  - attr(*, "agr")= Factor w/ 3 levels "constant","aggregate",..: NA NA NA NA NA NA
##   ..- attr(*, "names")= chr  "AREA" "PERIMETER" "COUNTYP010" "STATE" ...

class(nc)
## [1] "sf"         "data.frame"

class(nc$geometry)
## [1] "sfc_MULTIPOLYGON" "sfc"

class(nc$geometry[[1]])
## [1] "XY"           "MULTIPOLYGON" "sfg"
```

Projections

```
st_crs(nc)
## Coordinate Reference System:
##   EPSG: 4269
##   proj4string: "+proj=longlat +ellps=GRS80 +towgs84=0,0,0,0,0,0,0 +no_def
st_crs(air)
## Coordinate Reference System:
##   EPSG: 4269
##   proj4string: "+proj=longlat +ellps=GRS80 +towgs84=0,0,0,0,0,0,0 +no_def
st_crs(hwy)
## Coordinate Reference System:
##   EPSG: 26915
##   proj4string: "+proj=utm +zone=15 +ellps=GRS80 +towgs84=0,0,0,0,0,0,0 +u
```

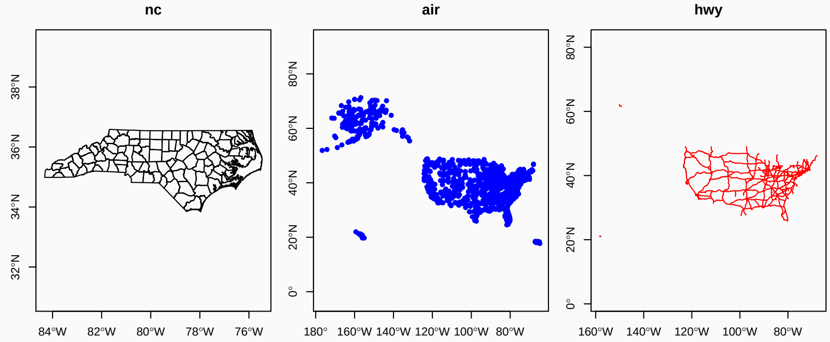


UTM Zones


USA Lower-48 UTM Zones
10
11
12
13
14
15
16
17
18
19
126°W
120°W
114°W
108°W
102°W
96°W
90°W
84°W
78°W
72°W
66°W
50°N
45°N
40°N
35°N
30°N
25°N
USGS

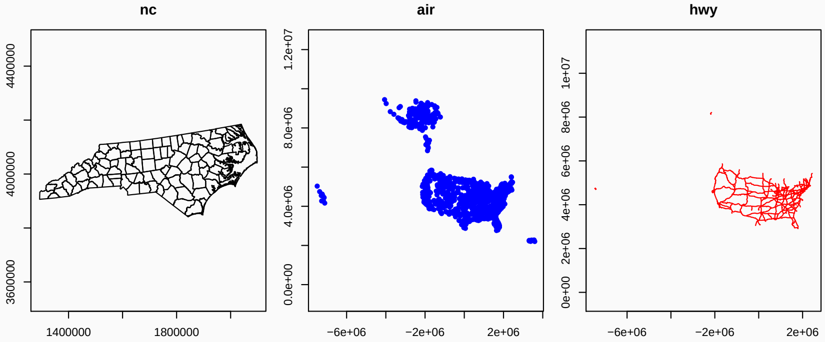
Lat/Long

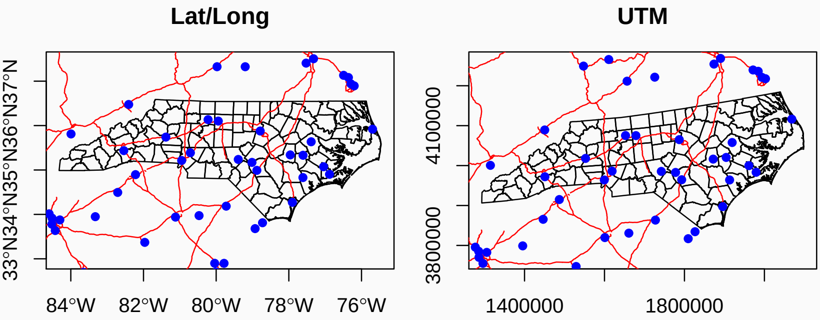
```
nc_ll = nc
air_ll = air
hwy_ll = st_transform(hwy, st_crs(nc))
```



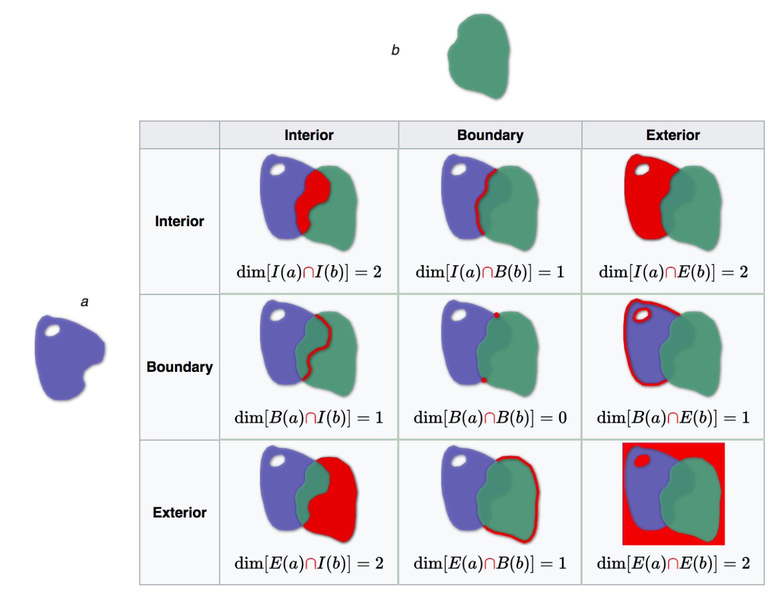
UTM

```r
nc_utm = st_transform(nc, st_crs(hwy))
air_utm = st_transform(air, st_crs(hwy))
hwy_utm = hwy
```

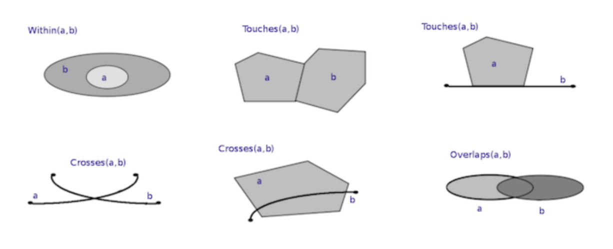



Lat/Long
33°N34°N35°N36°N37°N
84°W 82°W 80°W 78°W 76°W
UTM
3800000 4100000
1400000 1800000

Geometry Predicates


b
a
Interior
Boundary
Exterior
Interior
Boundary
Exterior
$\dim[I(a)\cap I(b)] = 2$
$\dim[I(a)\cap B(b)] = 1$
$\dim[I(a)\cap E(b)] = 2$
$\dim[B(a)\cap I(b)] = 1$
$\dim[B(a)\cap B(b)] = 0$
$\dim[B(a)\cap E(b)] = 1$
$\dim[E(a)\cap I(b)] = 2$
$\dim[E(a)\cap B(b)] = 1$
$\dim[E(a)\cap E(b)] = 2$

Spatial predicates



st_within(a,b)

$$\begin{bmatrix} T & * & F \\ * & * & F \\ * & * & * \end{bmatrix}$$

st_touches(a,b)

$$\begin{bmatrix} F & T & * \\ * & * & * \\ * & * & * \end{bmatrix} \cup \begin{bmatrix} F & * & * \\ T & * & * \\ * & * & * \end{bmatrix} \cup \begin{bmatrix} F & * & * \\ * & T & * \\ * & * & * \end{bmatrix}$$

st_crosses(a,b)

If dim(a)<dim(b)

$$\begin{bmatrix} T & * & T \\ * & * & * \\ * & * & * \end{bmatrix}$$

If dim(a)>dim(b)

$$\begin{bmatrix} T & * & * \\ * & * & * \\ T & * & * \end{bmatrix}$$

If dim$(any)=1$

$$\begin{bmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

st_overlaps(a,b) (dim(a) = dim(b))

If dim$\in\{0,2\}$

$$\begin{bmatrix} T & * & T \\ * & * & * \\ T & * & * \end{bmatrix}$$

If dim$=1$

$$\begin{bmatrix} 1 & * & T \\ * & * & * \\ T & * & * \end{bmatrix}$$

Sparse vs Full Results

```
st_intersects(nc[20:30,], air) %>% str()
## although coordinates are longitude/latitude, st_intersects assumes that t
## List of 11
##  $ : int(0)
##  $ : int(0)
##  $ : int(0)
##  $ : int(0)
##  $ : int(0)
##  $ : int 268
##  $ : int 717
##  $ : int(0)
##  $ : int(0)
##  $ : int(0)
##  $ : int(0)
##  - attr(*, "predicate")= chr "intersects"
##  - attr(*, "region.id")= chr [1:11] "20" "21" "22" "23" ...
##  - attr(*, "ncol")= int 940
##  - attr(*, "class")= chr "sgbp"
```

```
st_intersects(nc, air, sparse=FALSE) %>% str()
## although coordinates are longitude/latitude, st_intersects assumes that t
##  logi [1:100, 1:940] FALSE FALSE FALSE FALSE FALSE FALSE ...
```

Examples

- Which counties are adjacent to Durham County?
- Which counties have more than 4 neighbors?
- Which counties have an airport?