# Lecture 20

## Point referenced data (pt. 2)

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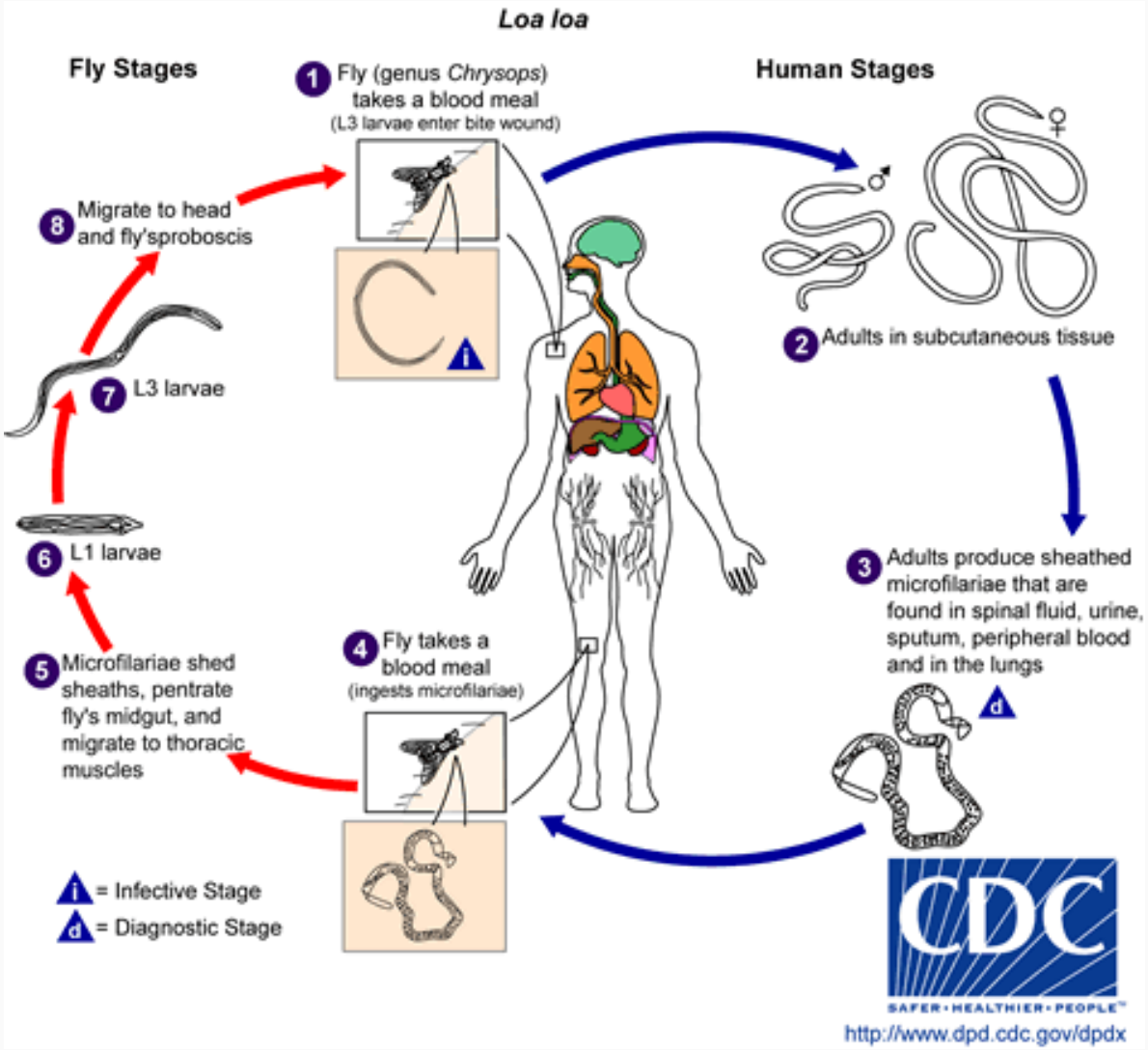
11/14/2018

# Loa Loa Example

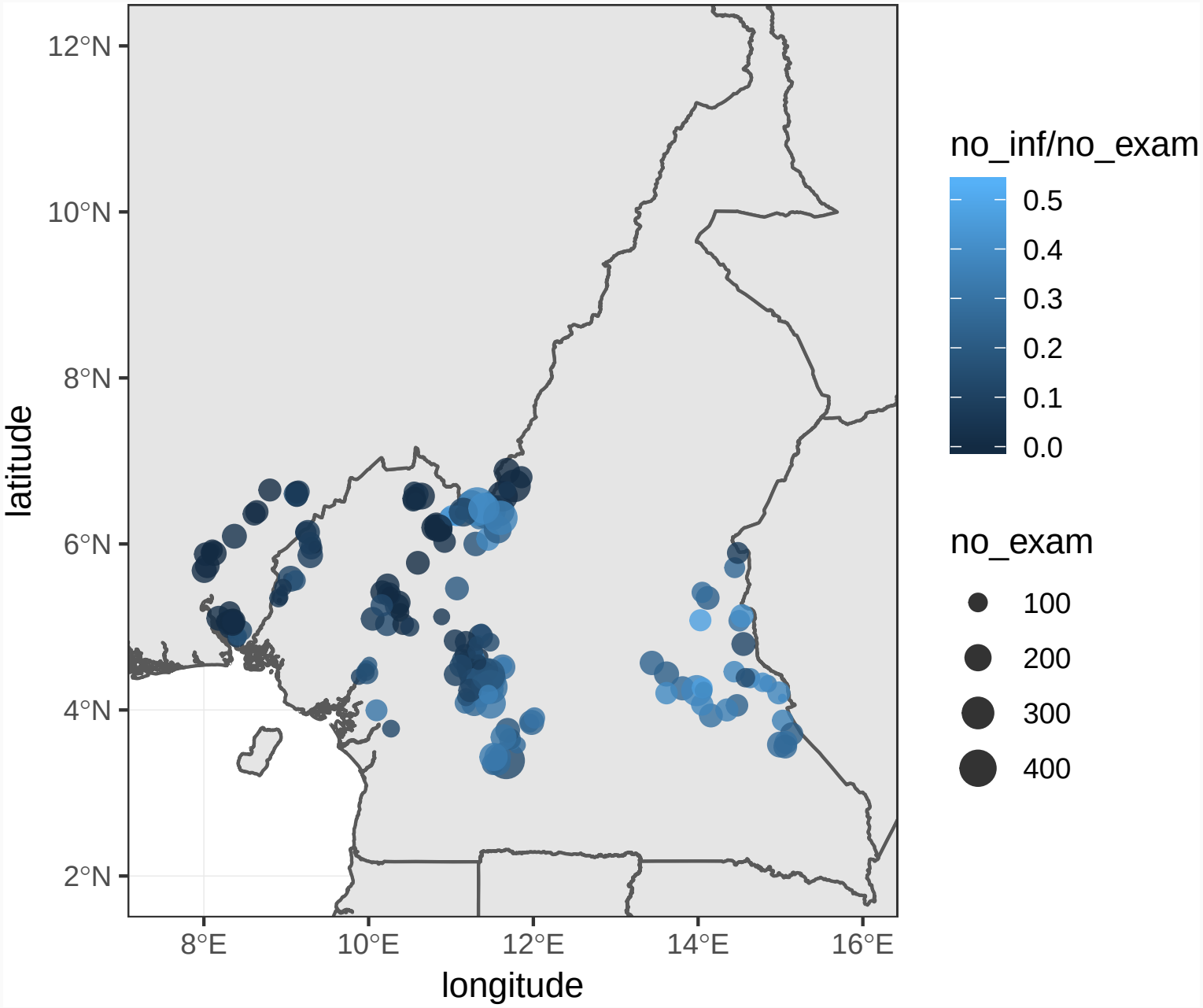
# Loa Loa


Loa loa
Fly Stages
Human Stages
1 Fly (genus Chrysops) takes a blood meal
(L3 larvae enter bite wound)
2 Adults in subcutaneous tissue
3 Adults produce sheathed microfilariae that are found in spinal fluid, urine, sputum, peripheral blood and in the lungs
4 Fly takes a blood meal
(ingests microfilariae)
5 Microfilariae shed sheaths, pentrate fly's midgut, and migrate to thoracic muscles
6 L1 larvae
7 L3 larvae
8 Migrate to head and fly'sproboscis
i = Infective Stage
d = Diagnostic Stage
CDC
SAFER·HEALTHIER·PEOPLE™
http://www.dpd.cdc.gov/dpdx

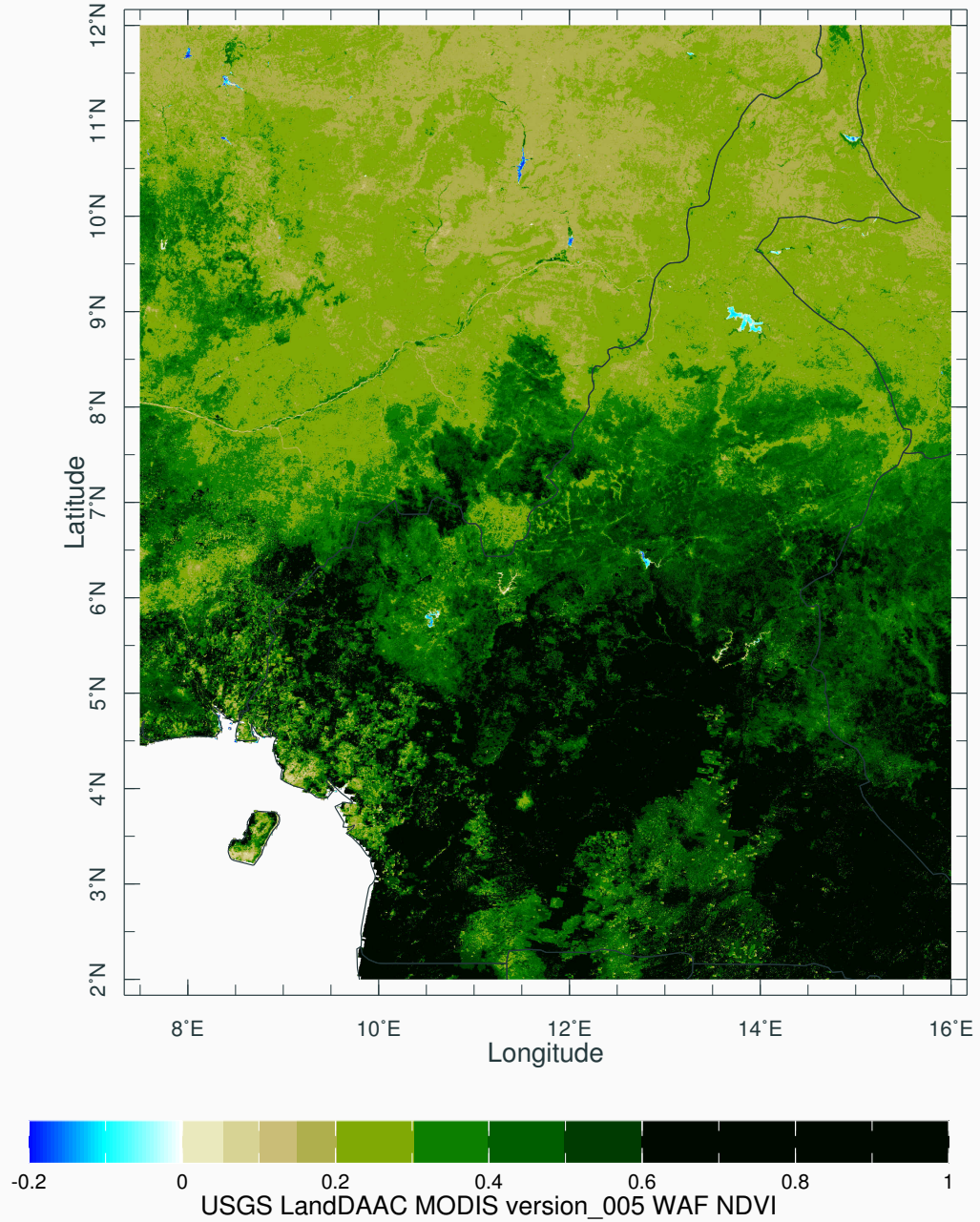
# Data

```
loaloa = tbl_df(PrevMap::loaloa) %>% setNames(., tolower(names(.))) %>%
  rename(elev=elevation)

loaloa
## # A tibble: 197 x 11
##      row villcode longitude latitude no_exam no_inf  elev mean9901 max990
##    <int>    <int>     <dbl>    <dbl>   <int>  <int> <int>    <dbl>   <dbl
##  1     1      214      8.04     5.74     162      0   108    0.439    0.6
##  2     2      215      8.00     5.68     167      1    99    0.426    0.7
##  3     3      118      8.91     5.35      88      5   783    0.491    0.7
##  4     4      219      8.10     5.92      62      5   104    0.432    0.6
##  5     5      212      8.18     5.10     167      3   109    0.415    0.8
##  6     6      116      8.93     5.36      66      3   909    0.436    0.8
##  7     7       16     11.4      4.88     163     11   503    0.502    0.7
##  8     8      217      8.07     5.90      83      0   103    0.373    0.6
##  9     9      112      9.02     5.59      30      4   751    0.481    0.8
## 10    10      104      9.31     6.00      57      4   268    0.487    0.8
## # ... with 187 more rows, and 2 more variables: min9901 <dbl>,
## #   stdev9901 <dbl>
```

# Spatial Distribution



# Normalized Difference Vegetation Index (NDVI)



# Paper / Data summary

Original paper - Diggle, et. al. (2007). *Spatial modelling and prediction of Loa loa risk: decision making under uncertainty*. Annals of Tropical Medicine and Parasitology, 101, 499-509.

- `no_exam` and `no_inf` - Collected between 1991 and 2001 by NGOs (original paper mentions 168 villages and 21,938 observations)
- `elev` - USGS gtopo30 (1km resolution)
- `mean9901` to `stdev9901` - aggregated data from 1999 to 2001 from the Flemish Institute for Technological Research (1 km resolution)

# Diggle's Model

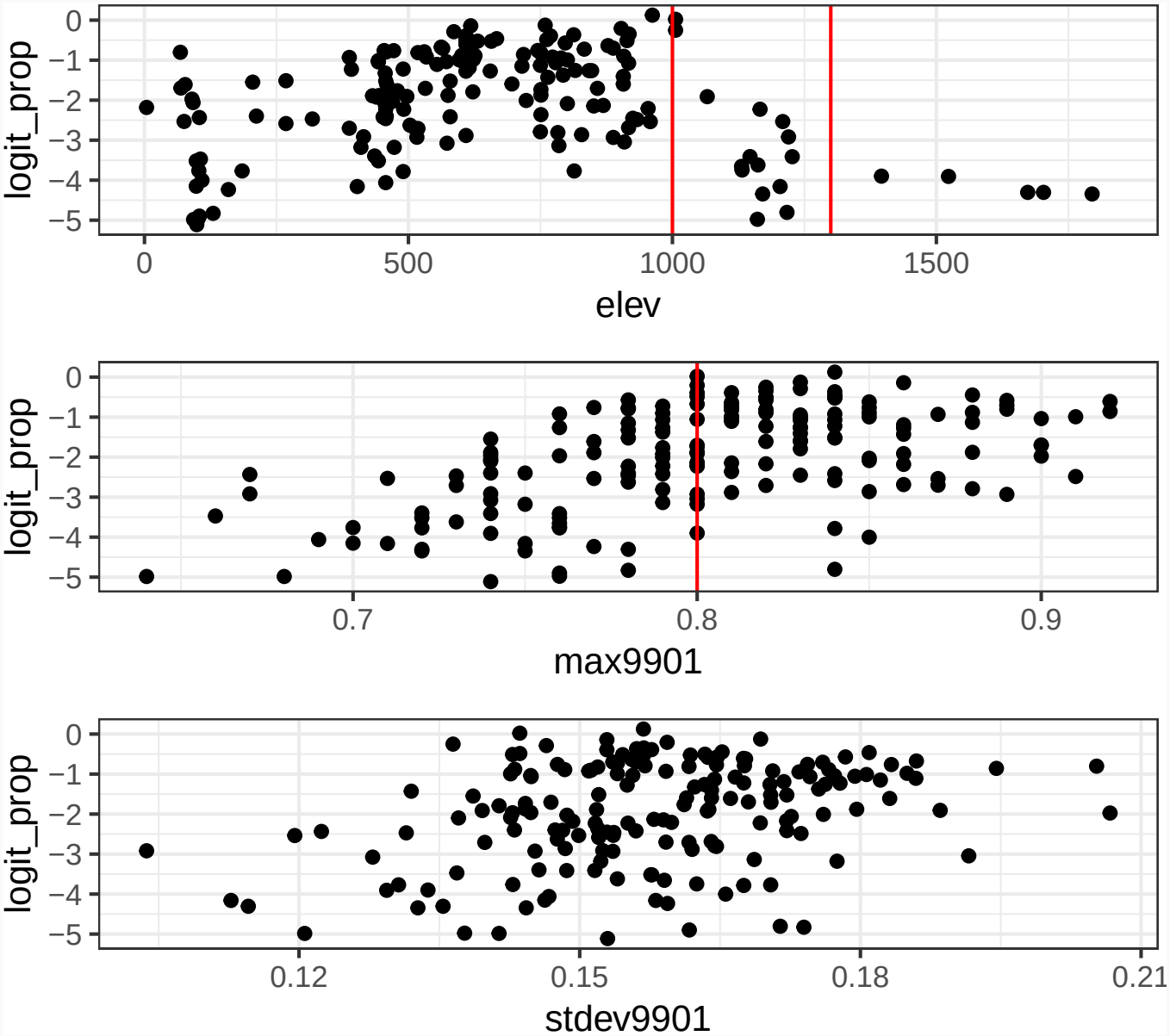
$$\log\left(\frac{p(s)}{1-p(s)}\right) = \alpha + f_1(\text{elev}(s)) + f_2(\text{MAX.NDVI}(s)) + f_3(\text{SD.NDVI}(s)) + w(s)$$

where

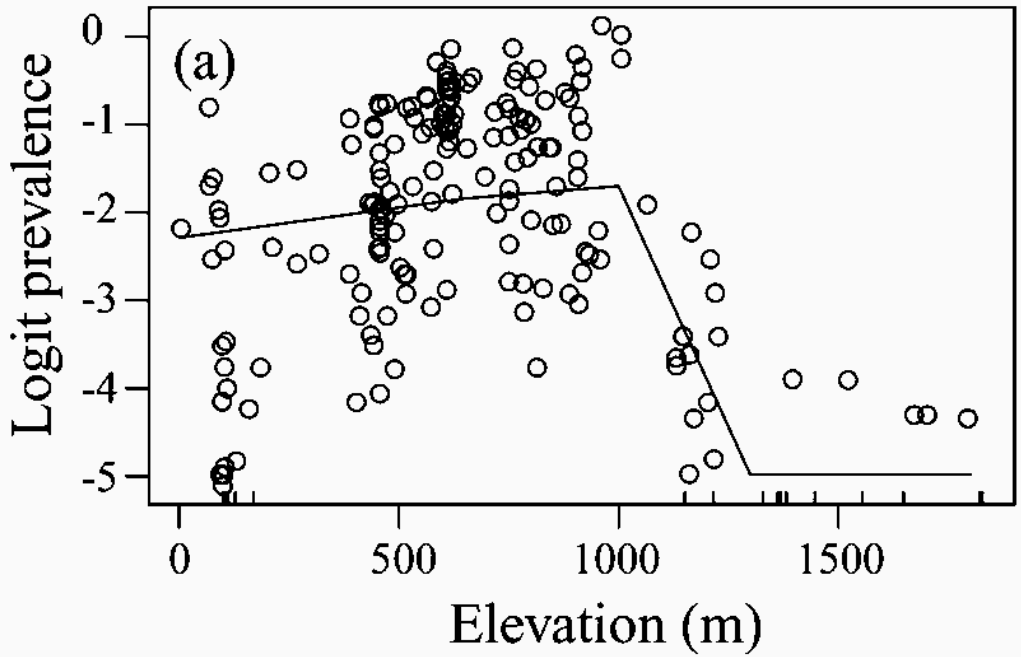
$$w(s) \sim \mathcal{N}(0, \Sigma)$$

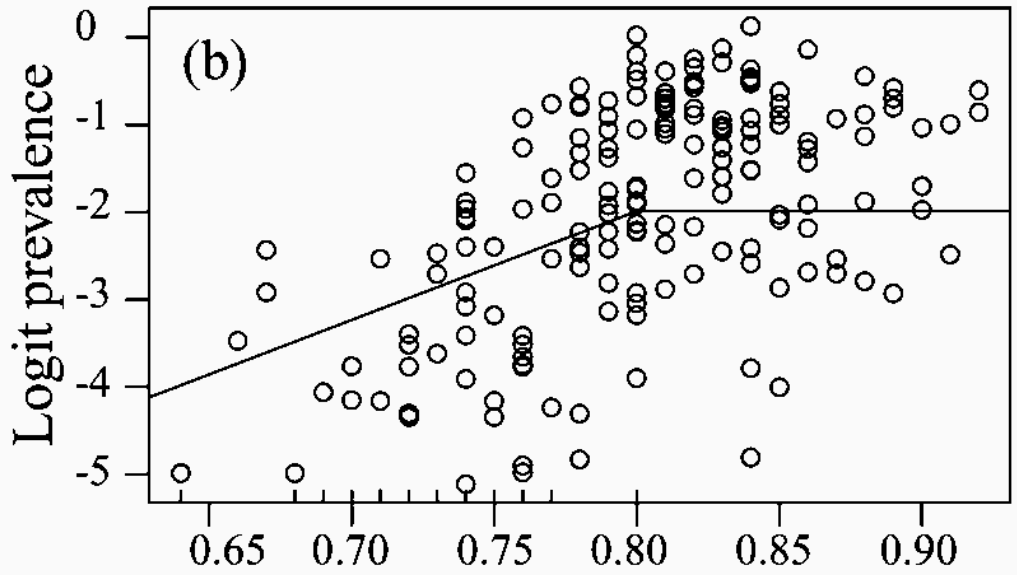
$$\{\Sigma\}_{ij} = \sigma^2 \exp(-d\,\phi)$$

# EDA


logit_prop
0
−1
−2
−3
−4
−5
0
500
1000
1500
elev
logit_prop
0
−1
−2
−3
−4
−5
0.7
0.8
0.9
max9901
logit_prop
0
−1
−2
−3
−4
−5
0.12
0.15
0.18
0.21
stdev9901

# Diggle's EDA


(a)
Logit prevalence
0
-1
-2
-3
-4
-5
0
500
1000
1500
Elevation (m)


(b)
Logit prevalence
0
-1
-2
-3
-4
-5
0.65
0.70
0.75
0.80
0.85
0.90

# Data Augmentation

```
loaloa = loaloa %>%
  mutate(
    elev_f = cut(elev, breaks=c(0,1000,1300,2000), dig.lab=5),
    max_f  = cut(max9901, breaks=c(0,0.8,1))
  )

loaloa %>% select(elev, elev_f, max9901, max_f)
## # A tibble: 197 x 4
##     elev elev_f   max9901 max_f
##    <int> <fct>      <dbl> <fct>
##  1   108 (0,1000]    0.69 (0,0.8]
##  2    99 (0,1000]    0.74 (0,0.8]
##  3   783 (0,1000]    0.79 (0,0.8]
##  4   104 (0,1000]    0.67 (0,0.8]
##  5   109 (0,1000]    0.85 (0.8,1]
##  6   909 (0,1000]    0.8  (0,0.8]
##  7   503 (0,1000]    0.78 (0,0.8]
##  8   103 (0,1000]    0.69 (0,0.8]
##  9   751 (0,1000]    0.8  (0,0.8]
## 10   268 (0,1000]    0.84 (0.8,1]
## # ... with 187 more rows
```

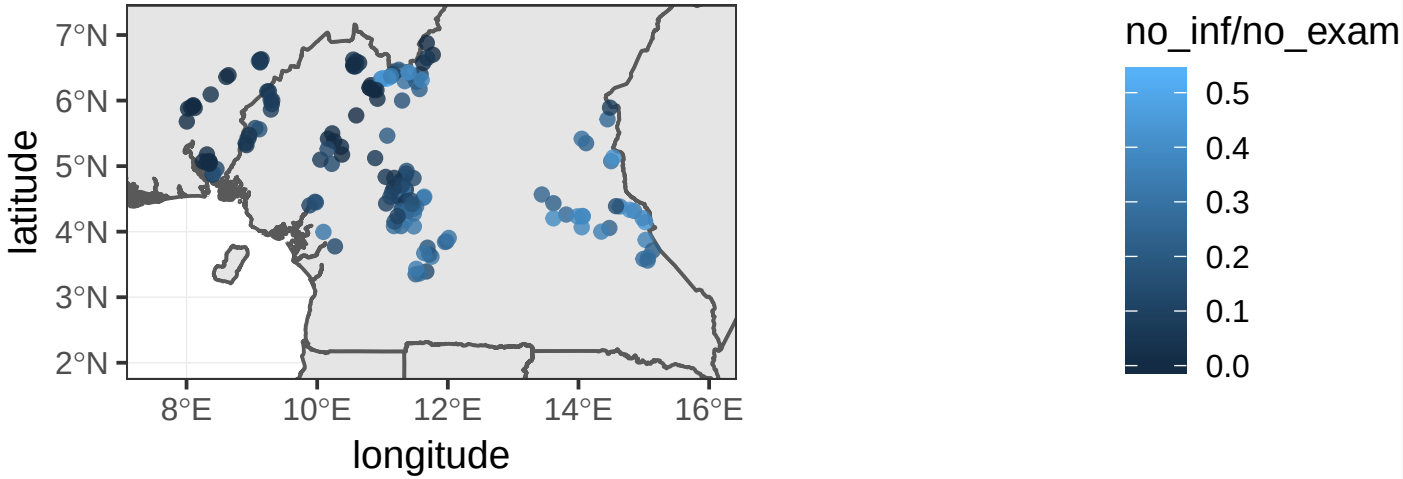
# Model Matrix {.}

```
model.matrix(
  ~ elev:elev_f - 1,
  data = loaloa
) %>%
  as_data_frame()
## # A tibble: 197 x 3
##    `elev:elev_f(0,1000]` `elev:elev_f(1000,1300]` `elev:elev_f(1300,2000]`
##                    <dbl>                    <dbl>                    <dbl>
##  1                   108                        0                        0
##  2                    99                        0                        0
##  3                   783                        0                        0
##  4                   104                        0                        0
##  5                   109                        0                        0
##  6                   909                        0                        0
##  7                   503                        0                        0
##  8                   103                        0                        0
##  9                   751                        0                        0
## 10                   268                        0                        0
## # ... with 187 more rows
```

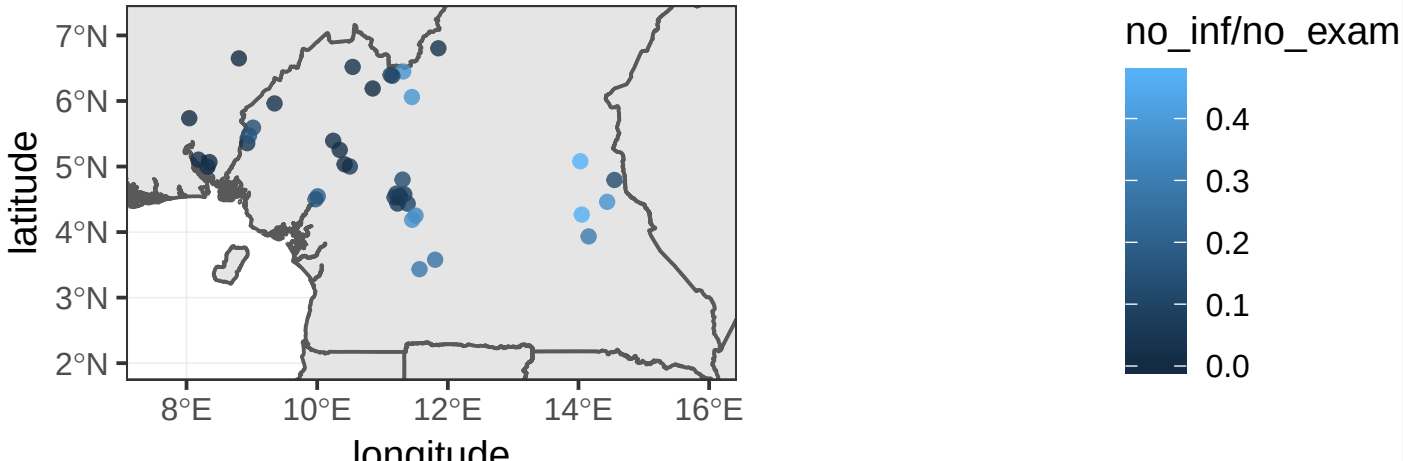
# OOS Validation

```
loaloa_test = loaloa %>% sample_frac(0.20)
loaloa = anti_join(loaloa, loaloa_test, quiet=TRUE)
```

Training




Testing

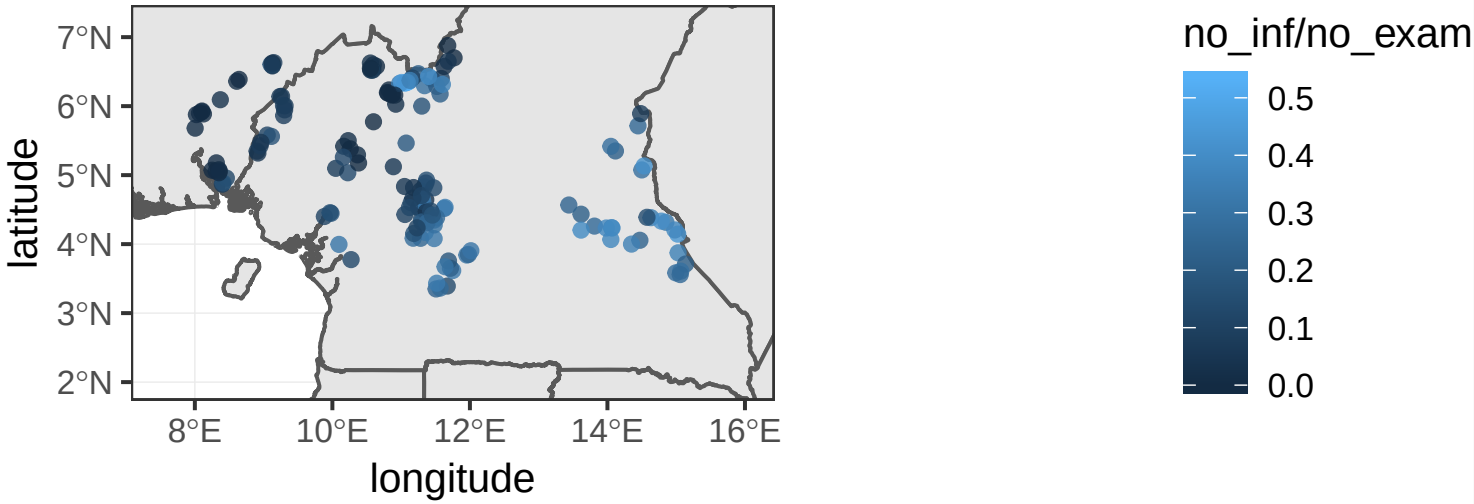


# Model

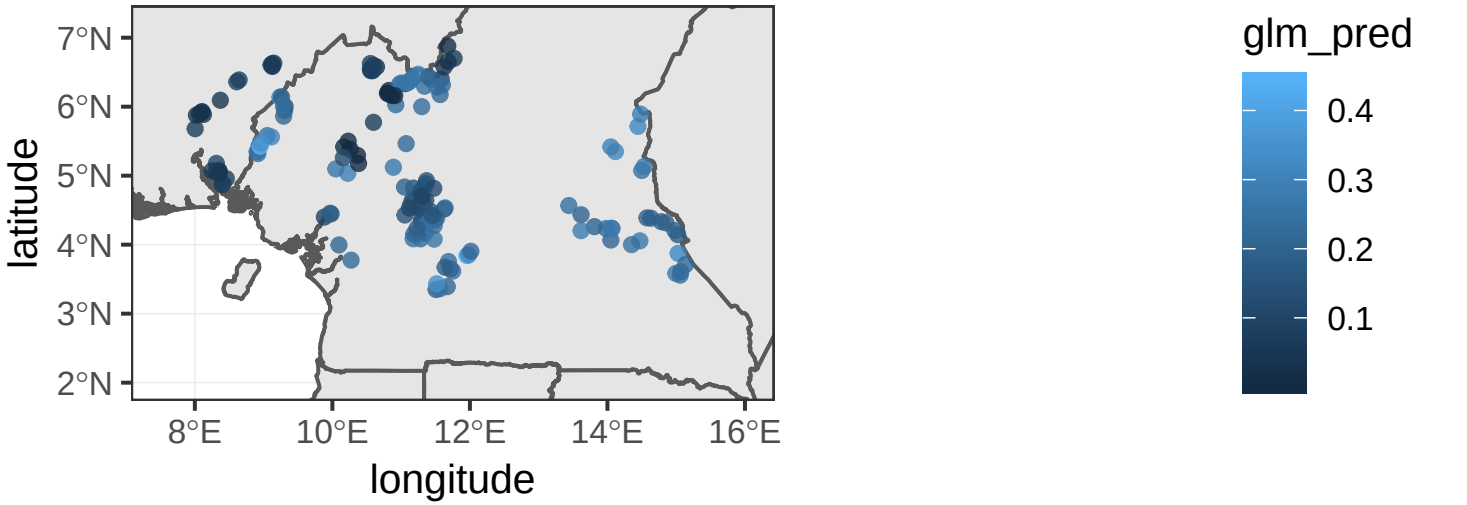
```
g = glm(no_inf/no_exam ~ elev:elev_f + max9901:max_f + stdev9901,
        data=loaloa, family=binomial, weights=loaloa$no_exam)

summary(g)
##
## Call:
## glm(formula = no_inf/no_exam ~ elev:elev_f + max9901:max_f +
##     stdev9901, family = binomial, data = loaloa, weights = loaloa$no_exam)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -6.9522  -2.5662  -0.4621   1.6720  10.1809
##
## Coefficients:
##                           Estimate Std. Error z value Pr(>|z|)
## (Intercept)             -8.5735389  0.5333413 -16.075  < 2e-16 ***
## stdev9901               11.9141737  1.3070028   9.116  < 2e-16 ***
## elev:elev_f(0,1000]      0.0015951  0.0001018  15.660  < 2e-16 ***
## elev:elev_f(1000,1300]   0.0003343  0.0000953   3.507 0.000453 ***
## elev:elev_f(1300,2000] -0.0016964  0.0002513  -6.750 1.48e-11 ***
## max9901:max_f(0,0.8]     5.2697375  0.6918702   7.617 2.60e-14 ***
## max9901:max_f(0.8,1]     5.2632126  0.6362108   8.273  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 3210.2  on 157  degrees of freedom
```

# Predictions - Training

## Data

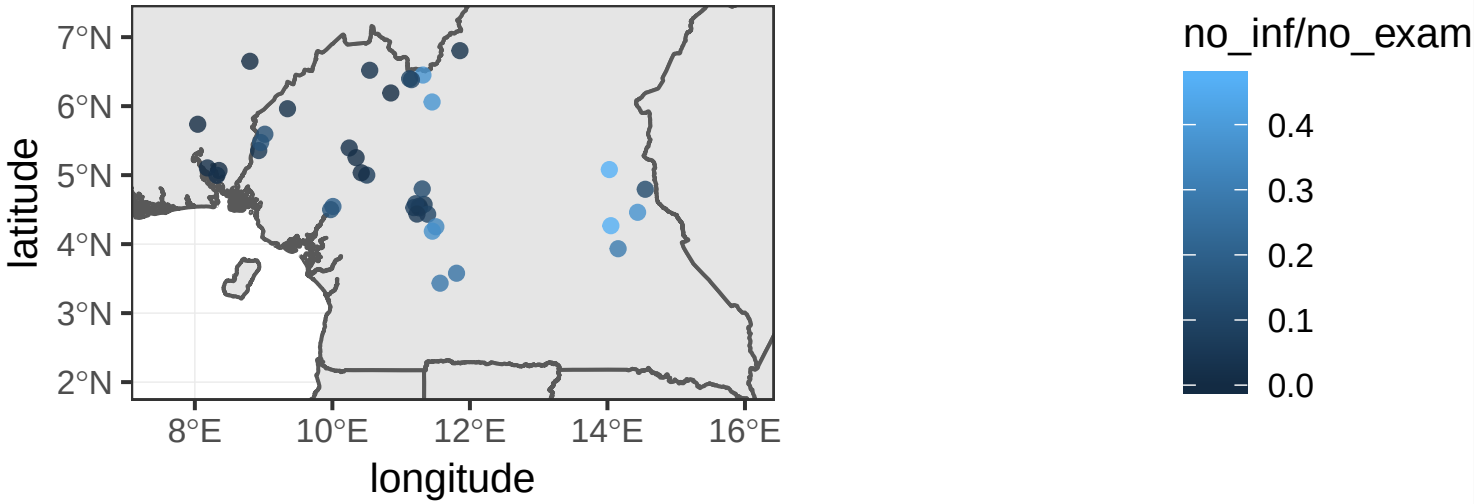



## GLM Prediction

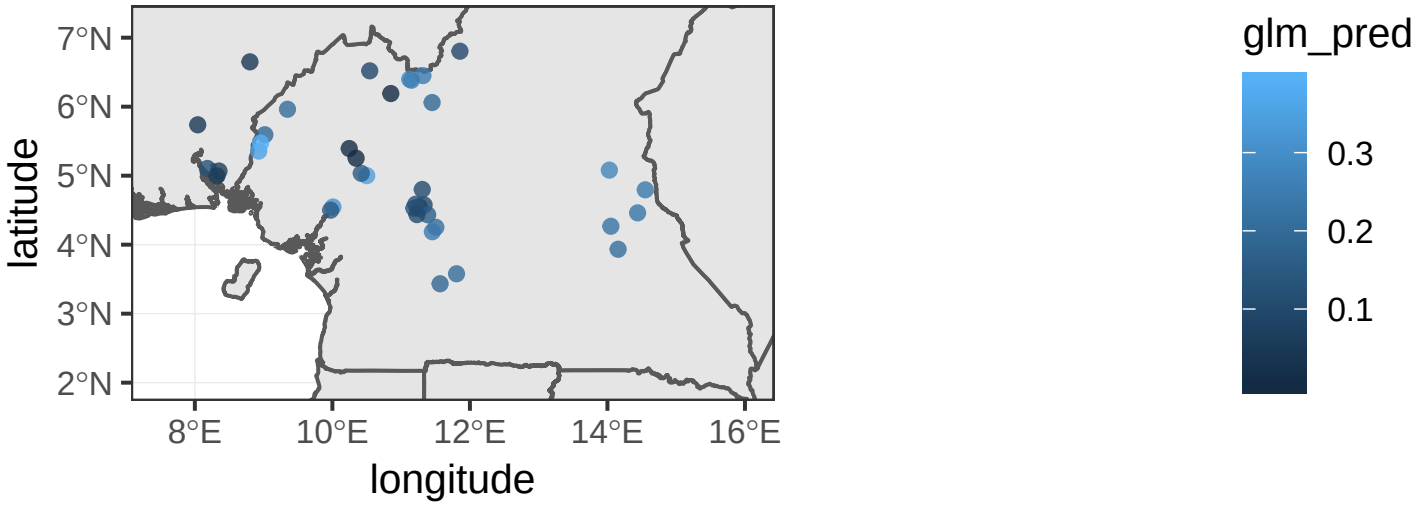


# Predictions - Testing

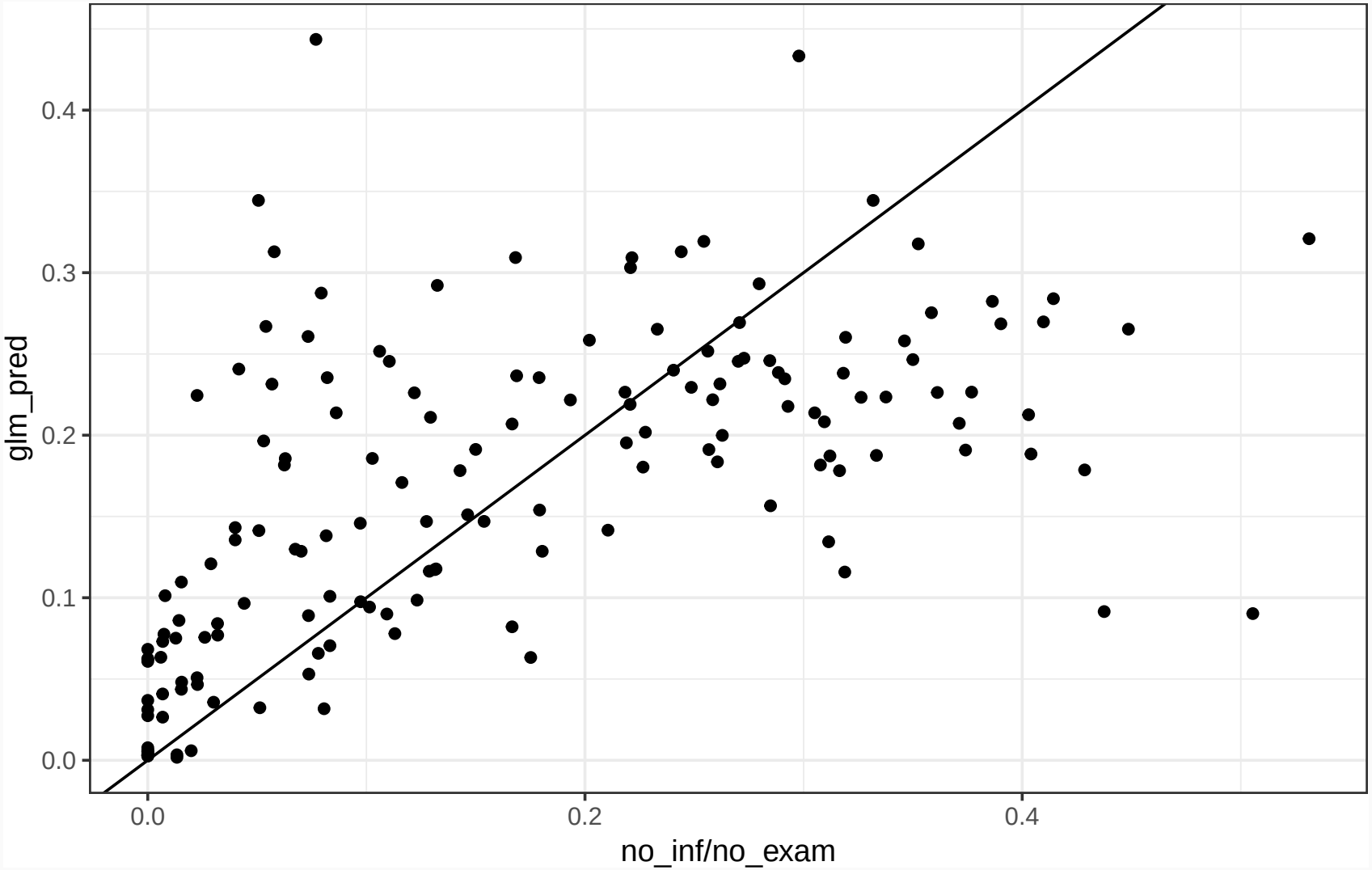
## Data



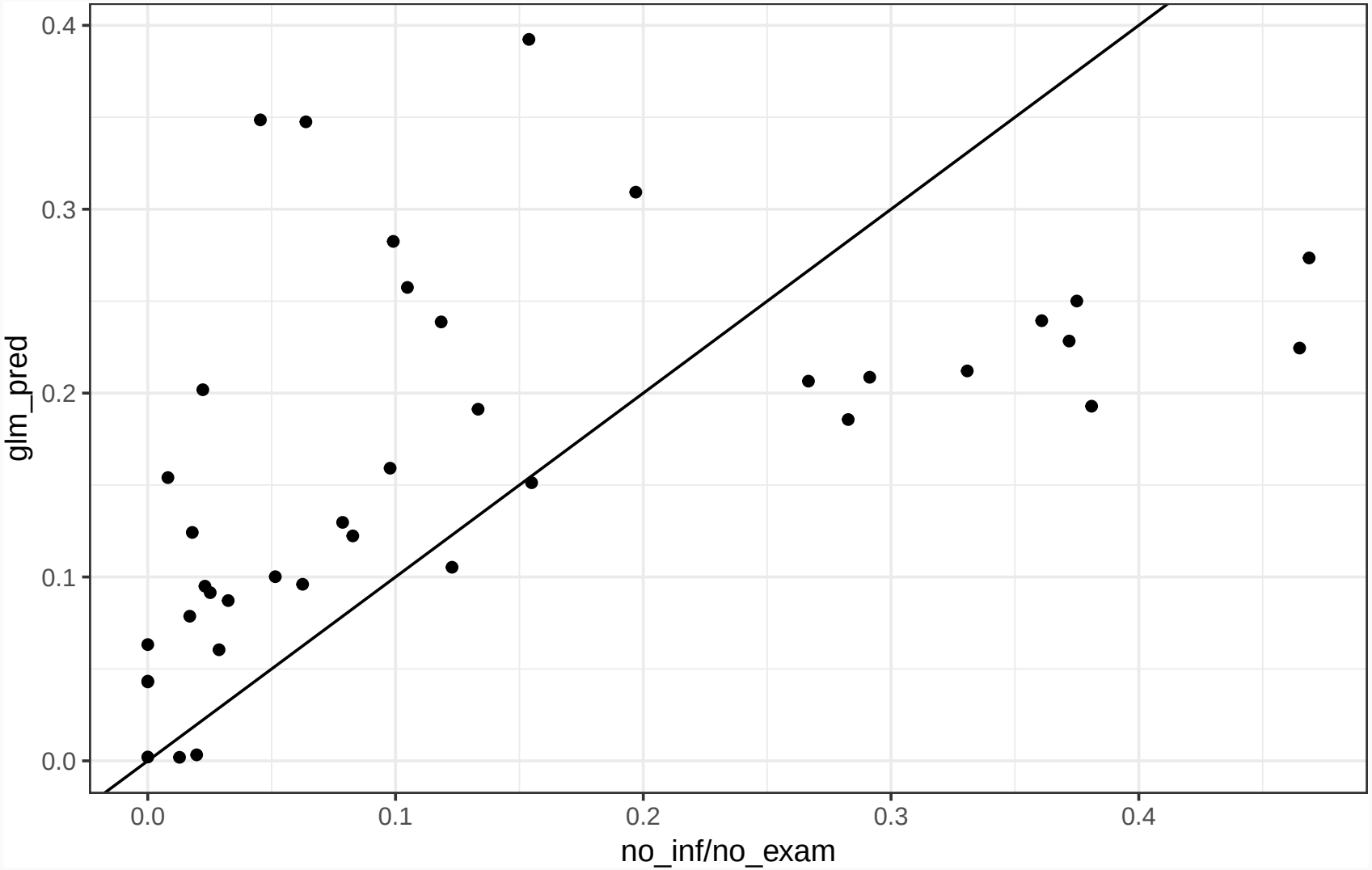

## GLM Prediction



# Fit - Training

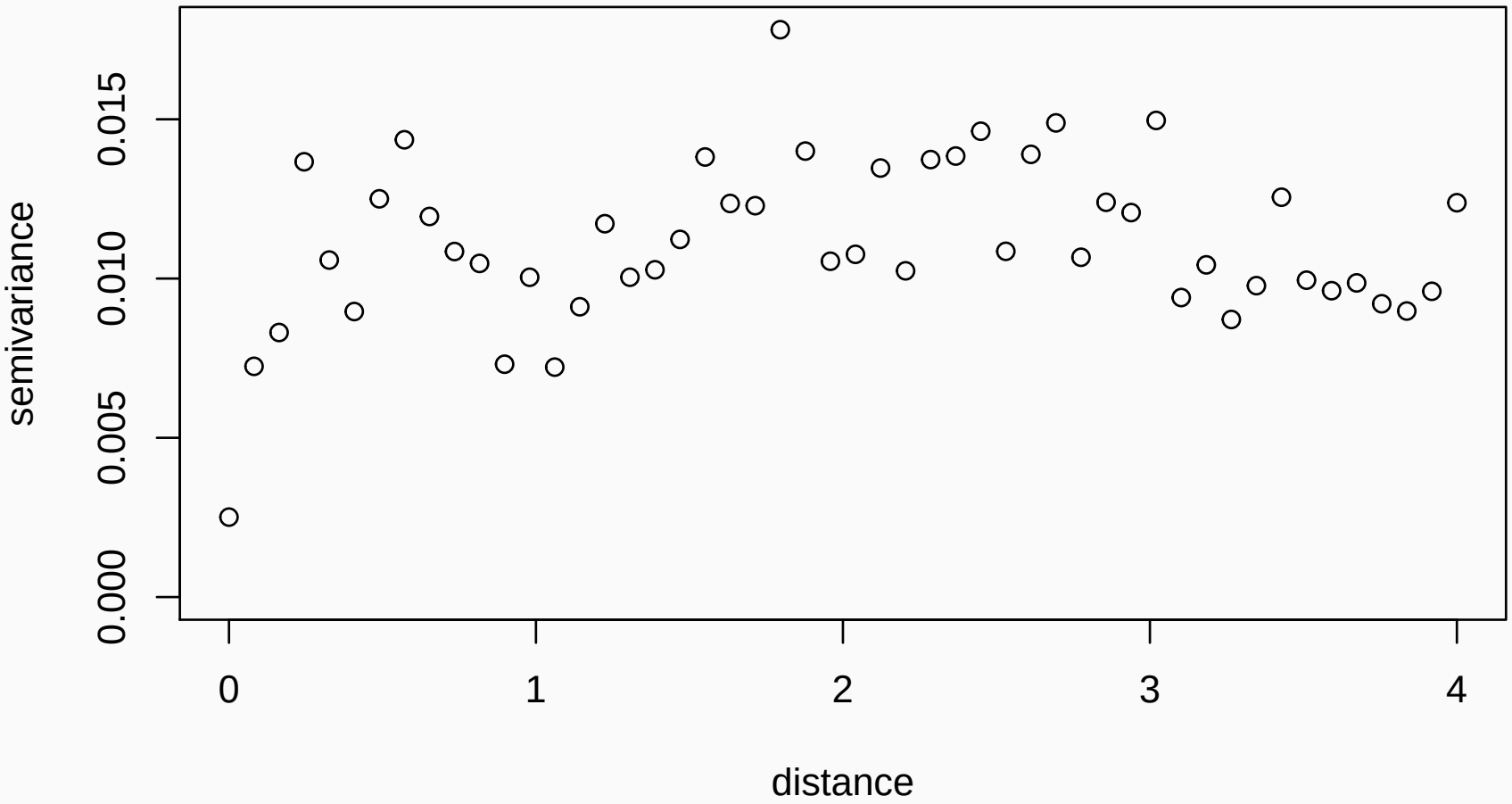


# Fit - Testing



# Spatial Structure?

```
geoR::variog(coords = cbind(loaloa$longitude, loaloa$latitude),
       data = loaloa$prop - loaloa$glm_pred,
       uvec = seq(0, 4, length.out = 50)) %>% plot()
## variog: computing omnidirectional variogram
```



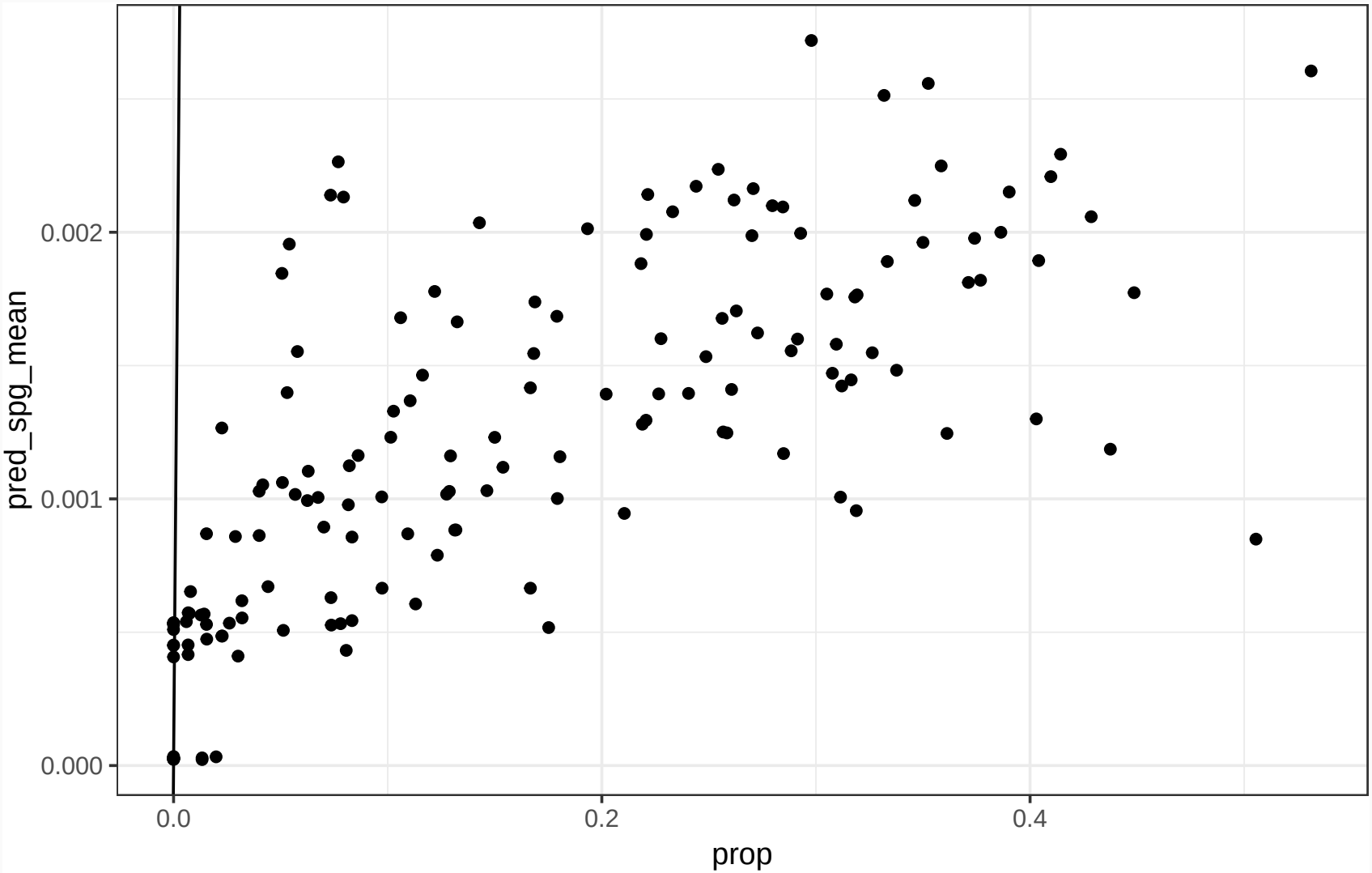
# spBayes GLM Model

```
spg = spBayes::spGLM(
  no_inf/no_exam ~ elev:elev_f + max9901:max_f + stdev9901,

  data=loaloa, family="binomial", weights=loaloa$no_exam,

  coords=cbind(loaloa$longitude, loaloa$latitude),

  cov.model="exponential", n.samples=20000,

  starting=list(beta=rep(0,7), phi=3, sigma.sq=1, w=0),

  priors=list(phi.unif=c(0.1, 10), sigma.sq.ig=c(2, 2)),

  amcmc=list(n.batch=1000, batch.length=20, accept.rate=0.43))

save(spg, loaloa, file="loaloa.Rdata")
```

```
spg$p.beta.theta.samples %>%
  post_summary() %>%
  knitr::kable(digits=5)
```

| param | post_mean | post_med | post_lower | post_upper |
|---|---|---|---|---|
| (Intercept) | -7.62467 | -7.10607 | -15.33201 | -1.56786 |
| stdev9901 | 1.77896 | -0.26705 | -19.15846 | 24.59887 |
| elev:elev_f(0,1000] | 0.00010 | 0.00065 | -0.00780 | 0.00316 |
| elev:elev_f(1000,1300] | -0.00059 | -0.00035 | -0.00471 | 0.00176 |
| elev:elev_f(1300,2000] | -0.01448 | -0.01064 | -0.04942 | -0.00030 |
| max9901:max_f(0,0.8] | 0.08517 | -0.78200 | -6.96111 | 9.06059 |
| max9901:max_f(0.8,1] | 0.69926 | -0.25813 | -5.79400 | 9.08833 |
| sigma.sq | 0.45277 | 0.39071 | 0.14322 | 1.17856 |
| phi | 2.12385 | 1.44856 | 0.12026 | 8.46872 |

# Prediction

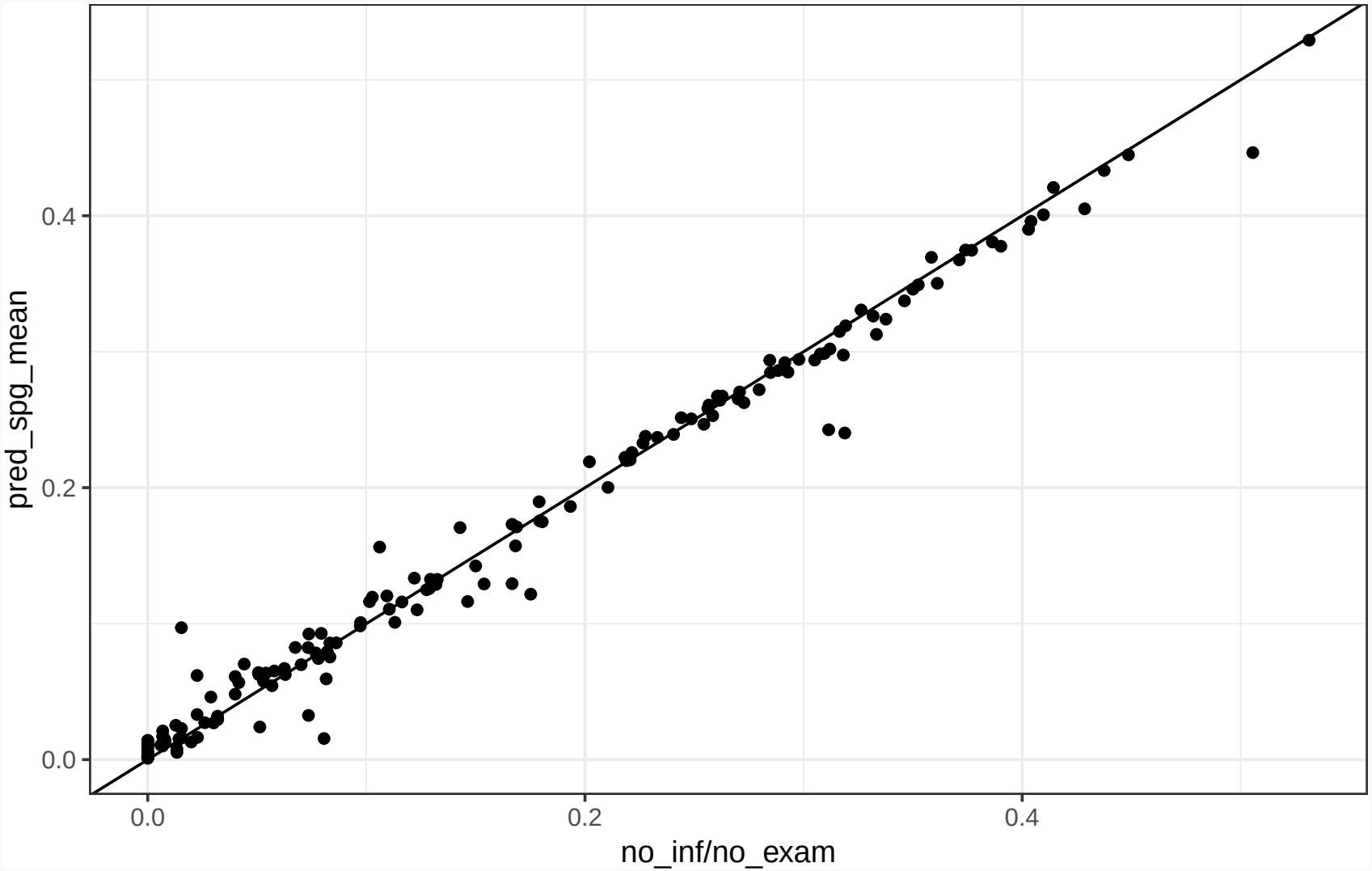


# spBayes GLM Model - Fixed?

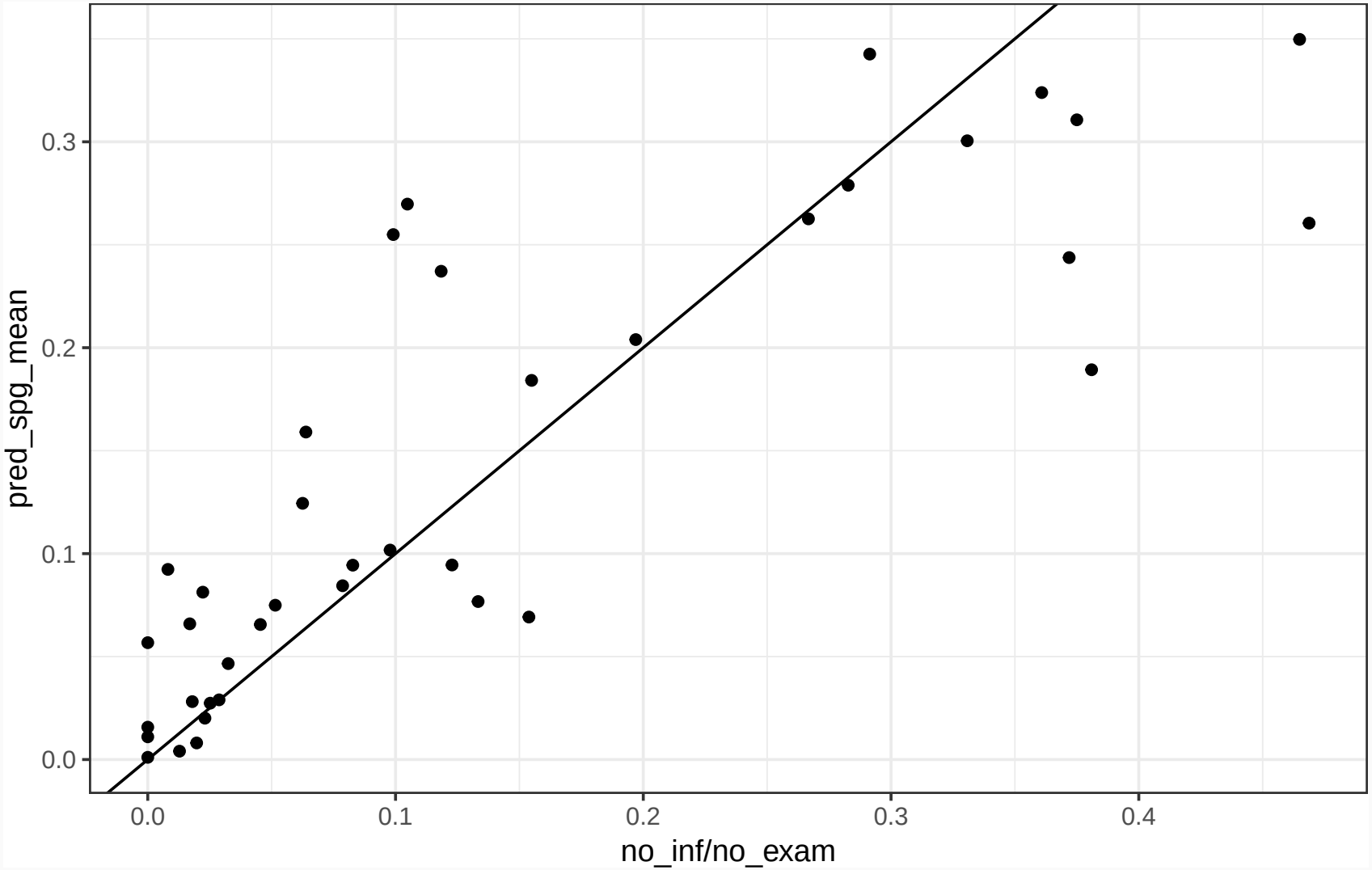
```
spg_fix = spBayes::spGLM(
  no_inf ~ elev:elev_f + max9901:max_f + stdev9901,

  data=loaloa, family="binomial", weights=loaloa$no_exam,

  coords=cbind(loaloa$longitude, loaloa$latitude),

  cov.model="exponential", n.samples=20000,

  starting=list(beta=rep(0,7), phi=3, sigma.sq=1, w=0),

  priors=list(phi.unif=c(0.1, 10), sigma.sq.ig=c(2, 2)),

  amcmc=list(n.batch=1000, batch.length=20, accept.rate=0.43)
)

save(spg_fix, loaloa, file="loaloa_fix.Rdata")
```

| param | post_mean | post_med | post_lower | post_upper |
|---|---|---|---|---|
| (Intercept) | -3.14223 | -3.43877 | -4.38140 | -1.01108 |
| stdev9901 | 1.88811 | 1.02957 | -5.28818 | 9.04674 |
| elev:elev_f(0,1000] | 0.00036 | 0.00048 | -0.00069 | 0.00114 |
| elev:elev_f(1000,1300] | -0.00036 | -0.00031 | -0.00127 | 0.00039 |
| elev:elev_f(1300,2000] | -0.00209 | -0.00206 | -0.00310 | -0.00131 |
| max9901:max_f(0,0.8] | 0.74129 | 0.55728 | -0.98971 | 2.78417 |
| max9901:max_f(0.8,1] | 1.15469 | 0.92740 | -0.18829 | 2.89406 |
| sigma.sq | 1.26052 | 1.21204 | 0.32891 | 2.36502 |
| phi | 2.51439 | 2.38441 | 1.08064 | 4.86766 |

# Fit - Training



# Fit - Testing



# Diggle's Predictive Surface

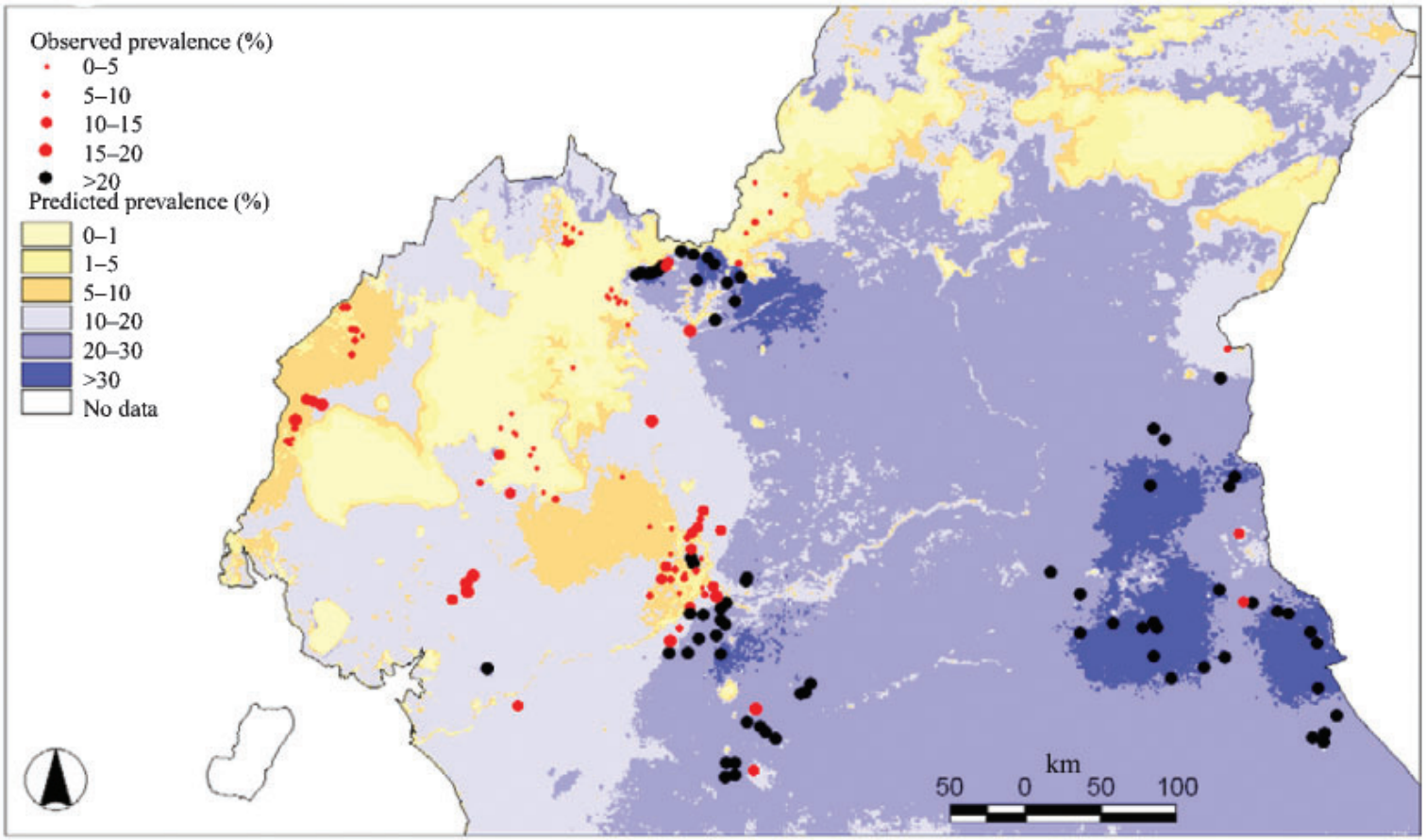
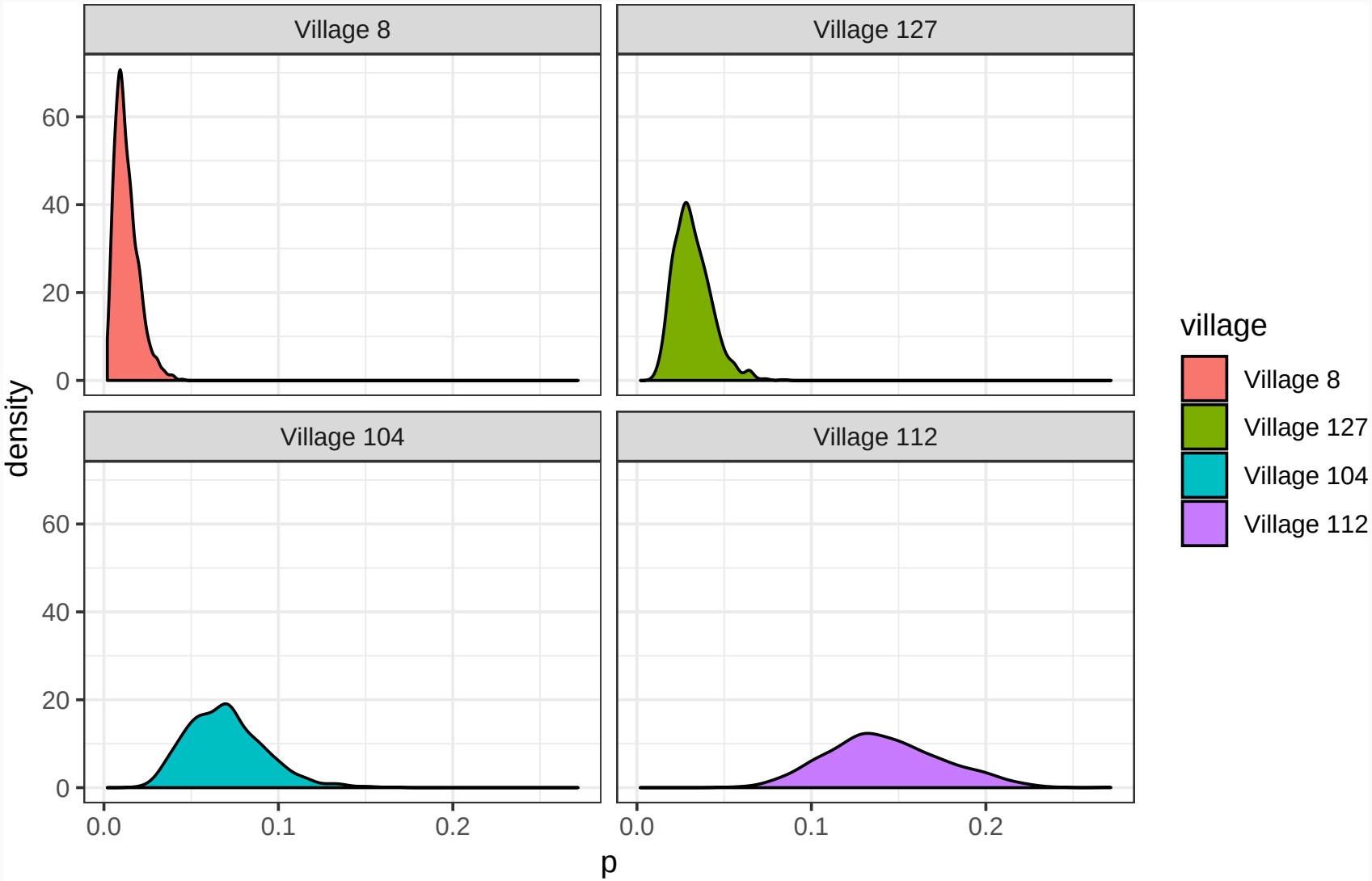



FIG. 2. Point estimates of the prevalence of *Loa loa* microfilaraemia, over-laid with the prevalences observed in field studies.

# Exceedance Probability - Posterior Summary



# Exceedance Probability Predictive Surface

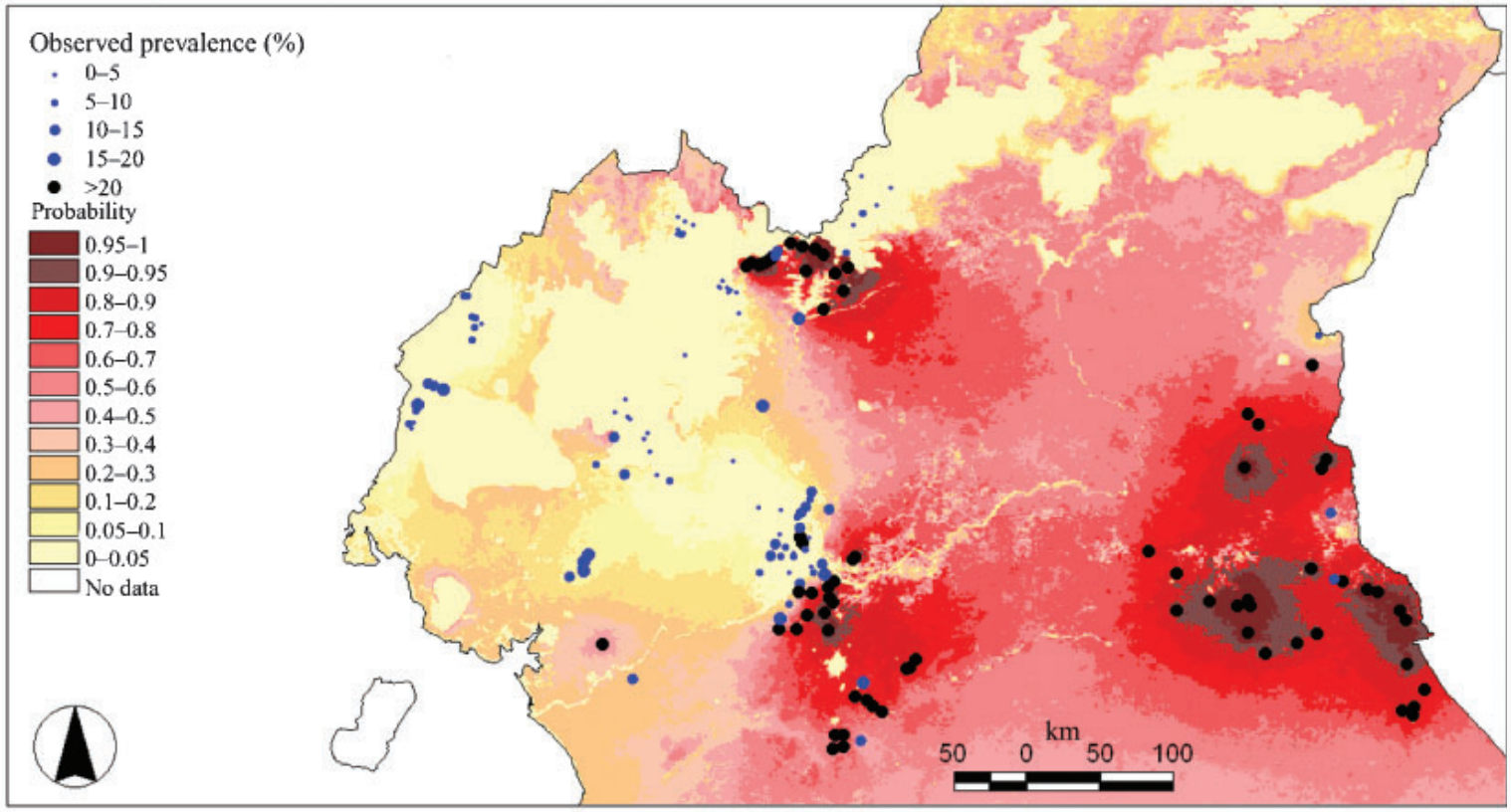



FIG. 4. A probability contour map, indicating the probability that the prevalence of *Loa loa* microfilaraemia in each area exceeds 20%, over-laid with the prevalences observed in field studies.

# Spatial Assignment of Migratory Birds

# Background

Using intrinsic markers (genetic and isotopic signals) for the purpose of inferring migratory connectivity.

- Existing methods are too coarse for most applications
- Large amounts of data are available ( >150,000 feather samples from >500 species)
- Genetic assignment methods are based on Wasser, et al. (2004)
- Isotopic assignment methods are based on Wunder, et al. (2005)

# Data - DNA microsatellites and $\delta^2\mathrm{H}$

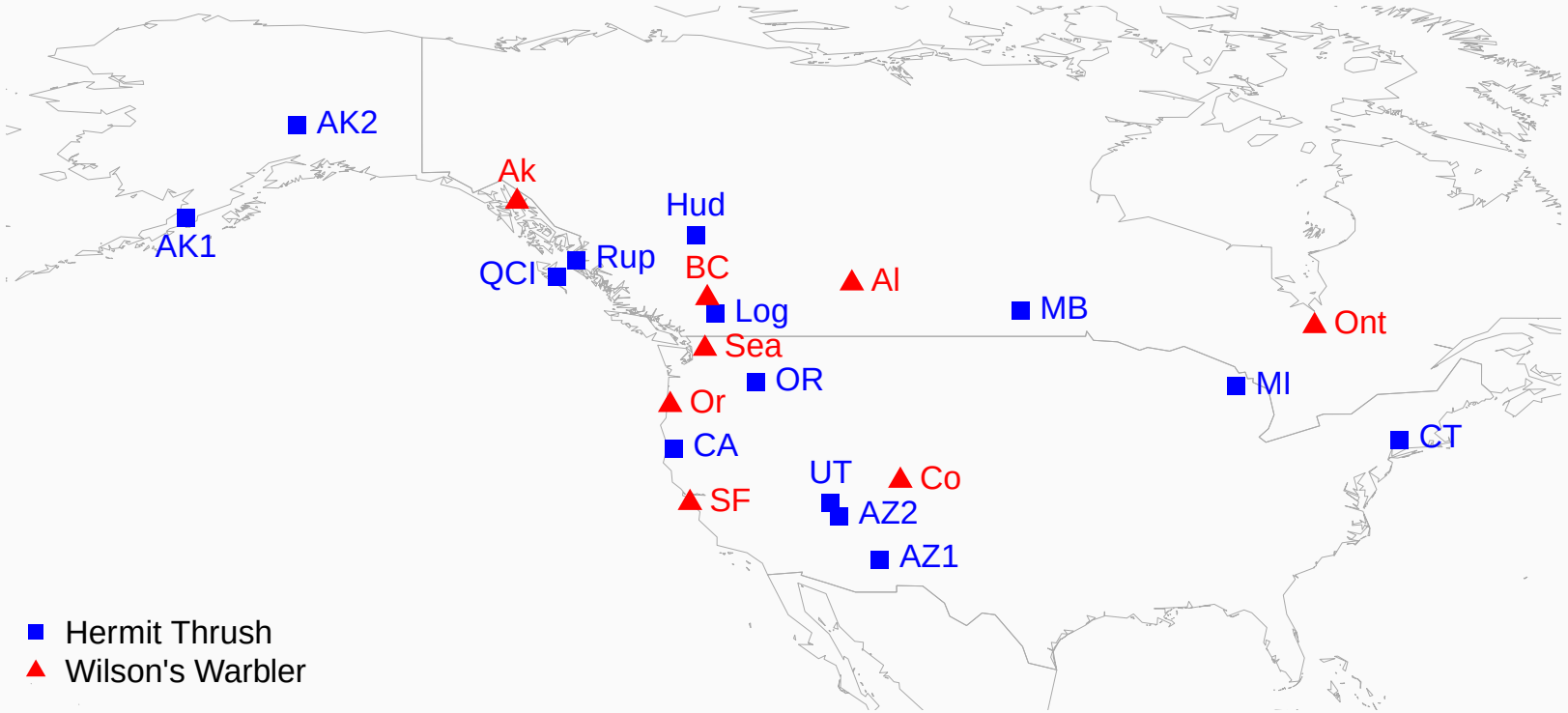
Hermit Thrush (*Catharus guttatus*)

- 138 individuals
- 14 locations
- 6 loci
- 9-27 alleles / locus


Wilson's Warbler (*Wilsonia pusilla*)

- 163 individuals
- 8 locations
- 9 loci
- 15-31 alleles / locus

# Sampling Locations



# Allele Frequency Model

For the allele $i$, from locus $l$, at location $k$

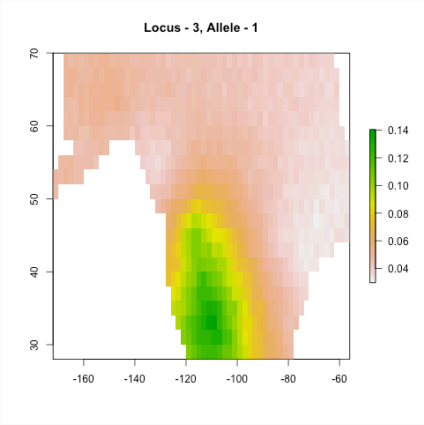
$$\mathbf{y}_{\cdot lk} | \boldsymbol{\Theta} \sim \mathcal{N}\left(\textstyle\sum_i y_{ilk},\ \mathbf{f}_{\cdot lk}\right)$$

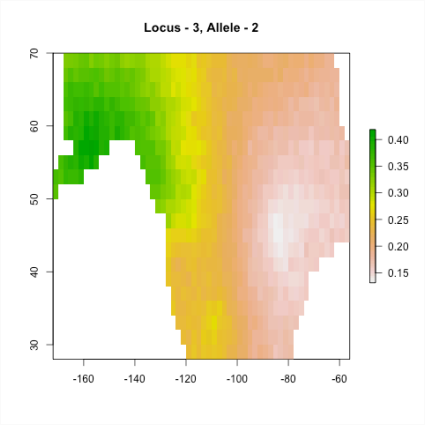
$$f_{ilk} = \frac{\exp(\Theta_{ilk})}{\sum_i \exp(\Theta_{ilk})}$$

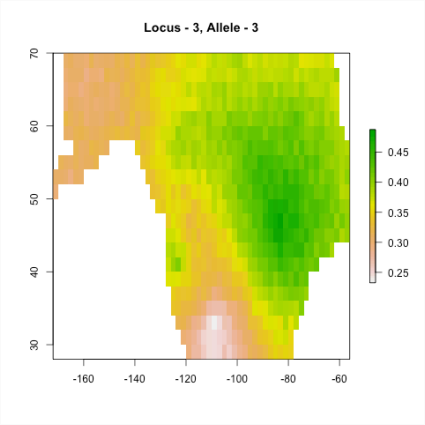
$$\boldsymbol{\Theta}_{il} | \boldsymbol{\alpha}, \boldsymbol{\mu} \sim \mathcal{N}(\boldsymbol{\mu}_{il},\ \boldsymbol{\Sigma})$$

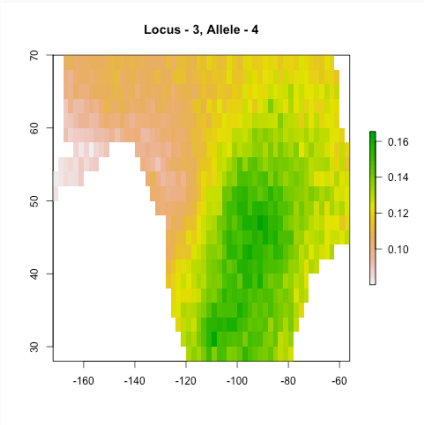
$$\{\Sigma\}_{ij} = \sigma^2\ \exp\Big( -\left(\{d\}_{ij}\, r\right)^{\psi}\Big) + \sigma_n^2\, 1_{i=j}$$

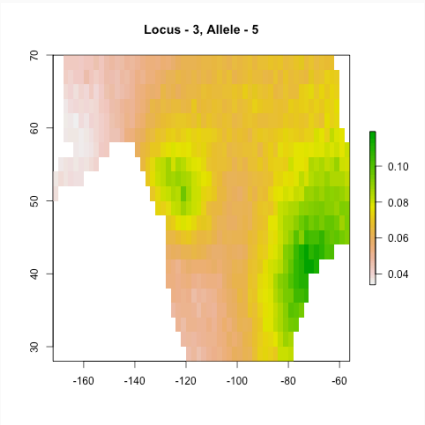
# Predictions by Allele (Locus 3)

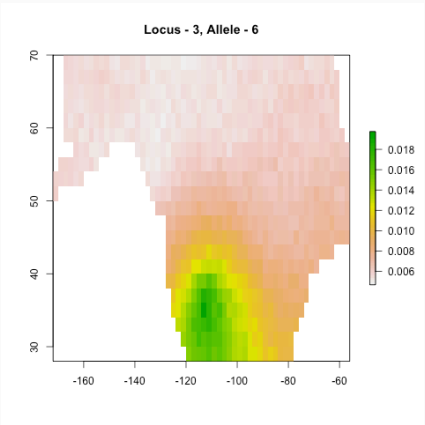


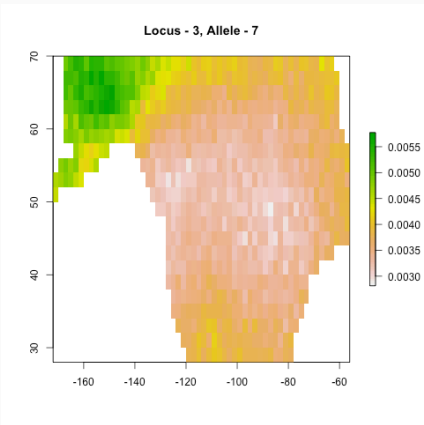



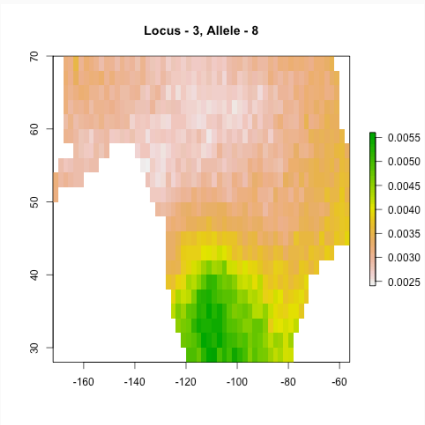



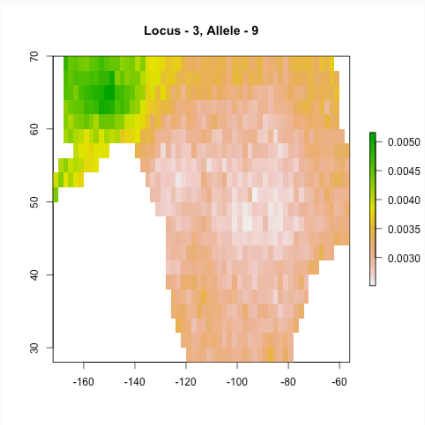



# Genetic Assignment Model

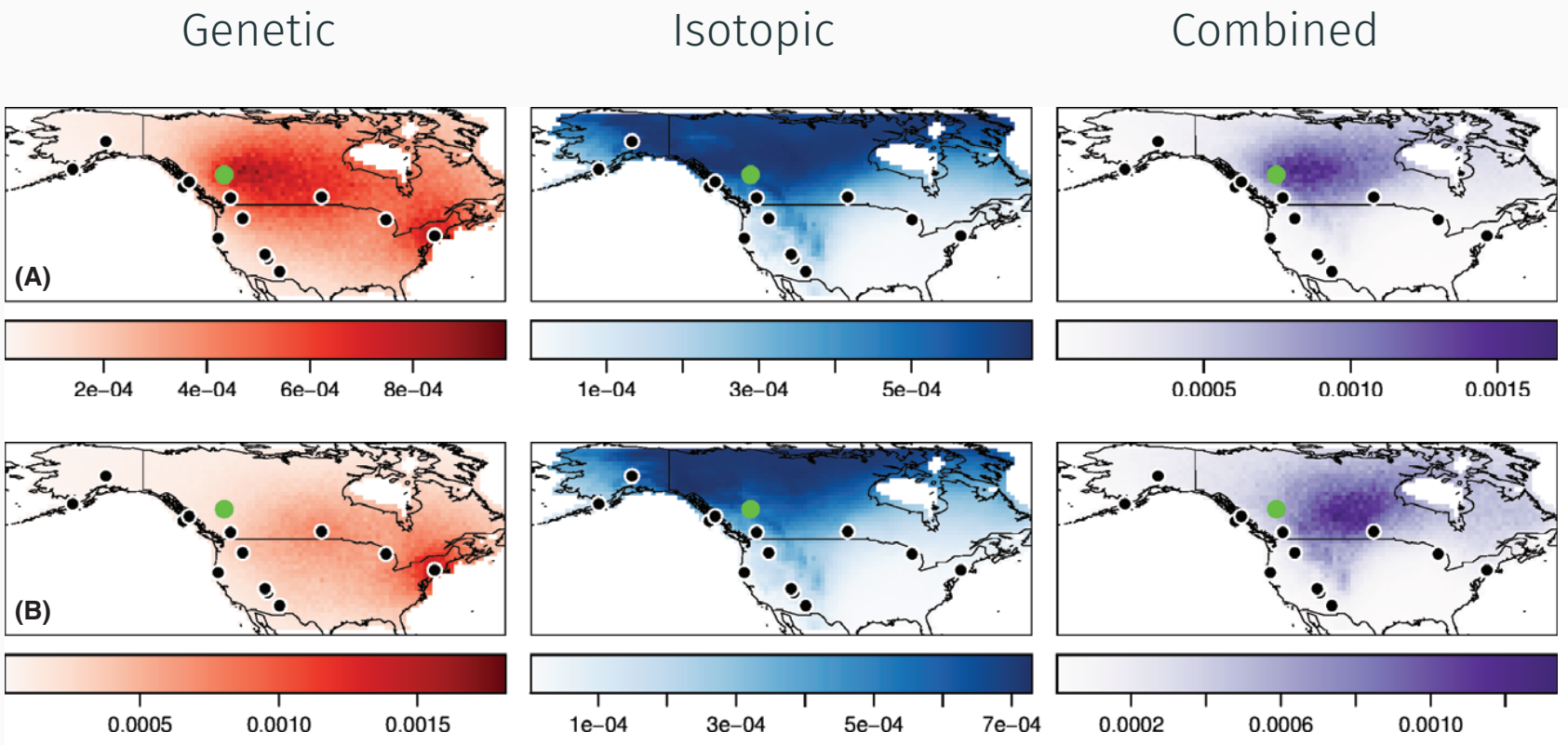
Assignment model assuming Hardy-Weinberg equilibrium and allowing for genotyping ($\delta$) and single amplification ($\gamma$) errors.

$$P(S_G|\mathbf{f},k) = \prod_l P(i_l, j_l|\mathbf{f},k)$$

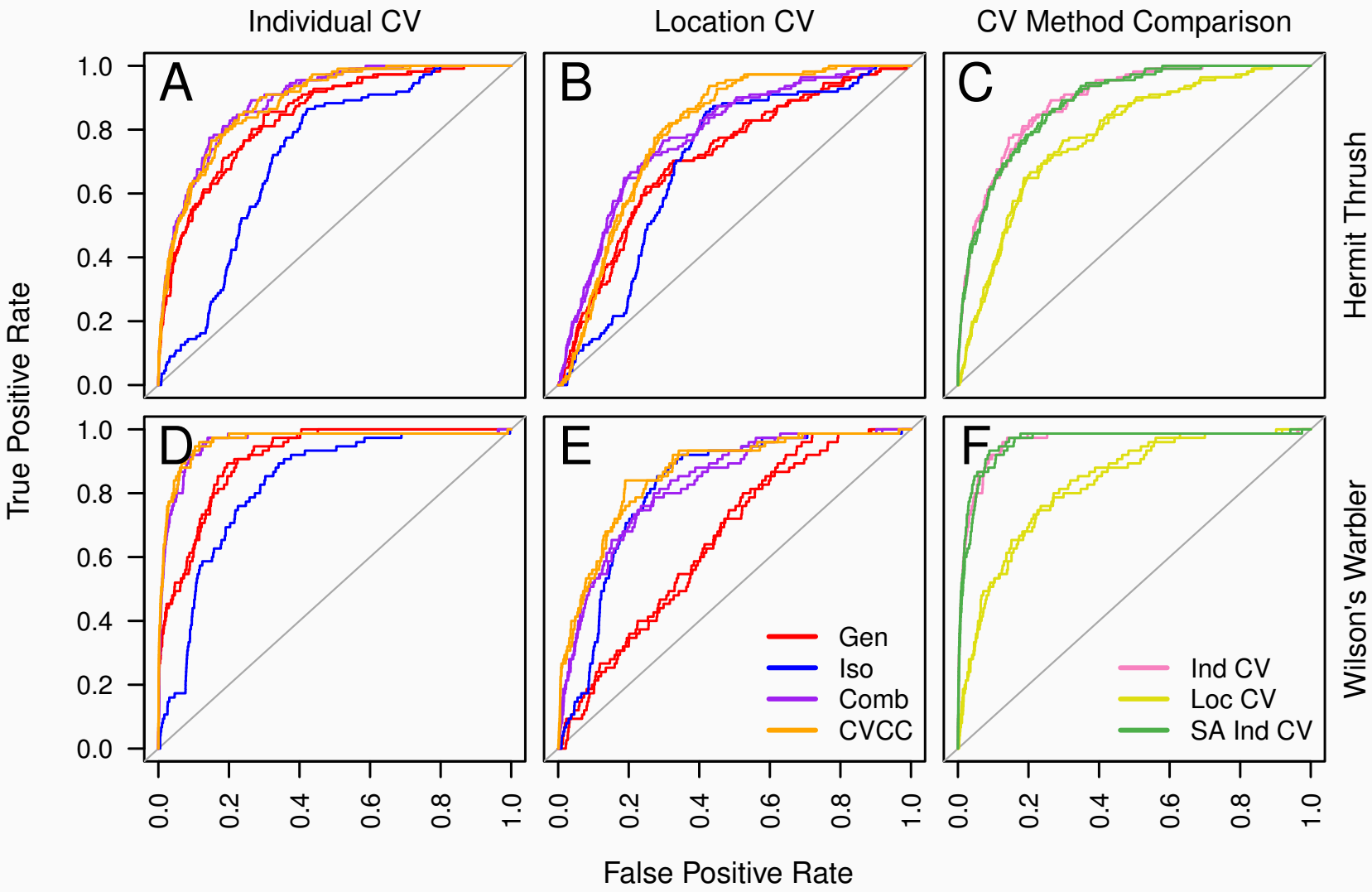
$$P(i_l, j_l|\mathbf{f},k) = \begin{cases} \gamma P(i_l|\mathbf{f},k) + (1-\gamma)P(i_l|\tilde{\mathbf{f}},k)^2 & \text{if } i = j \\ (1-\gamma)P(i_l|\mathbf{f},k)P(j_l|\mathbf{f},k) & \text{if } i \neq j \end{cases}$$

$$P(i_l|\mathbf{f},k) = (1-\delta)f_{lik} + \delta/m_l$$

# Combined Model



# Model Assessment



# Migratory Connectivity