Lecture 11

Seasonal ARIMA

Colin Rundel

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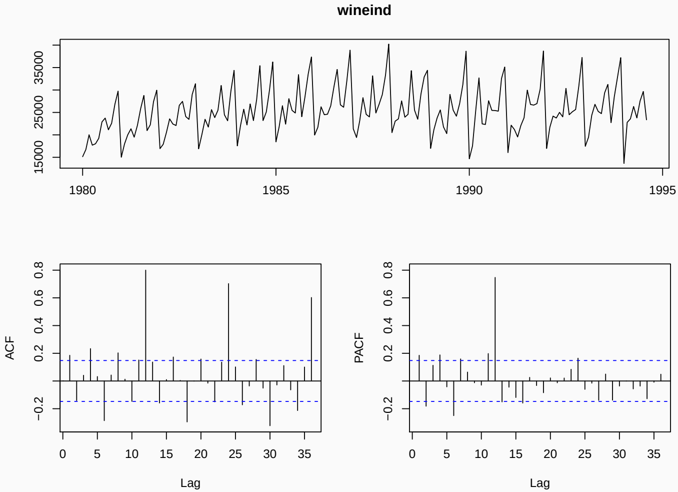
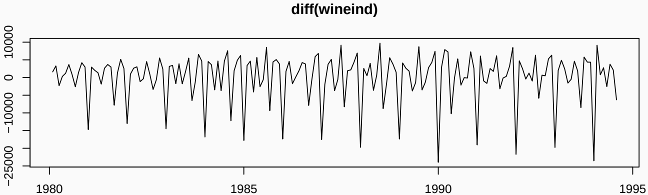
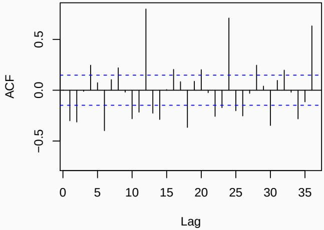
Seasonal Models

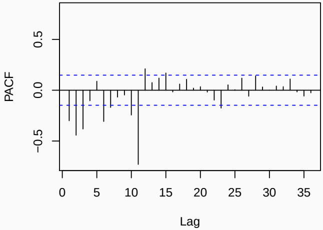
Australian total wine sales by wine makers in bottles <= 1 litre. Jan 1980 – Aug 1994.



Differencing







We can extend the existing ARIMA model to handle these higher order lags (without having to include all of the intervening lags).

Seasonal ARIMA $(p, d, q) \times (P, D, Q)_s$:

$$\Phi_P(L^s)\,\phi_p(L)\,\Delta_s^D\,\Delta^d\,y_t = \delta + \Theta_Q(L^s)\,\theta_q(L)\,w_t$$

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$$\Phi_P(L^s)\,\phi_p(L)\,\Delta_s^D\,\Delta^d\,y_t = \delta + \Theta_Q(L^s)\,\theta_q(L)\,w_t$$

where

$$\begin{aligned}\phi_p(L) &= 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p \\ \theta_q(L) &= 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_p L^q \\ \Delta^d &= (1 - L)^d\end{aligned}$$

$$\begin{aligned}\Phi_P(L^s) &= 1 - \Phi_1 L^s - \Phi_2 L^{2s} - \ldots - \Phi_P L^{Ps} \\ \Theta_Q(L^s) &= 1 + \Theta_1 L + \Theta_2 L^{2s} + \ldots + \theta_p L^{Qs} \\ \Delta_s^D &= (1 - L^s)^D\end{aligned}$$

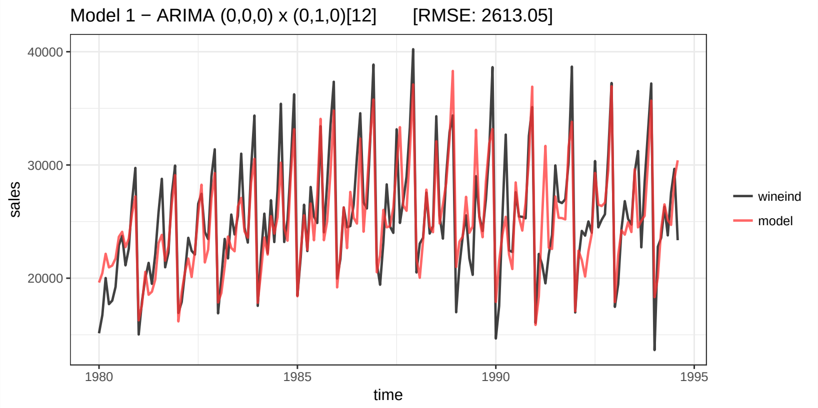
Seasonal ARIMA for `wineind`

Lets consider an ARIMA$(0,0,0) \times (1,0,0)_{12}$:

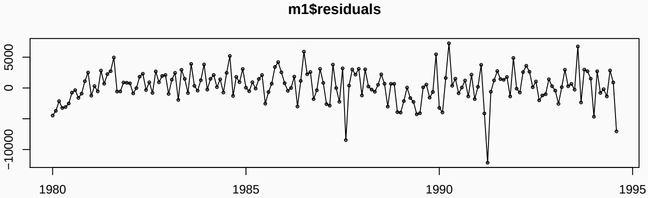
$$
\begin{aligned}
\left(1 - \Phi_1 L^{12}\right) y_t &= \delta + w_t \\
y_t &= \Phi_1 y_{t-12} + \delta + w_t
\end{aligned}
$$

```
(m1 = Arima(wineind, seasonal=list(order=c(1,0,0), period=12)))
## Series: wineind
## ARIMA(0,0,0)(1,0,0)[12] with non-zero mean
##
## Coefficients:
##          sar1       mean
##        0.8780  24489.243
## s.e.   0.0314   1154.487
##
## sigma^2 estimated as 6906536:  log likelihood=-1643.39
## AIC=3292.78   AICc=3292.92   BIC=3302.29
```

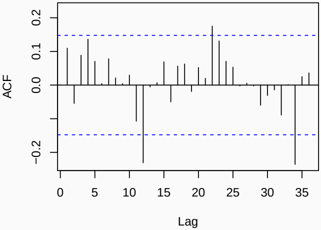
Fitted model



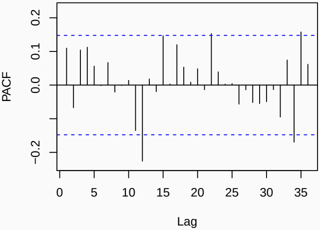
Residuals



m1$residuals
5000
0
−10000
1980
1985
1990
1995



ACF
0.2
0.1
0.0
−0.2
0
5
10
15
20
25
30
35
Lag



PACF
0.2
0.1
0.0
−0.2
0
5
10
15
20
25
30
35
Lag

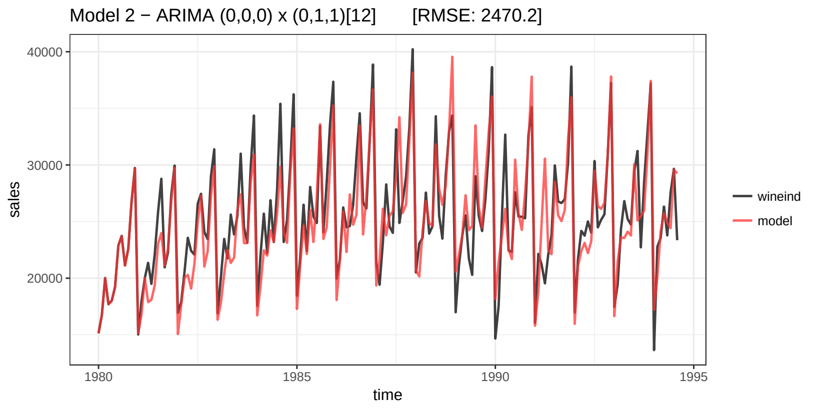
Model 2

ARIMA$(0,0,0) \times (0,1,1)_{12}$:

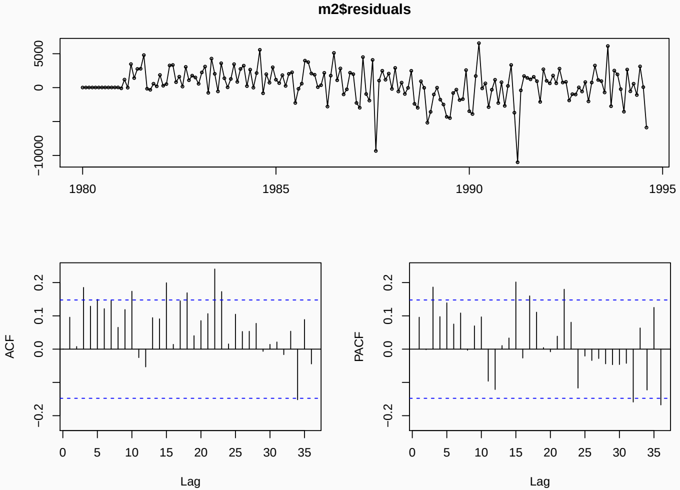
$$
\begin{aligned}
(1 - L^{12}) y_t &= \delta + (1 + \Theta_1 L^{12}) w_t \\
y_t - y_{t-12} &= \delta + w_t + \Theta_1 w_{t-12} \\
y_t &= \delta + y_{t-12} + w_t + \Theta_1 w_{t-12}
\end{aligned}
$$

```
(m2 = Arima(wineind, order=c(0,0,0), seasonal=list(order=c(0,1,1), period=12
## Series: wineind
## ARIMA(0,0,0)(0,1,1)[12]
##
## Coefficients:
##          sma1
##       -0.3246
## s.e.   0.0807
##
## sigma^2 estimated as 6588531:  log likelihood=-1520.34
## AIC=3044.68   AICc=3044.76   BIC=3050.88
```

Fitted model



Residuals



m2$residuals
5000
0
−10000
1980
1985
1990
1995
ACF
0.2
0.1
0.0
−0.2
0
5
10
15
20
25
30
35
Lag
PACF
0.2
0.1
0.0
−0.2
0
5
10
15
20
25
30
35
Lag

Model 3

ARIMA$(3,0,0) \times (0,1,1)_{12}$

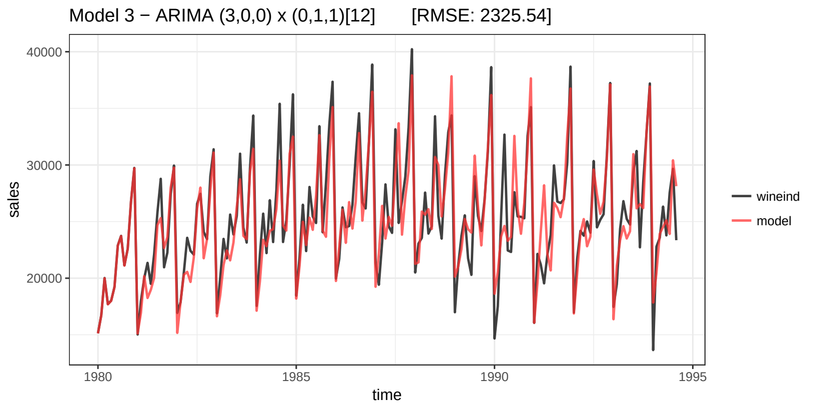
$$(1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3)\,(1 - L^{12}) y_t = \delta + (1 + \Theta_1 L) w_t$$

$$(1 - \phi_1 L - \phi_2 L^2 - \phi_3 L^3)\,(y_t - y_{t-12}) = \delta + w_t + w_{t-12}$$

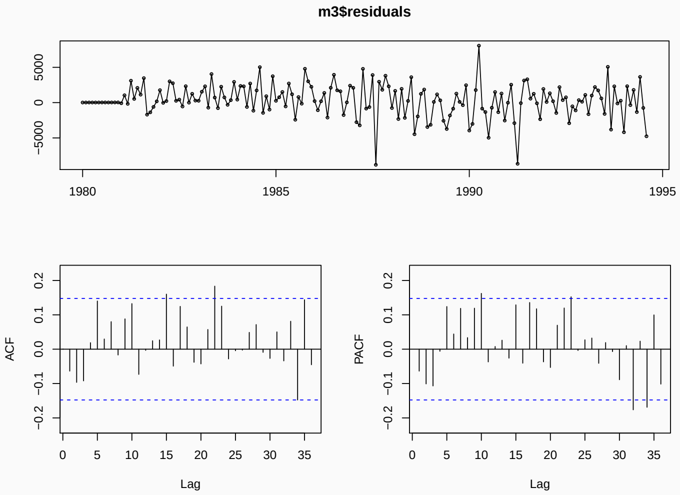
$$y_t = \delta + \sum_{i=1}^{3} \phi_i y_{t-1} + y_{t-12} - \sum_{i=1}^{3} \phi_i y_{t-12-i} + w_t + w_{t-12}$$

```r
(m3 = Arima(wineind, order=c(3,0,0),
            seasonal=list(order=c(0,1,1), period=12)))
## Series: wineind
## ARIMA(3,0,0)(0,1,1)[12]
##
## Coefficients:
##          ar1     ar2     ar3     sma1
##       0.1402  0.0806  0.3040  -0.5790
## s.e.  0.0755  0.0813  0.0823   0.1023
##
## sigma^2 estimated as 5948935:  log likelihood=-1512.38
## AIC=3034.77   AICc=3035.15   BIC=3050.27
```

Fitted model



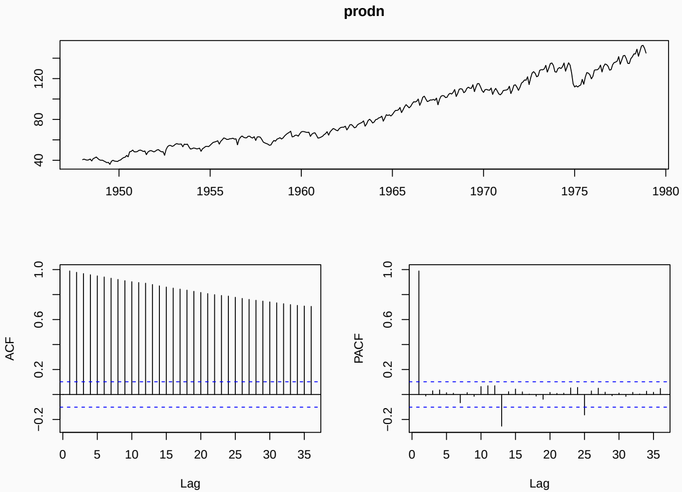
Model - Residuals



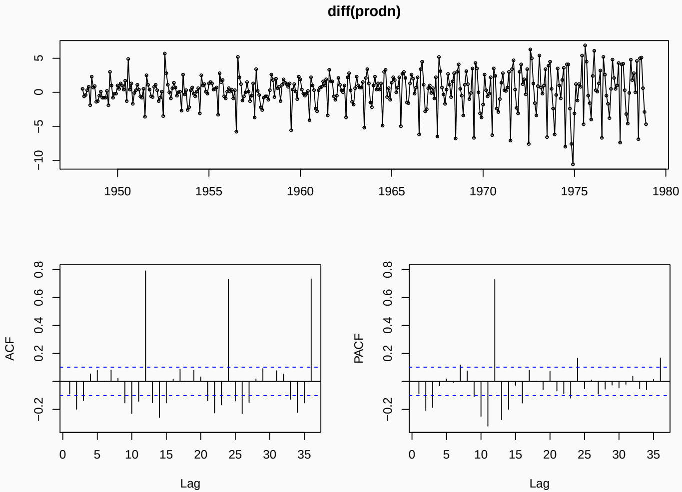
Federal Reserve Board Production Index

`prodn` from the `astsa` package

Monthly Federal Reserve Board Production Index (1948-1978)



Based on the ACF it seems like standard differencing may be required



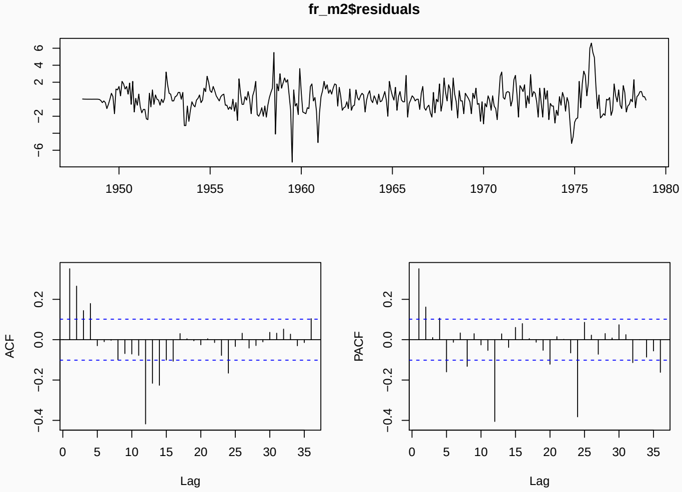
Differencing + Seasonal Differencing

Additional seasonal differencing also seems warranted

```
(fr_m1 = Arima(prodn, order = c(0,1,0),
            seasonal = list(order=c(0,0,0), period=12)))
## Series: prodn
## ARIMA(0,1,0)
##
## sigma^2 estimated as 7.147:  log likelihood=-891.26
## AIC=1784.51   AICc=1784.52   BIC=1788.43
```

```
(fr_m2 = Arima(prodn, order = c(0,1,0),
            seasonal = list(order=c(0,1,0), period=12)))
## Series: prodn
## ARIMA(0,1,0)(0,1,0)[12]
##
## sigma^2 estimated as 2.52:  log likelihood=-675.29
## AIC=1352.58   AICc=1352.59   BIC=1356.46
```

Residuals



fr_m2$residuals
6
4
2
−2
−6
1950
1955
1960
1965
1970
1975
1980
ACF
0.2
0.0
−0.2
−0.4
0
5
10
15
20
25
30
35
Lag
PACF
0.2
0.0
−0.2
−0.4
0
5
10
15
20
25
30
35
Lag

Adding Seasonal MA

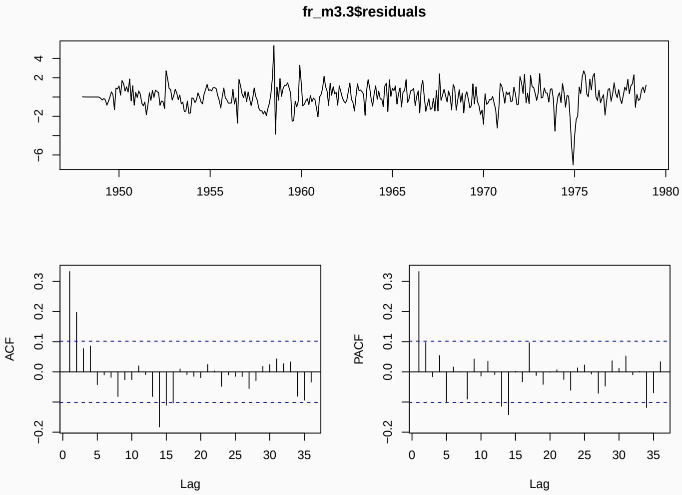
```
(fr_m3.1 = Arima(prodn, order = c(0,1,0),
            seasonal = list(order=c(0,1,1), period=12)))
## Series: prodn
## ARIMA(0,1,0)(0,1,1)[12]
##
## Coefficients:
##          sma1
##       -0.7151
## s.e.   0.0317
##
## sigma^2 estimated as 1.616:  log likelihood=-599.29
## AIC=1202.57   AICc=1202.61   BIC=1210.34
```

```
(fr_m3.2 = Arima(prodn, order = c(0,1,0),
            seasonal = list(order=c(0,1,2), period=12)))
## Series: prodn
## ARIMA(0,1,0)(0,1,2)[12]
##
## Coefficients:
##          sma1    sma2
##       -0.7624  0.0520
## s.e.   0.0689  0.0666
##
## sigma^2 estimated as 1.615:  log likelihood=-598.98
## AIC=1203.96   AICc=1204.02   BIC=1215.61
```

Adding Seasonal MA (cont.)

```
(fr_m3.3 = Arima(prodn, order = c(0,1,0),
           seasonal = list(order=c(0,1,3), period=12)))
## Series: prodn
## ARIMA(0,1,0)(0,1,3)[12]
##
## Coefficients:
##          sma1     sma2    sma3
##       -0.7853  -0.1205  0.2624
## s.e.   0.0529   0.0644  0.0529
##
## sigma^2 estimated as 1.506:  log likelihood=-587.58
## AIC=1183.15   AICc=1183.27   BIC=1198.69
```

Residuals - Model 3.3

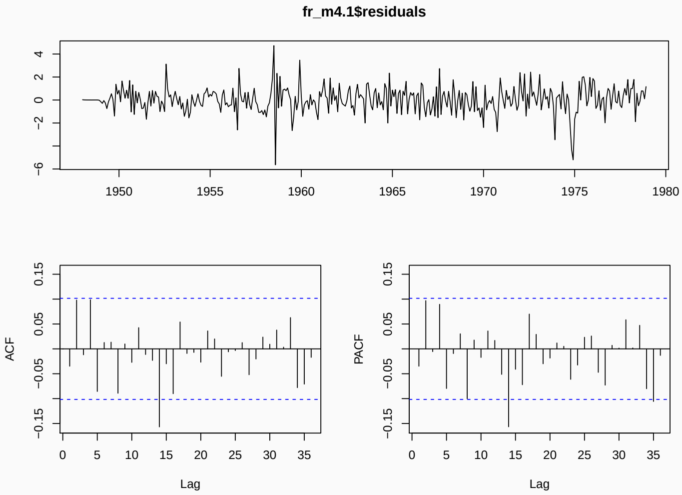


Adding AR

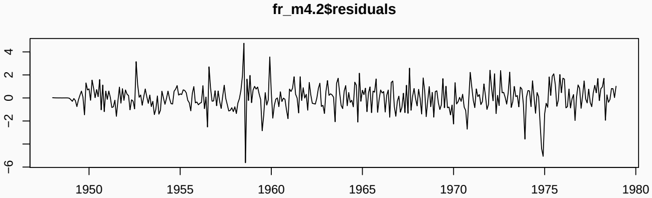
```
(fr_m4.1 = Arima(prodn, order = c(1,1,0),
            seasonal = list(order=c(0,1,3), period=12)))
## Series: prodn
## ARIMA(1,1,0)(0,1,3)[12]
##
## Coefficients:
##          ar1     sma1     sma2    sma3
##       0.3393  -0.7619  -0.1222  0.2756
## s.e.  0.0500   0.0527   0.0646  0.0525
##
## sigma^2 estimated as 1.341:  log likelihood=-565.98
## AIC=1141.95   AICc=1142.12   BIC=1161.37
```

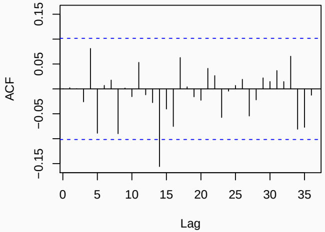
```
(fr_m4.2 = Arima(prodn, order = c(2,1,0),
            seasonal = list(order=c(0,1,3), period=12)))
## Series: prodn
## ARIMA(2,1,0)(0,1,3)[12]
##
## Coefficients:
##          ar1     ar2     sma1     sma2    sma3
##       0.3038  0.1077  -0.7393  -0.1445  0.2815
## s.e.  0.0526  0.0538   0.0539   0.0653  0.0526
##
## sigma^2 estimated as 1.331:  log likelihood=-563.98
## AIC=1139.97   AICc=1140.2   BIC=1163.26
```

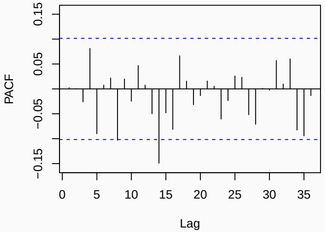
Residuals - Model 4.1



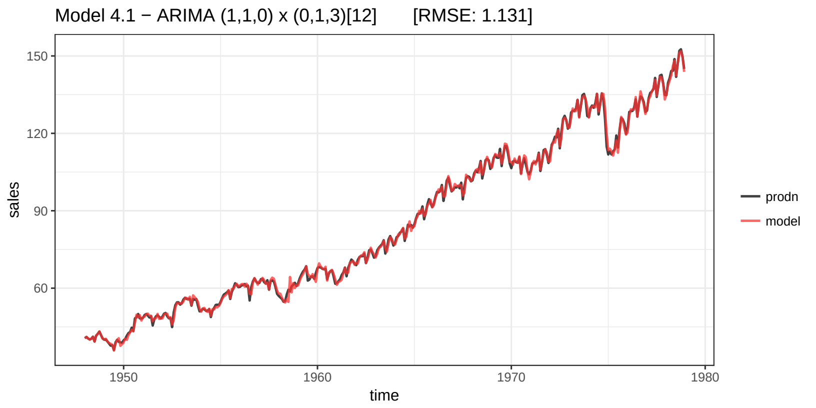
Residuals - Model 4.2



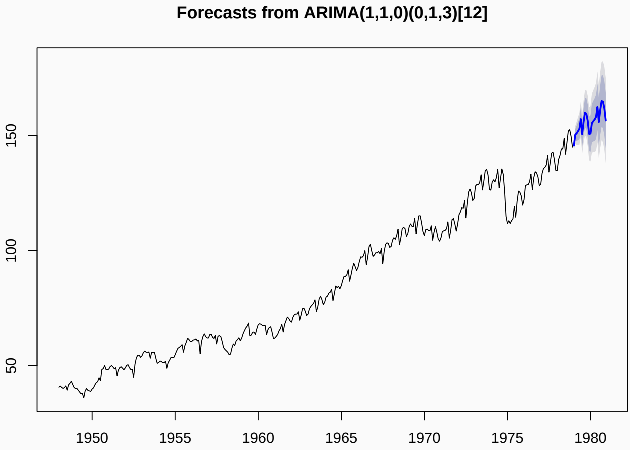




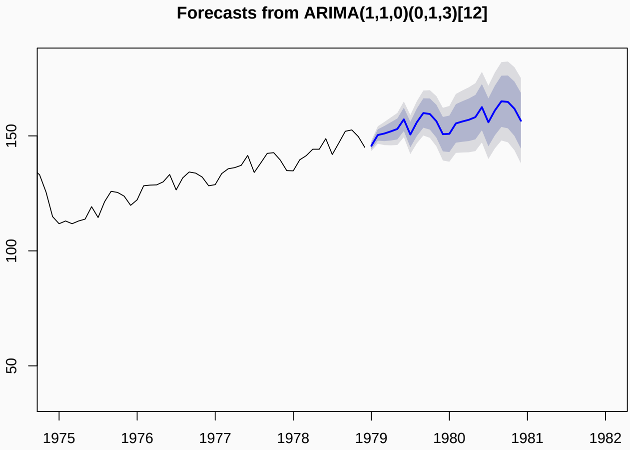
Model Fit


Model 4.1 − ARIMA (1,1,0) x (0,1,3)[12]   [RMSE: 1.131]
sales
150
120
90
60
1950
1960
1970
1980
time
prodn
model

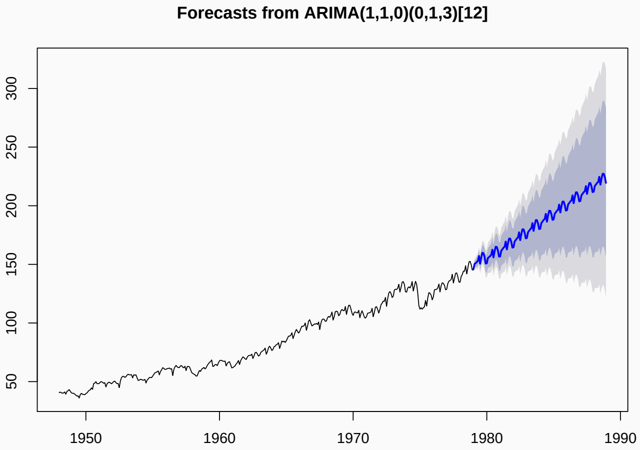
Model Forecast



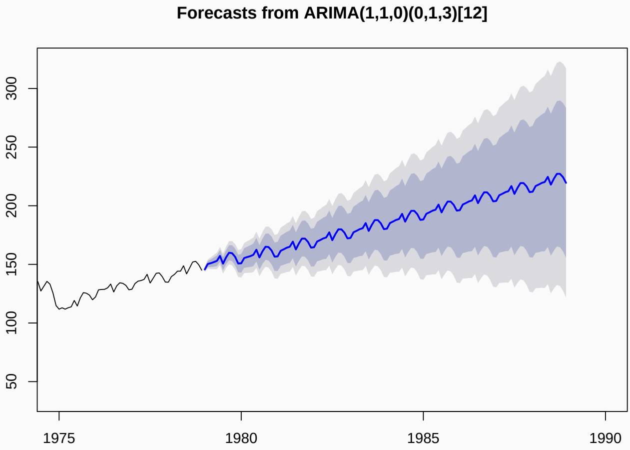
Model Forecast (cont.)



Model Forecast (cont.)

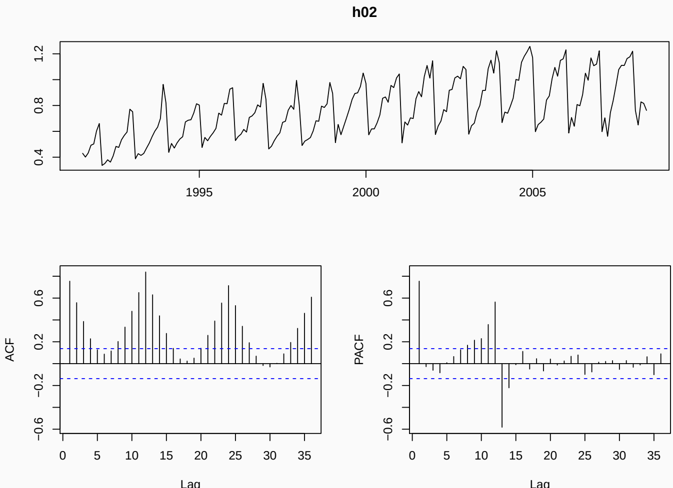


Model Forecast (cont.)



Monthly cortecosteroid drug sales in Australia from 1992 to 2008.

```r
library(fpp)
tsdisplay(h02,points=FALSE)
```



Hint

```
ts.intersect(h02, log(h02)) %>% plot()
```

.