Agent-Based Methods for Dynamic Social Networks

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Outline

- Introduction
- Social Network Models
- Agent-based Models
- Our Approach
- Results
- Statistical Challenges
Scientific Interest in Dynamic Social Networks

How do networks change over time?
How do we identify patterns?
How do we make predictions?

Examples:

- Baboon Fissions
- Antarctic Research Stations
- Terrorist Networks
- Elephants
Once in a lifetime (≈10 years) baboons will permanently fission from one group into two.

Before the final fission baboons repeatedly split into “test” groups. After a few hours or days apart they come back together. Some time later they (temporarily) split again.

What does this changing social structure tell us about how baboons make decisions?
How could one build cohesiveness in an Antarctic research station?

What early characteristics of the group will lead to “perfect” clustering, cliques, loners? Can these aspects of the social structure be predicted or changed by studying its dynamics?

What is the effect of having a few charismatic personalities, or someone everybody hates?
How will the structure of Al Qaeda look in the future?

What could dynamic data tell us about characteristics of the network?

How would the elimination of key individuals in the network affect the evolution of the network?
How do changing resources affect elephant social structure (Wet Season v. Dry Season)?

Do elephants change their preferences over time? How might this affect the social structure?

What effect might an elephant’s death have on the group structure?
Models of Social Networks

Social network analysis models relationships between actors.

- The attributes of individual actors are not as important as the attributes of their relationships, or ties, with other actors.
  - Presence/absence of a friendship (0 or 1)
  - Money exchanged between actors ($0.55)
  - a Win, Loss, or Draw (1, -1, 0) in a competition.

- A social network can be represented as a graph where the nodes are the actors and the edges represent the ties between the actors.

- A social network can also be represented by its matrix of ties:

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<thead>
<tr>
<th></th>
<th>Amy</th>
<th>Ang</th>
<th>Ali</th>
<th>Ast</th>
</tr>
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<tr>
<td>Amy</td>
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*Social Network Models*

- **Holland and Leinhardt (1981) \( p_1 \) model**
  - Assumes independence of dyads
  - Models the ties as functions of individual relational attributes (expansiveness, attractiveness) as well as features of the graph (density, overall tendency toward reciprocity)

- **Wasserman and Pattison (1986) \( p^* \) model**
  - Models the probabilities of ties conditional on all other ties in the network.
  - Explanatory variables are the differences in the network statistics (...) if a tie \( x_{ij} \) were to change from 1 to 0.

- **Hoff, Raftery, and Handcock (2002)**
Similar to the model I use for elephants!

- Sender and receiver random effects, as well as positions in a latent social space, account for the dependence between dyads.

\[
\text{logit}(p_{ij}) = \beta_0 + s_i + r_j + \beta_d X_{ij} - |z_i - z_j|
\]

- Common intercept $\beta_0$, a baseline probability
- Sender sociability or “expansiveness” $s_i$ random effect
- Receiver “attractiveness” $r_j$ random effect
- Vector of dyad-specific (observable) covariates $X_{ij}$

- The distance between $i$ and $j$ in “Social Space” affects the probability of a tie from $i \rightarrow j$.
  - Actors close together in social space are more likely to form ties.
Agent-Based Models

- Impose a few simple rules on agents, then study the aggregate effects of the resulting interactions.

- Complex social phenomena can be generated by individual agents acting according to the simple rules.

- Sugarscape (Epstein and Axtell 1996) is a classic example.
Agents collect sugar according to “vision” rules and burn it at an individual rate called “metabolism”.

The Sugarscape has rules for where and how fast sugar regrows.

Patterns of migration and skewed distributions of wealth emerge.

New rules produce new phenomena:

<table>
<thead>
<tr>
<th>Rules</th>
<th>Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vision, Metabolism, Regrowth rates</td>
<td>Migration, Wealth distribution</td>
</tr>
<tr>
<td>Sex, Reproduction</td>
<td>Evolution, Inheritance</td>
</tr>
<tr>
<td>Spice</td>
<td>Trade, Price equilibria</td>
</tr>
<tr>
<td>Cultural traits</td>
<td>Tribes, War, Migration</td>
</tr>
</tbody>
</table>
Students arrive at a boarding school having no friends and not knowing anyone.

Each student occupies a position in Social Space.

Students make friends at each time step according to a specified set of simple rules.
Social Space is a latent variable. It is a useful proxy for that which we cannot measure.

Perhaps the axes are **IQ** and **Social Status**.

**Key idea:** Students move towards their friends in Social Space.
The basic p-friendship model:

$$\log\left(\frac{p_{ij}}{1 - p_{ij}}\right) = \beta_0 + \beta_s s_i + \beta_r r_j + \delta X_{ij} - |z_i - z_j|$$

* p-Model:

$$\logit(p_{ij}) = \beta_0 + \delta_1 \left(I(Sex_i = Sex_j) - \overline{I(Sex_i = Sex_j)}\right) + \beta_s (s_i + s_j) - |z_i - z_j|$$

- $\logit(p_{ij})$ is the degree of friendship between agents $i$ and $j$. $p_{ii}$ is undefined.
- $\beta_0$ is the baseline degree of friendship between any two agents.
- $I(Sex_i = Sex_j)$ is an indicator whether agents $i$ and $j$ are of the same Sex. This dyadic covariate is centered about its mean to retain the interpretation of the baseline degree of friendship $\beta_0$.
- $\delta_1$ is the sensitivity of friendships to same Sex.
- **Charisma** $s_i \sim N(0, 1)$. Both the sender and the receiver have Charisma and both are equally weighted when making friendships.
- $\beta_s$ is the sensitivity of friendships to Charisma.
Rules for Agent Model:

- Rule 0. Twenty agents start randomly at time=1 on a (20 × 20) grid in 2-dimensional Social Space.

- Rule 1. At every time step each agent $i$ proffers a friendship to all agents $j \neq i$, and these proffers are accepted with probability $p_{ij}$.

- Rule 2. After new friendships are created, agents move a “move.fraction” towards the average of their friends’ locations in Social Space.

- Rule 3. Agents are split evenly between the sexes. $\delta_1$ is the sensitivity of friendships to same Sex.

- Rule 4. The Charisma (sociality $s_i$) of each agent is added to the model. $s_i \sim N(0, 1)$. $\beta_s$ is the sensitivity of friendships to Charisma.
Evaluation of Rules

How do changes in the set of rules change the results?

- Compare statistics of the network generated by different sets of rules
  - Average number of friends
  - Time until “perfect” clustering
  - Number of clusters
  - Number of completed triads
  - Number of opposite sex friends
  - Net distance moved by all agents in Social Space
**Evaluation of Rules: Model 1**

**p-Model 1:**

\[
\text{logit}(p_{ij}) = \beta_0 - |z_i - z_j|
\]

- \(\text{logit}(p_{ij})\) is the *degree* of friendship between agents \(i\) and \(j\).
- \(\beta_0\) is the baseline degree of friendship between any two agents.
- \(i = 1, \ldots, 20\). \(j = 1, \ldots, 20\). The degree of friendship between an agent and itself, \(\text{logit}(p_{ii})\), is undefined.
- \(z_i\) is the position of agent \(i\) in two-dimensional Social Space. \(|z_i - z_j|\) is the distance between agents \(i\) and \(j\).

**Rules for Agent Model 1:**

- Rule 0. Twenty agents start randomly at time=1 on a \((20 \times 20)\) grid in 2-dimensional Social Space.
- Rule 1. At every time step each agent \(i\) proffers a friendship to all agents \(j \neq i\), and these proffers are accepted with probability \(p_{ij}\).
- Rule 2. After new friendships are created, agents move a “move.fraction” towards the average of their friends’ locations in Social Space.
Evaluation of Rules: Model 1 and 1b

Initial Locations in Social Space for Model 1

Initial Locations in Social Space for Model 1b

\[ \text{logit}(p_{ij}) = \beta_0 - |z_i - z_j| \]

Rule 0. Size of Social Space
**Evaluation of Rules: Model 1 and 1b**

\[
\text{logit}(p_{ij}) = \beta_0 - |z_i - z_j|
\]

**Rule 0. Size of Social Space**
**Evaluation of Rules: Model 1 and 1b**

![Graph showing the average number of friends with different move fractions.](image)

\[ \text{logit}(p_{ij}) = \beta_0 - |z_i - z_j| \]

**Rule 2. Students “move.fraction” towards friends in Social Space**
Evaluation of Rules: Model 1 and 1b

Avg #Friends Model 1 with different Move Fractions

Time in 10s, max time is 2098

Average number of Friends

0 2 4 6 8 10

mf= 1
mf= 0.95
mf= 0.9
mf= 0.85
mf= 0.8
mf= 0.7
mf= 0.65
mf= 0.6
mf= 0.55
mf= 0.5
mf= 0.45
mf= 0.4
mf= 0.35
mf= 0.3
mf= 0.25
mf= 0.2
mf= 0.15
mf= 0.1
mf= 0.05
mf= 0
Evaluation of Rules: Model 1 and 1b

Avg #Friends Model 1 with different Move Fractions

Time in 10s, max time is 1098
Evaluation of Rules: Model 1b

**Average number of Triads Model 1b**

- Time in 10's
- Average number of Triads
- Different values of $b_0$: $-5, -3, -2, -1, 0, 1, 2, 3, 5$

**Average number of Clusters Model 1b**

- Time in 10's
- Average number of Clusters
- Different values of $b_0$: $-5, -3, -2, -1, 0, 1, 2, 3, 5$
★ p-Model 2:

\[ \text{logit}(p_{ij}) = \beta_0 + \delta_1 \left( I(Sex_i = Sex_j) - \overline{I(Sex_i = Sex_j)} \right) - |z_i - z_j| \]

- \( \beta_0 \) is the baseline degree of friendship between any two agents.
- \( I(Sex_i = Sex_j) \) is an indicator whether agents \( i \) and \( j \) are of the same Sex. This dyadic covariate is centered about its mean to retain the interpretation of the baseline degree of friendship \( \beta_0 \).

★ Rules for Agent Model 2:

- Rule 0. Twenty agents start randomly at time=1 on a (20 × 20) grid in 2-dimensional Social Space.

- Rule 1. At every time step each agent \( i \) proffers a friendship to all agents \( j \neq i \), and these proffers are accepted with probability \( p_{ij} \).

- Rule 2. After new friendships are created, agents move a “move.fraction” towards the average of their friends’ locations in Social Space.

- Rule 3. Agents are split evenly between the sexes. \( \delta_1 \) is the sensitivity of friendships to same Sex.
Evaluation of Rules: Model 2

Average number of Friends Model 2

Average number of Opposite Sex Friends Model 2

\[ \logit(p_{ij}) = 0 + \delta_1 \left( I(Sex_i = Sex_j) - I(Sex_i = Sex_j) \right) - |z_i - z_j| \]

Rule 3. $\delta_1$ is the sensitivity of friendships to same Sex.
Evaluation of Rules: Model 2

Average number of Triads Model 2

Time in 10’s

Average number of Clusters Model 2

Time in 10’s
Evaluation of Rules: Model 2

- As long as `move.fraction` > 0, all the agents will eventually move together to form one perfect cluster. But it could be practically impossible based on $\beta_0, \delta_1$, and the size of Social Space.

- Often one agent (starting in a corner of Social Space) will fail to make friends early and become isolated (nearly forever).

- Even when $\delta_1$ was large, clusters of opposite sexes emerged in Social Space.

- Students in the same location in Social Space are not necessarily friends.

- In general, the larger $\delta_1$ in absolute value, the higher the average number of friends.

- Adding sex into the equation drastically changes the dynamic behavior of the system. No longer do we observe one perfect cluster of agents at the end of the run. Sub-clusters in Social Space form and persist.
Agent Model 2 Example

- Social Space after 1098 iterations $\delta_1 = 2$ (preference for same sex):

- Six clusters of students emerge: A big cluster of 7 males, a cluster of 4 females, a smaller cluster of 2 females, two mixed clusters of 1 male and 2 females, and one lone male who never made any friends.
Charisma is Added to Agent Model 3

p-Model 3:

\[ \text{logit}(p_{ij}) = \beta_0 + \delta_1 \text{Sex}_{ij} + \beta_s(s_i + s_j) - |z_i - z_j| \]

- \( \beta_s \) is the sensitivity of friendships to Charisma.
- Charisma \( s_i \sim N(0, 1) \). Both the sender and the receiver have Charisma and both are equally weighted when making friendships.

Rules for Agent Model 3:

- Rule 0. Twenty agents start randomly at time=1 on a (20 \times 20) grid in 2-dimensional Social Space.
- Rule 1. At every time step each agent \( i \) proffers a friendship to all agents \( j \neq i \), and these proffers are accepted with probability \( p_{ij} \).
- Rule 2. After new friendships are created, agents move a “move.fraction” towards the average of their friends’ locations in Social Space.
- Rule 3. Agents are split evenly between the sexes. \( \delta_1 \) is the sensitivity of friendships to same Sex.
- Rule 4. The Charisma \( s_i \sim N(0, 1) \) of each agent is added to the model.
Evaluation of Rules: Model 3

\[ \text{logit}(p_{ij}) = 0 + \delta_1 \text{Sex}_{ij} + \beta_s (s_i + s_j) - |z_i - z_j| \]

Rule 4. $\beta_s$ is the sensitivity of friendships to Charisma.
Evaluation of Rules: Model 3

Average number of Triads Model 3

Average number of Clusters Model 3
Implications of Rules: Model 3

\[
\text{logit}(p_{ij}) = \beta_0 + \delta_1 \text{Sex}_{ij} + \beta_s s_i + \beta_r r_j - |z_i - z_j|
\]

\[
\text{logit}(p_{ij}) = 0 + \delta_1 \text{Sex}_{ij} + \beta_s (s_i + s_j) - |z_i - z_j|
\]

- Charisma of the sender \(s_i\) and the receiver \(r_j = s_j\) are equally important in making friendships since the coefficients are equal (\(\beta_s = \beta_r\)) and the ties are undirected. What if there are differential sender and receiver effects?

- Students with high Charisma make lots of friends and live in large clusters. They might even bring everybody together into a perfect cluster.

- The distribution of Charisma might make a difference in the behavior of the model.

- What results when Charisma \(\beta_s\) and Sex \(\delta_1\) both vary?
Varying Charisma and Sex in Model 3

Average # of Friends Model 3b, $d_1 = 0$

Average # of Friends Model 3b, $d_1 = 2$

Average # of Friends Model 3b, $d_1 = 5$

Average # of Friends Model 3b, $d_1 = 10$
Varying Charisma and Sex in Model 3

\[ \text{logit}(p_{ij}) = 0 + \delta_1 \text{Sex}_{ij} + \beta_s (s_i + s_j) - |z_i - z_j| \]

1. Only when \( \delta_1 = 10 \) is there a noticeable difference in **Average Number of Friends** for different values of \( \beta_s \).

2. For all values of \( \delta_1 \) (sensitivity to **Sex**), Average friends increases with an increase in \( \beta_s \).

3. Average number of **Opposite Sex Friends** seems very high for all values of \( \delta_1 \) when \( \beta_s = 10 \). For most values of \( \delta_1 \), opposite sex friends increases with an increase in \( \beta_s \).

4. The average final location is not a perfect cluster for any values of the parameters. Social Space seems to be big enough to get stable sub-clusters. Agents start far enough apart to have a high probability of remaining apart.

5. The **Number of Triads** is the same for all values of \( \delta_1 \) (except slightly when \( \delta_1 = 10 \)). There is a clear monotonic relationship between number of triads and \( \beta_s \) (for a given \( \delta_1 \)).

6. The **Number of Clusters** is also largely unaffected by \( \delta_1 \) and the relationship between clusters and \( \beta_s \) is fairly strong.
New Rules

- New rules could be imposed to add complexity to the model.
  - Introduce enmity or hatred between the students (a repulsive force)
  - Add jealousy to the model
  - Make movement in social space less likely as time goes on

- Model 4 bases movement in social space on all the probabilities of friendship $p_{ij}$ rather than on 0/1 friends.
  - Given the random assignment of genders and charisma, the model becomes deterministic.

- Could change the interpretation of social space from a distance model to Peter Hoff’s inner product social space, or could allow students to belong to specific “classes”.

Statistical Challenges

- Sufficient statistics for the social network

- Which summary statistics to use for the network?

- How does one summarize a story?

- Evaluating models and rules without data