Conditional probabilities

From Data to Insight

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DTap vaccine

DTaP vaccine

- Jackson et al. (2013) wanted to know whether it is better to give the diphtheria, tetanus and pertussis (DTaP) vaccine in either the thigh or the arm, so they collected data on severe reactions to this vaccine in children aged 3 to 6 years old.

- Is the probability of a severe reaction higher for vaccines in the thigh or the arm?

	No severe reaction	Severe reaction	Total
Thigh	4758	30	4788
Arm	8840	76	8916
Total	13598	106	13704

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$$P(\text{severe reaction} \mid \text{thigh}) = 30 / 4788 \approx 0.0063$$

$$P(\text{severe reaction arm}) = 76 / 8916 \approx 0.0085$$

Bayes' rule

$$P(A \mid B) = \frac{P(A \ and \ B)}{P(B)}$$

HIV testing with ELISA

HIV testing with ELISA

- ELISA
  - Sensitivity (true positive): 93%
  - Specificity (true negative): 99%
- Western blot
  - Sensitivity: 99.9%
  - Specificity: 99.1%
- Prevalance: 1.48 / 1000
- P(has HIV | ELISA +) = ?

$P(+ \mid HIV) = 0.93$

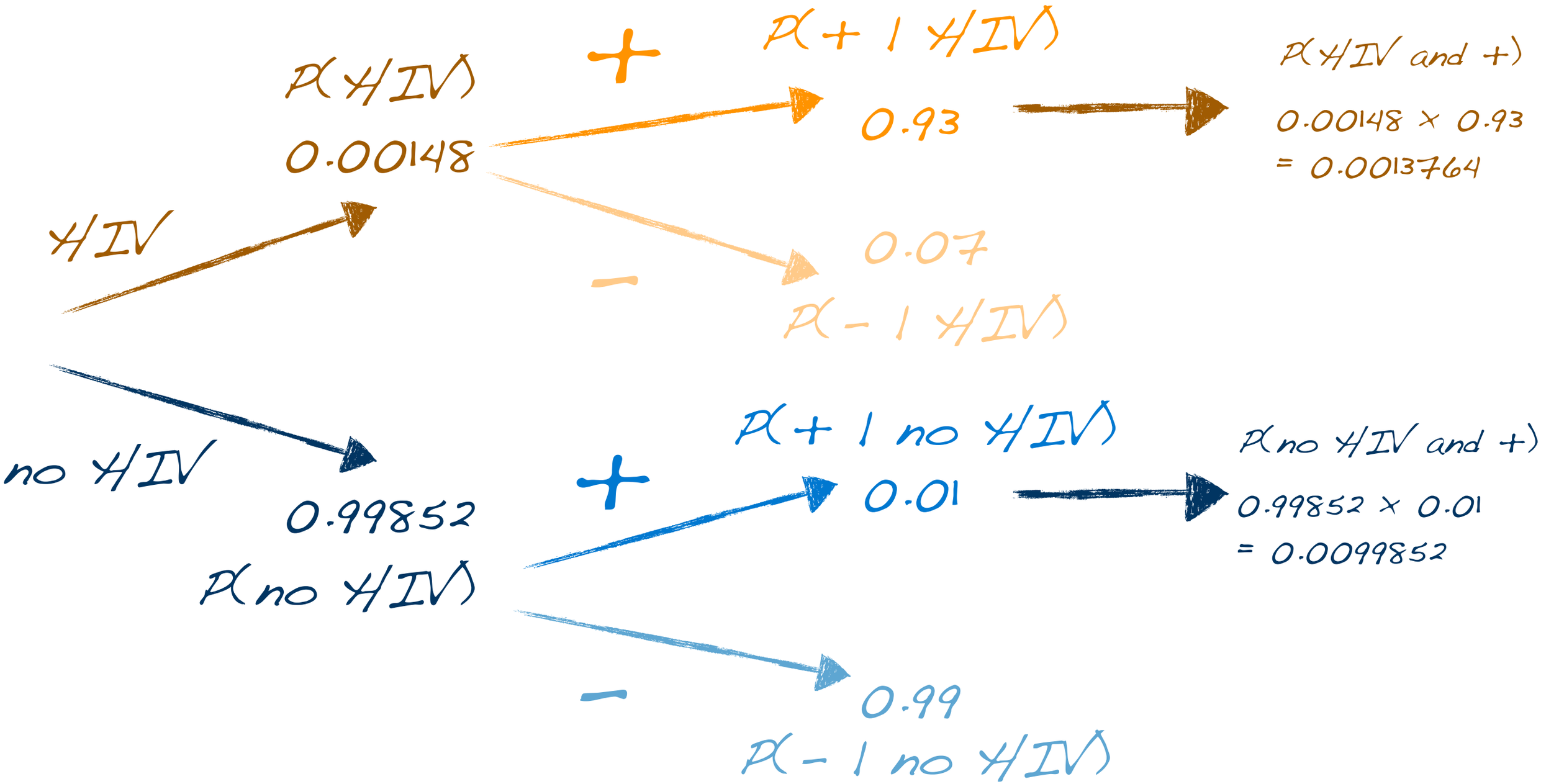
$P(- \mid no\ HIV) = 0.99$

$P(HIV) = 0.00148$

Prior to any testing,
what probability should be assigned to a recruit having HIV?

$$P(\text{HIV}) = 0.00148$$

When a recruit goes through HIV screening there are two competing claims: recruit has HIV and recruit doesn't have HIV. If the ELISA yields a positive result, what is the probability this recruit has HIV?



When a recruit goes through HIV screening there are two competing claims: recruit has HIV and recruit doesn't have HIV. If the ELISA yields a positive result, what is the probability this recruit has HIV?

$$P(HIV \mid +) = \frac{P(HIV \text{ and } +)}{P(+)}$$

$$= \frac{0.0013764}{0.0013764 + 0.0099852} \approx 0.12$$

Drug testing

[time permitting]

Drug testing [time permitting]

- Most companies drug test their employees before they start employment, and sometimes regularly during their employment as well.

- Suppose that a drug test for an illegal drugs is 97% accurate in the case of a user of that drug, and 92% accurate in the case of a non-user for that drug.

- Suppose also that 5% of the entire population uses that drug.

- You are the hiring manager at a company that drug tests their employees. You have recently decided to hire a new employee. The prospective employee gets drug tested, and the test comes out to be positive. What is the probability that they are actually a user for the drug?