STA103: Quiz III

This is an open book quiz (that is you can refer to any material you have available). It lasts 60 minutes. Attempt all questions

1. Suppose that $X$ has a binomial distribution with $p = 0.2$ and $n = 200$. Using the normal approximation to binomial what is the probability that $X \geq 45$?

2. Two coins are tossed and the total number of ‘heads’ is recorded. What is the probability after 500 throws the sum of these totals lies between 500 and 510?

3. In the coin tossing example of Chapter 5 what is the distribution of the sum of 300 coin tosses when, per throw, you win $2 when you win but lose $1.90 when you lose?

4. Convert the following relationships into linear ones by making transformations and defining new variables (a) $\exp(y + b) = ax$ (b) $x = ab^y$

5. Fit a line from $y = a + bx$ from the following data using the method of least squares

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.1</td>
<td>1.1</td>
<td>2.0</td>
<td>3.1</td>
<td>3.9</td>
</tr>
</tbody>
</table>

6. In Rice §14.8 Question 11 (a) the variance for $\hat{\mu}_0$ is found to be

$$Var(\hat{\mu}_0) = \sigma^2 \left[ \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right]$$

where $x_0$ is some new value of the explanatory variable and $\hat{\mu}_0 = \hat{\beta}_0 + \hat{\beta}_1x_0$, is the corresponding prediction of the response. Sketch the standard deviation of $\hat{\mu}_0$ as a function of $x_0 - \bar{x}$ and explain the significance of the shape of this curve.

7. Figure 1 (a), (b), (c) shows a regression model, a plot of residuals against explanatory variables and a histogram of the residuals respectively. Write one sentence on what you observe in each of these three plots regarding the quality of the fit of the model
8. The $R^2$ value for the model in Figure 1 is 0.64. Explain, in one sentence, what this means in terms of the variances of the histograms of Figure 1 (c), (d) where (d) is a histogram of the response variable. Comment on what the quality of the prediction would be for a new value of the explanatory variable equal to 0.5

![Figure 1: A Simple regression](image-url)