STA103: Quiz II Solutions

1. This is a Binomial(10, 1/6) so probability is
   \[
   \binom{10}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^7 = 0.155
   \]

2. Probability it happens in a single throw is Binomial(3, 1/2) which gives 1/8. So have a Geometric (1/8) distribution i.e. probability is 7/8 \times 1/8 = 0.109.

3. The integral of the density must be 1 so
   \[
   \int_{-1}^{1} cx^2 dx = \left[ \frac{1}{3}cx^3 \right]_{-1}^{1} = \frac{2}{3} c = 1
   \]
   so \( c = \frac{3}{2} \). The cdf is
   \[
   \int_{-1}^{x} \frac{3}{2}u^2 du = \frac{1}{2}x^3 + \frac{1}{2}
   \]

4. The cdf for an exponential \( \lambda \) is \( 1 - \exp(-\lambda x) \) so solve
   \[
   0.75 = 1 - \exp(-10 \times x) \quad \Rightarrow \quad x = -0.1 \log(0.25) = 0.138
   \]

5. Want the \( P(0 < z < (15 - 14.2)/\sqrt{(1.2)}) = P(0 < z < 0.73) \) This equals \( 0.7673 - 0.5 = 0.2673 \)

6. The number of emails in one hour is Poisson(0.5) so
   \[
   P(k = 0) = \exp(-0.5) = 0.6065
   \]

7. First find cdf , let \( X = \log(U) \)
   \[
   P(X < x) = P(\log(U) < x) = P(U < \exp(x)) = \exp(x)
   \]
   So density is also \( \exp(x) \) for \(-\infty < x < 0\)

8. The probability mass function is
   \[
   P(X = k) = P(k \leq T < k + 1) = \int_{k}^{k+1} \lambda \exp(-\lambda t) dt = [-\exp(-\lambda t)]_{k}^{k+1} = \exp(-\lambda(k)) - \exp(-\lambda(k + 1))
   \]