STA103: Quiz III Solutions

1. The mean is $0.2 \times 200 = 40$ and the variance is $0.2 \times 0.8 \times 200 = 32$, thus we want, using the normal approximation

$$P(z \geq \frac{45 - 40}{\sqrt{32}}) = Pr(z \geq 0.883) = 0.1894$$

2. The probability distribution for the $X$ the count on a single throw is

$$P(X = 0) = \frac{1}{4}, Pr(X = 1) = \frac{1}{2}, P(X = 2) = \frac{1}{4}$$

Thus the expectation is 1 and its variance is

$$(-1)^2 \frac{1}{4} + 0 \frac{1}{2} + 1^2 \frac{1}{4} = \frac{1}{2}$$

Thus, the mean and variance for the sum of 500 throw, $T$, is 500, and 250. Hence using the normal approximation

$$P(500 < T < 510) = \Pr\left(\frac{500 - 500}{\sqrt{250}} < Z < \frac{510 - 500}{\sqrt{250}}\right)$$

$$= \Pr(0 < Z < 0.6325) = 0.7357 - .5 = 0.2357$$

3. For a single throw the mean is $2\frac{1}{2} - 1.90\frac{1}{2} = 0.05$, while the variance is

$$(2 - 0.05)^2 \frac{1}{2} + (-1.9 - 0.05)^2 \frac{1}{2} = 3.8025$$

Thus by the CLT the distribution for the total after 300 tosses is $N(300 \times 0.05, 300 \times 3.802) = N(15, 1140.75)$

4. (a) $y = \log(x) + \log(a) - b$ (b)

$$y = \frac{\log(x)}{\log(b)} - \frac{\log(a)}{\log(b)}$$

5. $\hat{\beta}_0 = -0.84, \hat{\beta}_1 = 0.96$
6. The graph is quadratic in $x_0 - \bar{x}$ with a minimum at $x_0 = \bar{x}$, see Figure. Thus the uncertainty in the prediction for the new $x_0$ is at a minimum when $x_0$ is the mean of the observed data and increases as you move away from the mean.

Figure 1:

7. (a) There is a non-linear relationship between the variables. (b) There is a pattern in the residual plot showing that the linear model assumption do not hold. (c) The histogram for the residuals is a bit skewed and non-normal looking.

8. If $R^2 = 0.64$ then the ratio of the the explained variation (i.e variance of (d) ) to the total (i.e. the variance of (c) ) is 0.64. If a new value of $x = 0.5$ was chosen from (a) we see the fitted value will be on the line considerably above observed values of $y$ in that region.