STA103: Quiz IV Solutions

1. The random variables are (c) and (d)

2. All are true, for (a) it contains the sample mean 100% ≥ 95% of the time

3. Sampling without replacement gives there are \( \binom{4}{2} = 6 \) samples all equally likely these are

\[
\{-3, -1\}, \{-3, 1\}, \{-3, 3\}, \{-1, 1\}, \{-1, 3\}, \{1, 3\}
\]

which have sample means \{-2, -1, 0, 1, 2\} respectively. Thus the sampling distribution of \( \bar{X} \) is

\[
P(\bar{X} = -2) = \frac{1}{6}, P(\bar{X} = -1) = \frac{1}{6}, P(\bar{X} = 0) = \frac{2}{6}, P(\bar{X} = 1) = \frac{1}{6}, P(\bar{X} = 2) = \frac{1}{6}
\]

4. The standard error is \( \sqrt{0.85(1 - 0.85)} \) so solve

\[
\frac{0.85(1 - 0.85)}{n} = 0.1^2
\]

i.e. \( n = 12.75 \) or greater than 13

5. The sample mean is 11.039, the sample variance is 1.356, and sample size is 10, so using the \( t_9 \)-statistic which is 2.821 we get a 95% confidence interval \((11.039 - 2.262\sqrt{\frac{1.356}{10}}, 11.039 + 2.262\sqrt{\frac{1.356}{10}}) = (10.21, 11.87)\)

6. The mle for the variance is 1.2205 so the 99% confidence interval for \( \sigma^2 \) in a Normal sample is

\[
\left( \frac{10\hat{s}^2}{\chi^2_9(0.995)}, \frac{10\hat{s}^2}{\chi^2_9(0.005)} \right) = \left( \frac{12.205}{23.59}, \frac{12.205}{1.73} \right) = (0.5174, 7.055)
\]

7. The sample proportion is 0.7, and its standard error is

\[
\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \sqrt{1 - \frac{n - 1}{N - 1}} = \sqrt{\frac{0.7 \times 0.3}{10}} \sqrt{1 - \frac{9}{49}} = 0.1309
\]

so the 95% confidence interval is

\[
(0.7 - 1.96 \times 0.1309, 0.7 + 1.96 \times 0.1309) = (0.4434, 0.956)
\]

8. The data is \( x_1 = 3, x_2 = 3, x_3 = 1, x_4 = 4, x_5 = 2 \) and let \( P(x|p) \) be the probability mass function, so the log likelihood function is

\[
\ell(p) = \sum_{i=1}^{5} \log P(x_i|p)
\]

\[
= \log P(3|p) + \log P(3|p) + \log P(1|p) + \log P(4|p) + \log P(2|p)
\]

\[
= 2 \log \frac{2 - p}{6} + 2 \log \frac{1}{6} + \log \frac{2 + p}{6}
\]