1. Show how you can use the CDF F of a random variable Y to compute

   (a) $\Pr(Y \in (a, b])$;

   (b) $\Pr(Y \in (a, b))$;

   (c) $\Pr(Y \in [a, b])$.

2. Derive the density of W for the following cases:

   (a) $W = -\log Y$ where $p_Y(y) = 1$ for $y \in (0, 1)$.

   (b) $W = 1/Y$ where $p_Y(y) = \frac{1}{\pi(1+y^2)}$ for $y \in \mathbb{R}$.

   (c) $W = e^Y$ where $Y \sim N(0, 1)$.

   (d) $W = Y^2$ where Y has a t_ν distribution.

3. Let Y be a random variable with a continuous strictly increasing CDF, so in particular F^{-1} exists and $F^{-1}(F(y)) = y$.

   (a) Find the CDF of U where $U = F(Y)$;

   (b) Find the CDF of X, where $X = F^{-1}(U)$;

   (c) Explain how these results can be used to simulate a normal distribution in R using only the `runif` command and the `qnorm` command. Write out your computer code.

4. Let X and Y be real-valued continuous random variables defined on the same probability space. Define $\Pr(X \in A|Y = y)$ using $p_{X|Y}(x|y)$ as we did in class. Show that $\int \Pr(X \in A|Y = y)p_y(y)\, dy = \Pr(X \in A)$. Describe in words why this result makes intuitive sense, possibly by making an analogy to the discrete case.

5. Let X and Y be real-valued continuous random variables defined on the same probability space. Show that $\lim_{\epsilon \to 0} \Pr(X \in A|Y \in B_\epsilon) = \int_A p_{x|y}(x|y)\, dx$, where $B_\epsilon = (y - \epsilon, y]$.

6. Let X have a Gamma(a, b) distribution, with density $p(x) = b^a x^{a-1} e^{-bx} / \Gamma(a)$ for $x > 0$. Let Let $Y|X \sim$ Gamma(c, X). Derive the marginal density of Y and the conditional density of of X given Y.