1. Let $Y|\theta \sim N(\theta, \sigma^2)$ and let $\theta \sim N(\mu, \tau^2)$. Derive the marginal density of Y and the conditional density of θ.

2. Show that random variables X and Y are independent if and only if $p_{XY}(x, y) = p_X(x)p_Y(y)$ (or more precisely, if there exist versions of the pdfs satisfying the equality).

3. Without using the result from Exercise 2, show that random variables X and Y are independent if and only if the marginal and conditional pdfs satisfy $p_{X|Y}(x|y) = p_X(x)$.

4. Let X and Y be independent. Show that $U = g(X)$ and $V = h(Y)$ are independent.

5. Let $Y_1, \ldots, Y_n \sim$ i.i.d. P with continuous density $p(y)$. Find the CDFs and pdfs of $Y_{(1)} = \min\{Y_1, \ldots, Y_n\}$ and $Y_{(n)} = \max\{Y_1, \ldots, Y_n\}$.

6. Derive and identify the distribution of $Y_{(1)} = \min\{Y_1, \ldots, Y_n\}$ when $Y_1, \ldots, Y_n$ is an i.i.d. sample from

   (a) an exponential distribution with mean $1/\lambda$;

   (b) a uniform distribution on $[0, 1]$;

   (c) a discrete distribution with density $p(y) = \theta(1 - \theta)^{y-1}$ for $y \in \{1, 2, \ldots\}$.

7. Let $Y_1, Y_2 \sim$ i.i.d. with a density $p(y) = e^{-y/\lambda}/\lambda$ on $y \in (0, \infty)$.

   (a) Find the CDF that corresponds to $p(y)$, that is, find $\Pr(Y_i \leq y)$.

   (b) Let $S = Y_1 + Y_2$ and $D = Y_1 - Y_2$. For a given value of D, what are the possible values of S? Draw a picture with S on the horizontal axis and D on the vertical axis that shows the joint range of S and D.

   (c) Find the joint density of S and D, and the conditional density of D given S.

   (d) Find the marginal density of D and sketch a picture of it.