Stat 542
Homework 1
Assigned 01/11/10
Due 01/20/10

1. Multinomial distribution: Let $\boldsymbol{e}_j$ be a p-dimensional vector with a 1 in the jth slot and zeros elsewhere. Let $\boldsymbol{Y}$ be a random vector such that $\Pr(\boldsymbol{Y} = \boldsymbol{e}_j) = a_j$, for which $a_j > 0$ and $\sum a_j = 1$.
   (a) Obtain the expectation and covariance of $\boldsymbol{Y}$.
   (b) Let $\boldsymbol{X} = \sum_{i=1}^{n} \boldsymbol{Y}_i$, where $\{\boldsymbol{Y}_i, i = 1, \ldots, n\}$ are i.i.d. with the distribution described above. Obtain the form of $\mathrm{E}[\boldsymbol{X}]$, $\mathrm{Cov}[\boldsymbol{X}]$ and $p(\boldsymbol{X})$.
   (c) Is $\mathrm{Cov}[\boldsymbol{X}]$ positive definite? Explain why or why not.

2. Matrix algebra: Let $\boldsymbol{X}$ be a p-dimensional random vector with $\mathrm{E}[\boldsymbol{X}] = \boldsymbol{\theta}$ and $\mathrm{Cov}[\boldsymbol{X}] = \boldsymbol{\Sigma}$, and $\mathbf{A}$ a $p \times p$ symmetric matrix. Prove that

$$\mathrm{E}[\boldsymbol{X}^T \mathbf{A} \mathbf{X}] = \mathrm{tr}(\mathbf{A}\boldsymbol{\Sigma}) + \boldsymbol{\theta}^T \mathbf{A} \boldsymbol{\theta}.$$

3. Copula transformation: Let $\boldsymbol{Z} \sim$ multivariate normal$(\mathbf{0}, \boldsymbol{C})$ where $\boldsymbol{C}$ is a correlation matrix. Let $\boldsymbol{X}$ be the random vector defined by $X_j = g_j(Z_j)$, where g_j is a known strictly increasing function. Derive, step by step, the form of the density function for $\boldsymbol{X}$.

4. Multivariate normal simulation: In this problem you will write R-code to simulate a random normal vector.
   (a) Write R-code that will generate a multivariate normal random vector with mean `m` and covariance `S` for arbitrary values of `m` and `S`. Send me the code via email in plain text, so that I can cut and paste your code. Your code should not be identical to that of any other student.
   (b) Write a proof of why your code simulates from the multivariate normal distribution (this should be turned in with the rest of the homework, not emailed with your code).