Introduction

567 Statistical analysis of social networks

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Networks are relational data

Relationship: An irreducible property of two or more entities.

- Contrast to properties of entities alone (**attributes**).
- Relations are possibly affected, but not determined, by entity attributes.

Focus of RDA/SNA: The study of relational data arising from social entities.

- **Entities**: people, animals, groups, locations, organizations, regions, etc..
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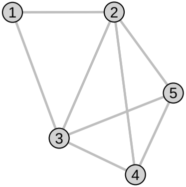
Some vocabulary

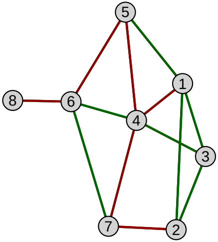
Relational data:
A collection of entities and a set of measured relations between them.

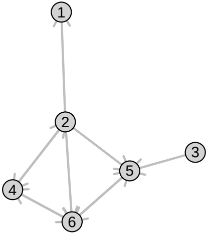
- Entities: nodes, actors, egos, units.
- Relations: ties, links, edges.

Relations can be

- directed or undirected;
- valued or dichotomous (binary).







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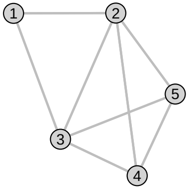

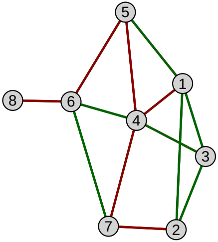

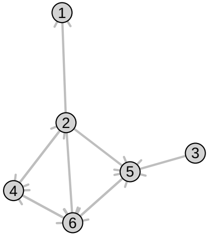

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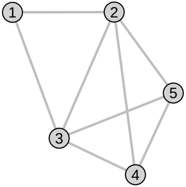

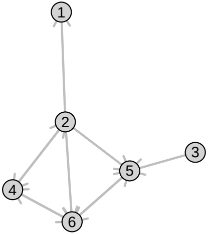

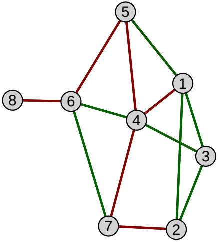

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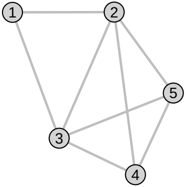

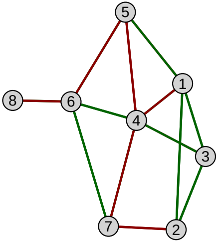

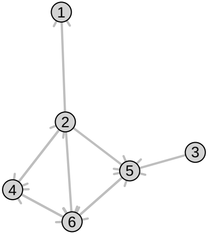

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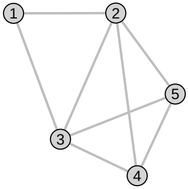

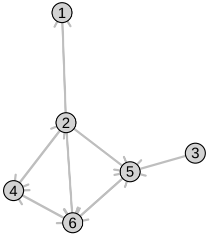

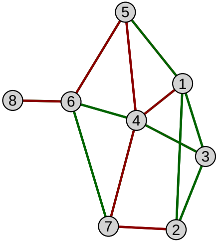

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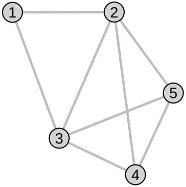

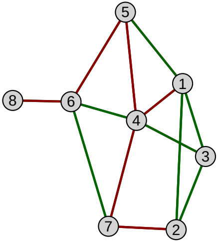

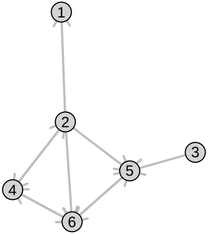

Different perspectives on network analysis

- Social sciences (social theory, description, survey design)
- Machine learning (clustering, prediction, computation)
- Physics and applied math (agent-based models, emergent features)
- Statistics (statistical modeling, estimation and testing, design based inference)

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Core areas of statistical network analysis

1. **Statistical modeling**: evaluation and fitting of network models.
   - Testing: evaluation of competing theories of network formation.
   - Estimation: evaluation of parameters in a presumed network model.
   - Description: summaries of main network patterns.
   - Prediction: prediction of missing or future network relations.
2. **Design-based inference**: Network inference from sampled data.
   - Design: survey and data-gathering procedures.
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Example: AddHealth friendships

- "Add Health" - The National Longitudinal Study of Adolescent Health:
  - A school-based study of adolescent health and social behaviors;
  - `www.cpc.unc.edu/projects/addhealth`.
- Data from 160 schools across the US:
  - The smallest had 69 adolescents in grades 7–12;
  - The largest had thousands of participants.
- Relational data:
  - Participants nominated and ranked up to 5 boys and 5 girls as friends.

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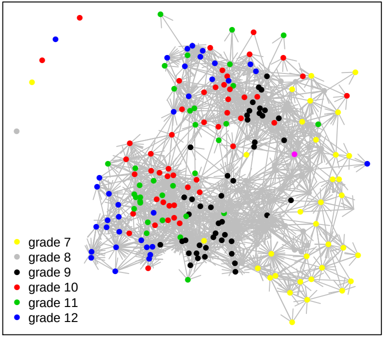

Notice: Homophily by nodal attributes.

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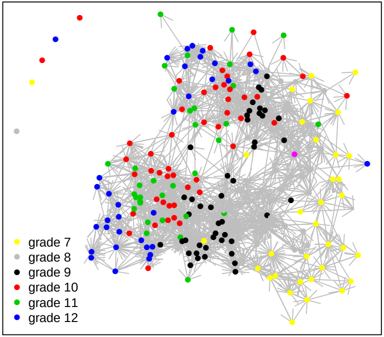

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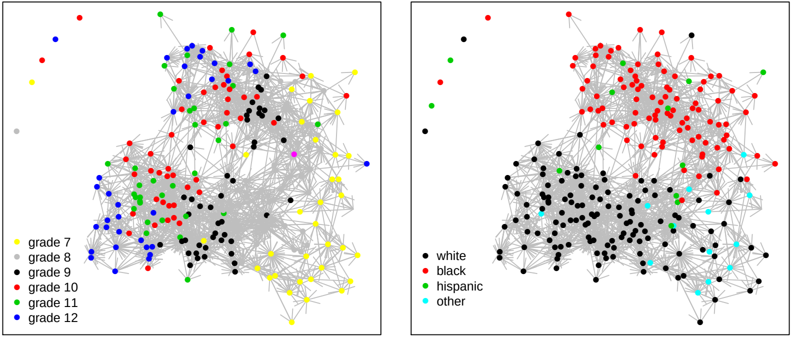

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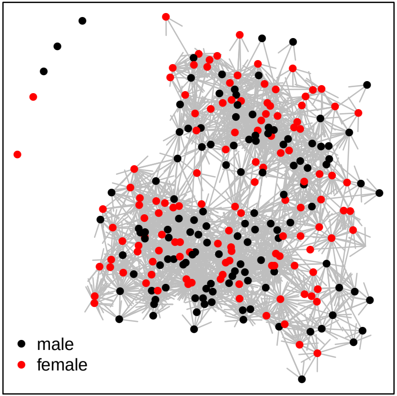
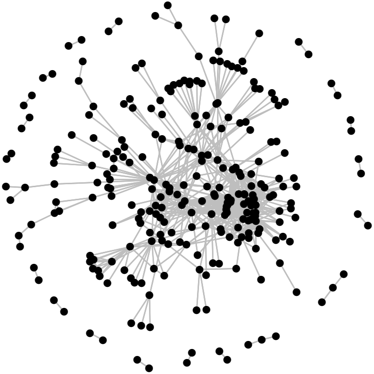


Question: Why might this plot be misleading?

Example: Protein interaction data



Notice: Network structure as compared to the friendship data.

Features of many social networks

In addition to associations to nodal and dyadic attributes, many networks exhibit the following features:

- **Reciprocity** of ties
- **Degree heterogeneity** in the propensity to form or receive ties
  - sociability
  - popularity
- **Homophily** by actor attributes
  - higher propensity to form ties between actors with similar attributes
  - attributes may be observed or unobserved
- **Transitivity** of relationships
  - friends of friends have a higher propensity to be friends
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- **Equivalence** of nodes
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Dependence:

Two outcomes are **dependent** if knowing one gives you information about the other.

Exercise:

How might network features give rise to statistical dependence?

Ubiquitous feature of network data:

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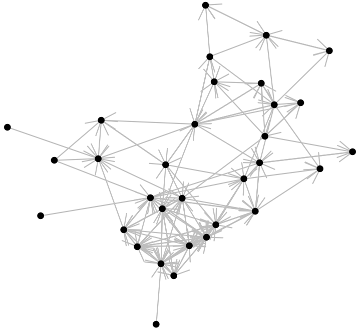
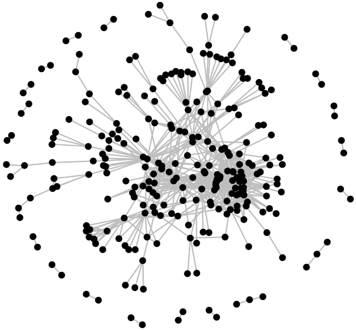
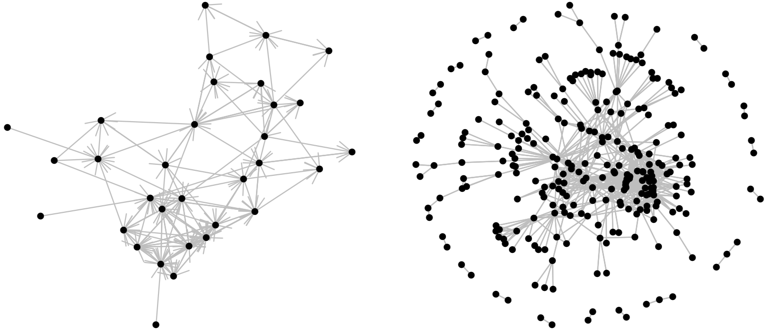
Data analysis goals

How do network features drive our data analysis?

1. How can we describe features of social relations?
   (reciprocity/sociability/popularity/transitivity : descriptive statistics)
2. How can we identify nodes with similar network roles?
   (stochastic equivalence : node partitioning)
3. How do we relate the network to covariate information?
   (homophily : regression modeling)

Data analysis goals



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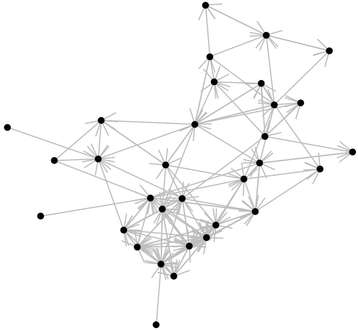

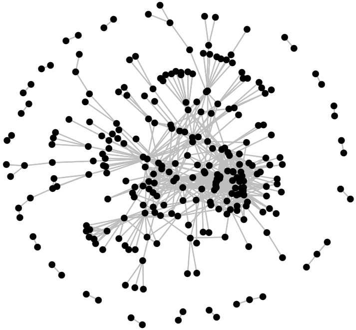

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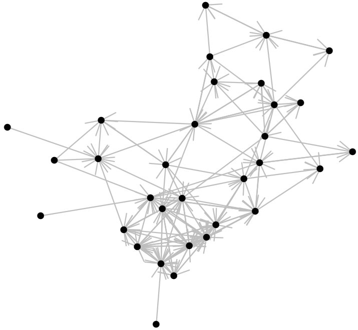

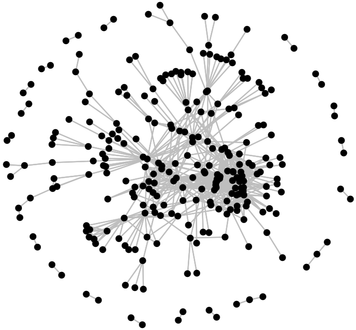

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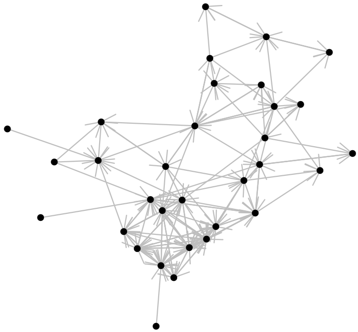

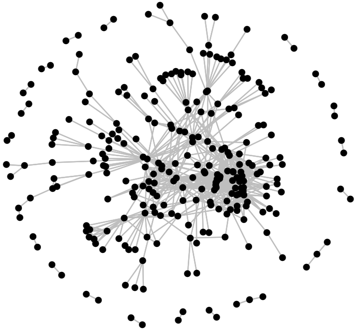

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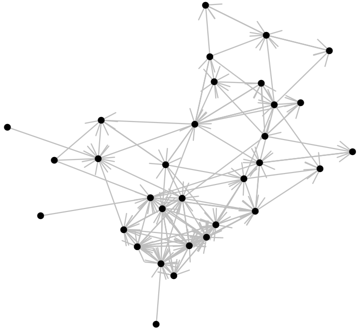

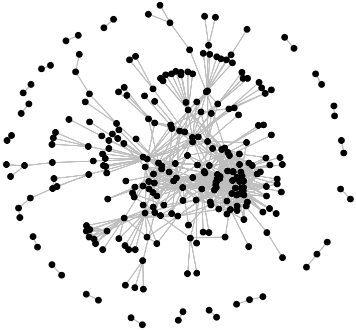

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Inferential goals in the regression framework

$y_{i,j}$ measures $i \rightarrow j$, $\mathbf{x}_{i,j}$ is a vector of explanatory variables.

$$\mathbf{Y} = \begin{pmatrix} y_{1,1} & y_{1,2} & y_{1,3} & \mathrm{NA} & y_{1,5} & \cdots \\ y_{2,1} & y_{2,2} & y_{2,3} & y_{2,4} & y_{2,5} & \cdots \\ y_{3,1} & \mathrm{NA} & y_{3,3} & y_{3,4} & \mathrm{NA} & \cdots \\ y_{4,1} & y_{4,2} & y_{4,3} & y_{4,4} & y_{4,5} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} \mathbf{x}_{1,1} & \mathbf{x}_{1,2} & \mathbf{x}_{1,3} & \mathbf{x}_{1,4} & \mathbf{x}_{1,5} & \cdots \\ \mathbf{x}_{2,1} & \mathbf{x}_{2,2} & \mathbf{x}_{2,3} & \mathbf{x}_{2,4} & \mathbf{x}_{2,5} & \cdots \\ \mathbf{x}_{3,1} & \mathbf{x}_{3,2} & \mathbf{x}_{3,3} & \mathbf{x}_{3,4} & \mathbf{x}_{3,5} & \cdots \\ \mathbf{x}_{4,1} & \mathbf{x}_{4,2} & \mathbf{x}_{4,3} & \mathbf{x}_{4,4} & \mathbf{x}_{4,5} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix}$$

Consider a basic (generalized) linear model

$$y_{i,j} \sim \boldsymbol{\beta}^T \mathbf{x}_{i,j} + e_{i,j}$$

A model can provide

- a measure of the association between $\mathbf{X}$ and $\mathbf{Y}$: $\hat{\beta}$, $\mathrm{se}(\hat{\beta})$
- imputations of missing observations: $p(y_{1,4}|\mathbf{Y}, \mathbf{X})$
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Course outline

1. Representations of relational data
    - matrix representations
    - graph representations
2. Descriptive statistics and summaries
    - matrix-based (row/column summaries, matrix decompositions)
    - graph-based (degrees, dyads, triads, paths)
    - covariates
3. Inference for complete relational data
    - model comparison via hypothesis testing
    - regression models
    - p_1 and ERGM models
    - social relations model
    - latent variable models (random effects, latent factors and blockmodels)
4. Inference for incomplete or sampled relational data
    - sampling designs (link tracing, egocentric)
    - sample-based inference
    - model-based inference
5. Longitudinal and multivariate relational data