Conditional tests

567 Statistical analysis of social networks

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Evaluating statistical models

$$H : \{y_{i,j}, i \neq j\} \sim \text{binary}(\theta), \text{ for some } \theta \in [0,1]$$

We would like to evaluate this model, but we don't know *precisely* what to expect from it, as we don't know which is the correct value of θ, if H were to be true.

Problem: The **null distribution** of $\mathbf{Y}$ depends on the unknown θ.

A solution:

- Perhaps some aspect of the null distribution doesn't depend on θ
- If so, then it could be used to evaluate the null model (for all $\theta \in [0,1]$).

The tool we need to develop this idea further is **conditional probability**

Conditional distributions

Let $y_1, y_2, y_3 \sim$ i.i.d. binary(θ).

Suppose we are told that $y_1 + y_2 + y_3 = 2$.

- What is the probability that $y_1 = 1$?
- What is the probability that $y_1 = y_2 = 1$?

Intuitive calculation

Consider all possible outcomes, before we are told the sum:

$$\begin{array}{c|cccccccc} y_1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ y_2 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ y_3 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}$$

Intuitive calculation

Compute the probabilities of each:

y_1	0	1	0	1	0	1	0	1
y_2	0	0	1	1	0	0	1	1
y_3	0	0	0	0	1	1	1	1
Pr	$(1-\theta)^3$	$\theta(1-\theta)^2$	$\theta(1-\theta)^2$	$\theta^2(1-\theta)$	$\theta(1-\theta)^2$	$\theta^2(1-\theta)$	$\theta^2(1-\theta)$	θ^3

Intuitive calculation

Now we are told that $y_1 + y_2 + y_3 = 2$:

y_1	0	1	0	1	0	1	0	1
y_2	0	0	1	1	0	0	1	1
y_3	0	0	0	0	1	1	1	1
Pr	$(1-\theta)^3$	$\theta(1-\theta)^2$	$\theta(1-\theta)^2$	$\theta^2(1-\theta)$	$\theta(1-\theta)^2$	$\theta^2(1-\theta)$	$\theta^2(1-\theta)$	θ^3

So it seems that having been told $y_1 + y_2 + y_3 = 2$,

- $(1,1,0), (1,0,1), (0,1,1)$ are the only possibilities;
- each of these was equally probable to begin with;
- they should be equally probable given the information.

Intuitive calculation

Hence,

$$\left.\begin{array}{l}\Pr((y_1, y_2, y_3) = (1,1,0) | y_1 + y_2 + y_3 = 2) \\ \Pr((y_1, y_2, y_3) = (1,0,1) | y_1 + y_2 + y_3 = 2) \\ \Pr((y_1, y_2, y_3) = (0,1,1) | y_1 + y_2 + y_3 = 2)\end{array}\right\} = 1/3$$

Let's answer our conditional probability questions:

Suppose we are told that $y_1 + y_2 + y_3 = 2$.

- What is the probability that $y_1 = 1$?
  - $2/3$
- What is the probability that $y_1 = y_2 = 1$?
  - $1/3$

Let A and B be two uncertain events.

$$\Pr(B|A) = \frac{\Pr(A \text{ and } B)}{\Pr(A)}$$

Example: Consider days with non-rainy mornings:

- $B =$ rainy in the evening;
- $A =$ cloudy in the morning.

	B	B^c
A	.4	.2
A^c	.1	.3

$$\begin{aligned}
\Pr(B) &= \Pr(B \cap A) + \Pr(B \cap A^c) = .4 + .1 = .5 \\
\Pr(A) &= \Pr(B \cap A) + \Pr(B^c \cap A) = .4 + .2 = .6 \\
\Pr(B|A) &= \Pr(B \cap A)/\Pr(A) = .4/.6 = 2/3
\end{aligned}$$

Conditional probability

Let A and B be two uncertain events.

$$\Pr(B|A) = \frac{\Pr(A \text{ and } B)}{\Pr(A)}$$

Example: A card deck is shuffled and a single card is dealt.

- $B =$ the card is the 3 of hearts.
- $A =$ the card is red.

$$\begin{aligned}
\Pr(B|A) &= \frac{\Pr(A \text{ and } B)}{\Pr(A)} \\
&= \frac{\Pr(\text{the card is the 3 of hearts})}{\Pr(\text{the card is red})} \\
&= \frac{1/52}{1/2} = 1/26
\end{aligned}$$

Example: Let $y_1, y_2, y_3 \sim$ i.i.d. binary(θ).

- $B = \{(y_1, y_2, y_3) = (0, 1, 1)\}$
- $A = \{y_1 + y_2 + y_3 = 2\}$

$$\begin{aligned} \Pr(B) &= (1-\theta) \times \theta \times \theta = \theta^2(1-\theta) \\ \Pr(A) &= \Pr((0,1,1)) + \Pr((1,0,1)) + \Pr((1,1,0)) = 3\theta^2(1-\theta) \end{aligned}$$

$$\Pr(B|A) = \frac{\theta^2(1-\theta)}{3\theta^2(1-\theta)} = \frac{1}{3}.$$

Note that this probability *doesn't depend* on the value of θ.

A **conditional probability distribution** is an assignment of conditional probabilities to a partition of the outcome space.

Let $B_1, \dots, B_K$ be a **partition**, so that

- $\Pr(B_j \text{ and } B_k) = 0$;
- $\Pr(B_1 \text{ or } \cdots \text{ or } B_K) = \Pr(B_1) + \cdots \Pr(B_K) = 1$

A conditional probability distribution over $B_1, \dots, B_K$ given A is simply the collection $\{\Pr(B_k|A), k = 1, \dots, K\}$.

Conditional probability distributions

Example: Let $y_1, y_2, y_3 \sim$ i.i.d. binary(θ).

Conditional on $A = \{y_1 + y_2 + y_3 = 2\}$, you should now be able to show that

- $\Pr((y_1, y_2, y_3) = B|A) = 1/3$ if B is either (0,1,1), (1,0,1) or (1,1,0) .
- $\Pr((y_1, y_2, y_3) = B|A) = 0$ otherwise

We say the distribution of $\mathbf{y} = (y_1, y_2, y_3)$ given $\sum y_i = 2$ is **uniform**, as it assigns equal probabilities to all possible events under the condition.

Conditioning binary sequences

$$y_1, \ldots, y_m \sim \text{ i.i.d. binary}(\theta)$$

What is the conditional distribution of

$\mathbf{y} = (y_1, \ldots, y_m)$ given

$s = \sum y_i$?

Let $\tilde{\mathbf{y}} = (\tilde{y}_1, \ldots, \tilde{y}_m)$ be a binary sequence.

$$\Pr(\mathbf{y} = \tilde{\mathbf{y}} | s) = \frac{\Pr(\mathbf{y} = \tilde{\mathbf{y}} \text{ and } \sum y_i = s)}{\Pr(\sum y_i = s)}$$

First note that this must equal zero if $\sum \tilde{y}_i \neq s$.

Conditioning binary sequences

$$y_1, \ldots, y_m \sim \text{ i.i.d. binary}(\theta)$$

Let $\tilde{\mathbf{y}} = (\tilde{y}_1, \ldots, \tilde{y}_m)$ be a binary sequence with $\sum \tilde{y}_i = s$.

$$\begin{aligned}
\Pr(\mathbf{y} = \tilde{\mathbf{y}} | s) &= \frac{\Pr(\mathbf{y} = \tilde{\mathbf{y}} \text{ and } \sum y_i = s)}{\Pr(\sum y_i = s)} \\
&= \frac{\Pr(\mathbf{y} = \tilde{\mathbf{y}})}{\Pr(\sum y_i = s)} \\
&= \frac{\theta^s (1-\theta)^{m-s}}{\binom{m}{s} \theta^s (1-\theta)^{m-s}} = \frac{1}{\binom{m}{s}}.
\end{aligned}$$

Recall, $\binom{m}{s}$ is the number of sequences that have s ones.

- the probability doesn't depend on θ;
- the probability is the same for *all* sequences $\tilde{\mathbf{y}}$ such that $\sum \tilde{y}_i = s$;

The distribution of $\mathbf{y}|s$ is called a **conditionally uniform** distribution - each possible sequence gets equal probability.

A simulation of $\mathbf{y}|s$ can be generated as follows:

0. Put s ones into a bucket, along with $m - s$ zeros.
1. Randomly select a number from the bucket, assign it to y_1, and throw it away.
2. Randomly select a number from the bucket, assign it to y_2, and throw it away.

$\vdots$

m. Select the last number from the bucket and assign it to y_m.

This is called (uniform) **sampling without replacement**.

Under this scheme, we will always have $\sum y_i = s$.

ER graphs

Let's return to our SRG model for a sociomatrix:

$$H : \mathbf{Y} = \{y_{i,j}, i \neq j\} \sim \text{binary}(\theta), \text{ for some } \theta \in [0,1]$$

Under this model, $(y_{1,2}, \ldots, y_{n-1,n})$ forms an i.i.d. binary sequence.

- The conditional distribution of this sequence given $\sum y_{i,j} = s$ is simply the conditional uniform distribution.
- Knowing $\sum y_{i,j}$ is the same as knowing $\bar{y}$.
- The conditional distribution of $\mathbf{Y}$ given s is sometimes called the Erdos-Reyni graph ($SRG(n,s)$).
- Conditioning on s is the same as conditioning on $\bar{y}$.

Simulating from $\mathbf{Y}|s$

```
rY.s<-function(n,s)
{
  Y<-matrix(0,n,n) ; diag(Y)<-NA
  Y[!is.na(Y)]<- sample( c(rep(1,s),rep(0,n*(n-1)-s )))
Y
}

rY.s(5,3)

##      [,1] [,2] [,3] [,4] [,5]
## [1,]   NA    0    0    1    0
## [2,]    0   NA    0    0    0
## [3,]    0    0   NA    0    0
## [4,]    0    0    0   NA    1
## [5,]    1    0    0    0   NA

rY.s(5,10)

##      [,1] [,2] [,3] [,4] [,5]
## [1,]   NA    0    1    0    0
## [2,]    1   NA    0    1    1
## [3,]    0    0   NA    0    0
## [4,]    1    0    1   NA    1
## [5,]    0    1    1    1   NA
```

Conditional tests

Suppose $\mathbf{Y} \sim SRG(n, \theta)$ for some $\theta \in [0, 1]$. Then

$\mathbf{Y}$ shoud "look like" another sample from $SRG(n, \theta)$

- (but we can't generate these).

$\mathbf{Y}$ should also "look like" another sample from $SRG(n, s)$, where $s = \sum y_{i,j}$

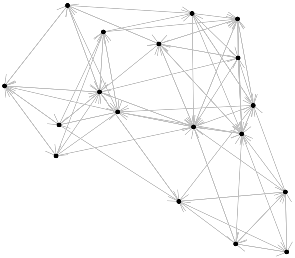
- (we can generate these).

Conditional evaluation of the SRG: Given a test statistic t,

$$\begin{array}{rccc} \text{compare} & t_{\text{obs}} & = & t(\mathbf{Y}) \\ \text{to} & \tilde{t} & = & t(\tilde{\mathbf{Y}}), \end{array}$$

where $\tilde{\mathbf{Y}} \sim SRG(n, \sum y_{i,j})$.

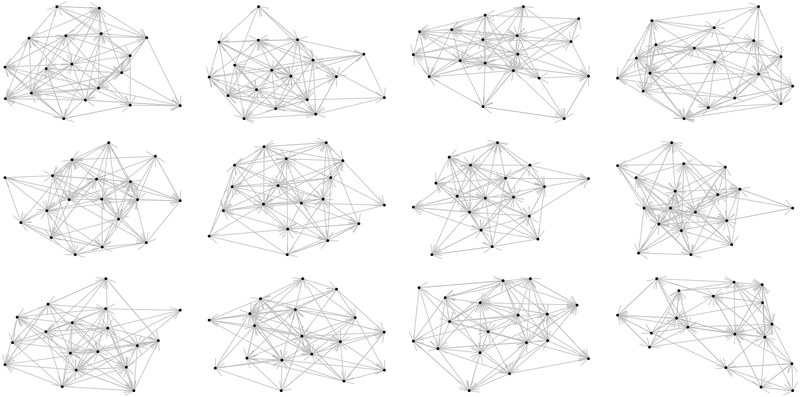
Example: Monk friendships



```
mean(Y,na.rm=TRUE)

## [1] 0.2875817

Cd(Y)

## [1] 0.07352941

Cd(t(Y))

## [1] 0.4044118

##

nrow(Y)

## [1] 18

sum(Y,na.rm=TRUE)

## [1] 88
```

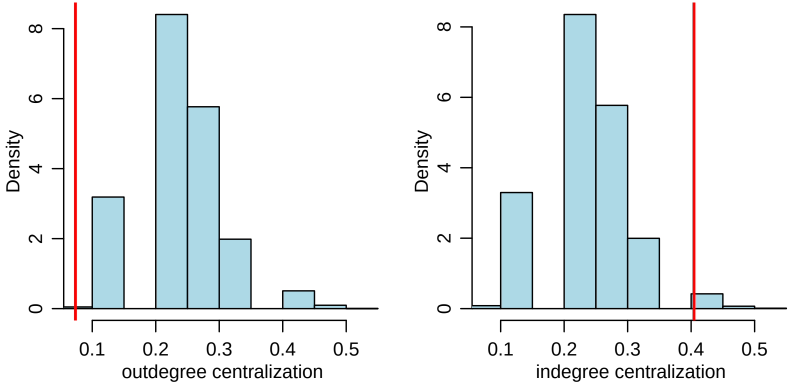
Simulated networks

```
Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
```



Example: Monk friendships

```
CD.H<-NULL
for(s in 1:S)
{
  Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
  CD.H<-rbind(CD.H, c(Cd(Ysim),Cd(t(Ysim))))
}
```



Example: Monk friendships

```
mean(CD.H[,1] <= Cd(Y))

## [1] 0.0024

mean(CD.H[,2] >= Cd(t(Y)))

## [1] 0.025
```

These can be interpreted as *p*-values, but a better thing to say is

- observed outdegree centralization was below the lower 1-percentile of the null distribution;
- observed indegree centralization was above the upper 3-percentile of the null distribution;

The interpretation is that both statistics show evidence against H.

Formal conditional testing

For any test statistic t, consider the following procedure:

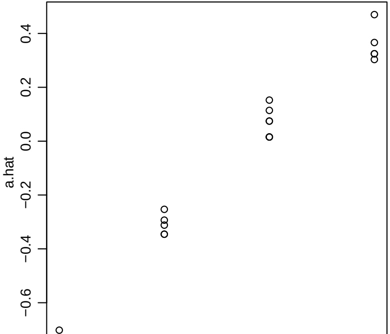
1. observe $\mathbf{Y}$
2. compute $p = \Pr(t(\tilde{\mathbf{Y}}) > t(\mathbf{Y}) | H, \sum \tilde{y}_{i,j} = \sum y_{i,j})$
3. reject H if $p < \alpha$.

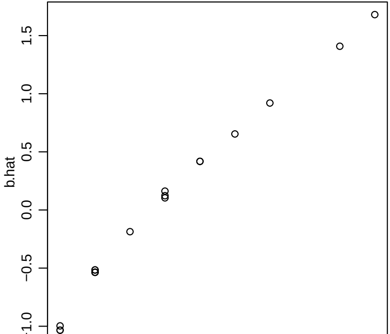
If $\mathbf{Y} \sim SRG(n, \theta)$ for some $\theta \in [0, 1]$ (i.e. H is true) , then

$$\Pr(\text{reject } H) = \alpha.$$

Comparison of principled to ad-hoc

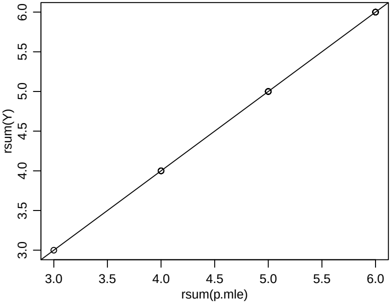
```
CD.H<-NULL
for(s in 1:S)
{
  Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
  #Ysim<-matrix(rbinom(nrow(Y)^2,1,mean(Y,na.rm=TRUE)),nrow(Y),nrow(Y))
  CD.H<-rbind(CD.H, c(Cd(Ysim),Cd(t(Ysim))))
}
mean(CD.H[,1] <= Cd(Y))

## [1] 0.0022

mean(CD.H[,2] >= Cd(t(Y)))

## [1] 0.0322
```

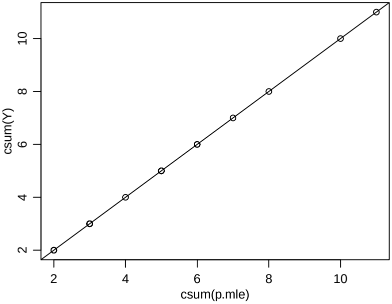




Comparison of principled to ad-hoc

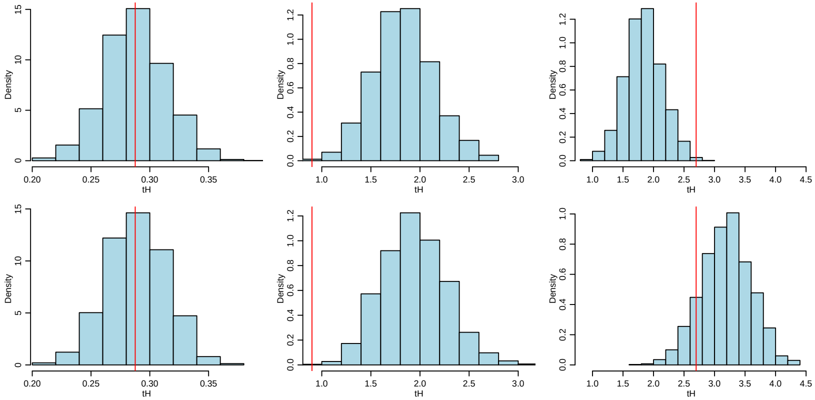
```
CD.H<-NULL
for(s in 1:S)
{
  #Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
  Ysim<-matrix(rbinom(nrow(Y)^2,1,mean(Y,na.rm=TRUE)),nrow(Y),nrow(Y))
  CD.H<-rbind(CD.H, c(Cd(Ysim),Cd(t(Ysim))))
}
mean(CD.H[,1] <= Cd(Y))

## [1] 6e-04

mean(CD.H[,2] >= Cd(t(Y)))

## [1] 0.0214
```





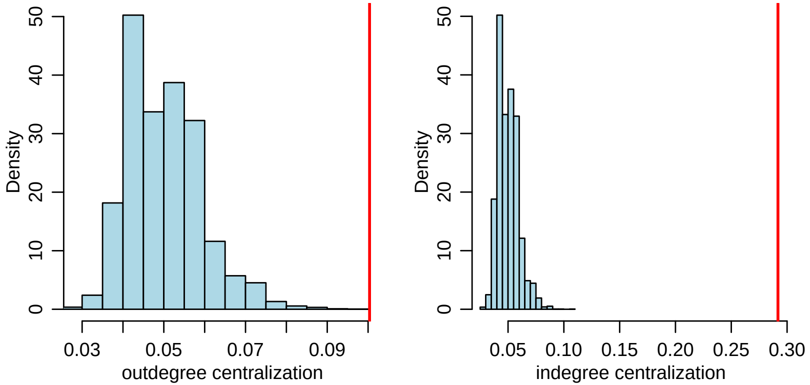
Example: High school friendships

```
CD.H<-NULL
for(s in 1:S)
{
  Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
  CD.H<-rbind(CD.H, c(Cd(Ysim),Cd(t(Ysim))))
}
```



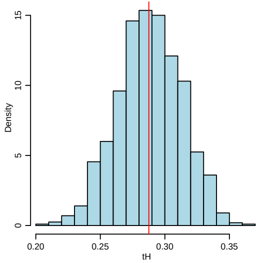
Example: High school friendships

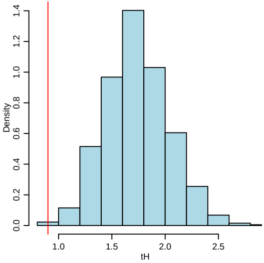
```
CD.H<-NULL
for(s in 1:S)
{
#  Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
  Ysim<-matrix(rbinom(nrow(Y)^2,1,mean(Y,na.rm=TRUE)),nrow(Y),nrow(Y))
  CD.H<-rbind(CD.H, c(Cd(Ysim),Cd(t(Ysim))))
}
```

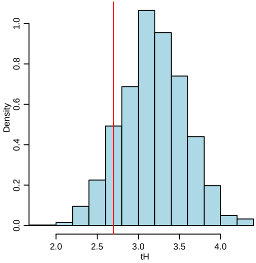


Example: High school friendships

```
Sd<-function(Y) { sd(apply(Y,1,sum,na.rm=TRUE)) }
SD.H<-NULL
for(s in 1:S)
{
  Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
  SD.H<-rbind(SD.H, c(Sd(Ysim),Sd(t(Ysim))))
}
```

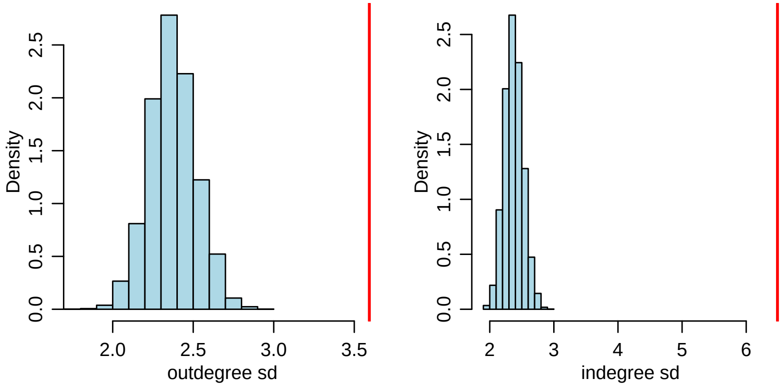






Example: High school friendships

```
SD.H<-NULL
for(s in 1:S)
{
#  Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
 Ysim<-matrix(rbinom(nrow(Y)^2,1,mean(Y,na.rm=TRUE)),nrow(Y),nrow(Y))
  SD.H<-rbind(SD.H, c(Sd(Ysim),Sd(t(Ysim))))
}
```



$SRG(n, \theta)$ for undirected graphs

```
n<-10 ; theta<-.1
Y<-matrix(0,n,n)
Y[upper.tri(Y)]<-rbinom( n*(n-1)/2 , 1 , theta )
Y<-Y+t(Y)
diag(Y)<-NA

Y

##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]   NA    0    0    0    0    0    0    0    0     0
##  [2,]    0   NA    0    0    0    0    0    0    0     0
##  [3,]    0    0   NA    0    1    1    1    0    0     0
##  [4,]    0    0    0   NA    0    0    0    0    0     0
##  [5,]    0    0    1    0   NA    0    0    1    0     0
##  [6,]    0    0    1    0    0   NA    0    0    0     0
##  [7,]    0    0    1    0    0    0   NA    0    0     0
##  [8,]    0    0    0    0    1    0    0   NA    0     0
##  [9,]    0    0    0    0    0    0    0    0   NA     0
## [10,]    0    0    0    0    0    0    0    0    0    NA

sum(Y)

## [1] NA
```

$SRG(n, s)$ for undirected graphs

```
n<-10 ; s<-10
Y<-matrix(0,n,n)
Y[upper.tri(Y)]<-sample( c(rep(1,s),rep(0,n*(n-1)/2 - s)) )
Y<-Y+t(Y)
diag(Y)<-NA

Y

##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]   NA    0    0    0    1    0    1    1    1     0
##  [2,]    0   NA    0    0    0    0    0    0    0     0
##  [3,]    0    0   NA    1    1    1    0    0    0     0
##  [4,]    0    0    1   NA    0    0    0    0    0     0
##  [5,]    1    0    1    0   NA    0    0    1    0     0
##  [6,]    0    0    1    0    0   NA    0    0    0     0
##  [7,]    1    0    0    0    0    0   NA    0    0     0
##  [8,]    1    0    0    0    1    0    0   NA    0     1
##  [9,]    1    0    0    0    0    0    0    0   NA     1
## [10,]    0    0    0    0    0    0    0    1    1    NA

sum(Y)

## [1] NA
```

Permutation tests for covariate effects

Recall the high-school girls friendship data:

- $y_{i,j}$ = indicator of friendship, among 144 students;
- x_i = indicator of above-average smoking behavior.

	xj=0	xj=1
xi=0	0.044	0.044
xi=1	0.031	0.045

Table : friendship rates between smoking categories

```
Xrow<-outer(x,rep(1,n))
Xcol<-outer(rep(1,n),x)
Xint<-outer(x,x)

fit<-glm( c(Y) ~ c(Xrow) + c(Xcol) + c(Xint) ,family=binomial)

fit$coef

##  (Intercept)      c(Xrow)      c(Xcol)      c(Xint)
## -3.072443033 -0.382355105 -0.006106814  0.400634157

exp(fit$coef)

## (Intercept)     c(Xrow)     c(Xcol)     c(Xint)
##  0.04630788  0.68225274  0.99391180  1.49277105
```

SRG null distribution

```
ebeta.obs<-exp( fit$coef )
ebeta.obs

## (Intercept)     c(Xrow)     c(Xcol)     c(Xint)
##  0.04630788  0.68225274  0.99391180  1.49277105

EBETA.sim<-NULL
for(s in 1:1000)
{
  Ysim<-rY.s( nrow(Y), sum(Y,na.rm=TRUE) )
  beta.sim<-glm( c(Ysim) ~ c(Xrow) + c(Xcol) + c(Xint) , family=binomial)$coef
  EBETA.sim<-rbind(EBETA.sim,exp(beta.sim) )
}

head(EBETA.sim)

##      (Intercept)   c(Xrow)   c(Xcol)   c(Xint)
## [1,]  0.04649499 0.8854832 0.9307677 1.0051026
## [2,]  0.04295135 1.0370672 0.9243420 1.0544853
## [3,]  0.04054054 1.0466761 1.1824768 0.7870772
## [4,]  0.03887804 1.1729487 0.9996460 1.0664013
## [5,]  0.04500000 0.9102323 0.9429514 1.0795469
## [6,]  0.04444048 1.0213677 0.8745243 1.0442391
```

SRG null comparisons

The null distribution we are using is a **conditional** null distribution:

- It is conditional on the observed number of ties $\sum i \neq j y_{i,j}$;
- It is *also* conditional on the observed values of $\mathbf{x}$, smoking behavior.

This distribution tests the following *joint* model for relational and nodal data:

Model: $\{\mathbf{Y}, \mathbf{x}\} \sim p(\mathbf{Y}, \mathbf{x})$

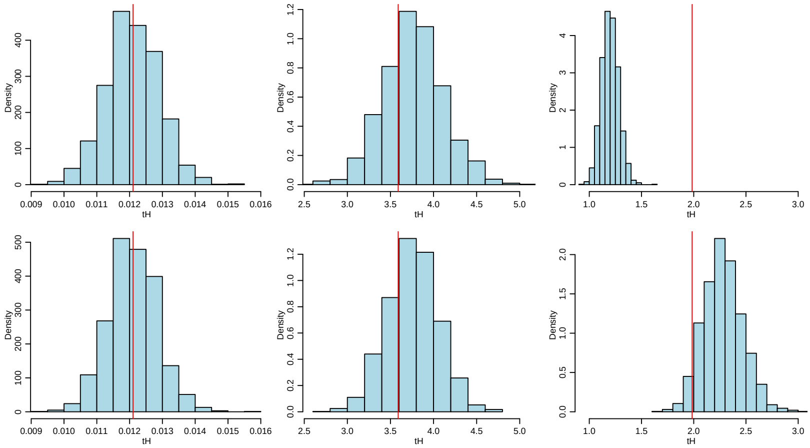
Null Hypothesis:

- $p(\mathbf{Y}, \mathbf{x}) = p(\mathbf{Y}) \times p(\mathbf{x})$, (ties are independent of covariates) **and**
- $p(\mathbf{Y})$ is a SRG(θ) distribution, for some θ.

Null Distribution: Test statistics are generated by

- simulating $\tilde{\mathbf{Y}}$ from the SRG conditional on $\sum \tilde{y}_{i,j} = \sum y_{i,j}$;
- simulating $\tilde{\mathbf{x}}$ conditional on $\tilde{\mathbf{x}} = \mathbf{x}$.

SRG null comparisons



SRG null comparisons

As compared to *any* SRG distribution for $\mathbf{Y}$ independent of $\mathbf{x}$,

- the data show lower smoker sociability (β_r)
- the data show higher homophily (β_{int}).

We reject the following hypothesis:

Null Hypothesis:

- $p(\mathbf{Y}, \mathbf{x}) = p(\mathbf{Y}) \times p(\mathbf{x})$, (ties are independent of covariates) **and**
- $p(\mathbf{Y})$ is a SRG(θ) distribution, for some θ.

But what are we rejecting?

- Are we rejecting because $\mathbf{x}$ and $\mathbf{Y}$ are not independent?
- Are we rejecting the SRG for $\mathbf{Y}$?

The rejection of the test isn't particularly compelling if we already suspect the SRG to be a poor model.

Tests based on exchangeability

Consider instead the following nonparametric null hypothesis:

Null Hypothesis:

- $p(\mathbf{Y}, \mathbf{x}) = p(\mathbf{Y}) \times p(\mathbf{x})$, (ties are independent of covariates) **and**
- $x_1, \ldots, x_n \sim$ i.i.d. $p(x)$ for some distributon $p(x)$.

Consider simulating values of $(\tilde{\mathbf{x}}, \tilde{\mathbf{Y}})$ the null distribution:

- Can't do it unconditionally, don't know $p(\mathbf{Y})$ or $p(\mathbf{x})$.
- What about conditionally?

Null Distribution: Condition on $\tilde{\mathbf{Y}} = \mathbf{Y}$, $\text{sort}(\tilde{x}_1, \ldots, \tilde{x}_n) = \text{sort}(x_1, \ldots, x_n)$.

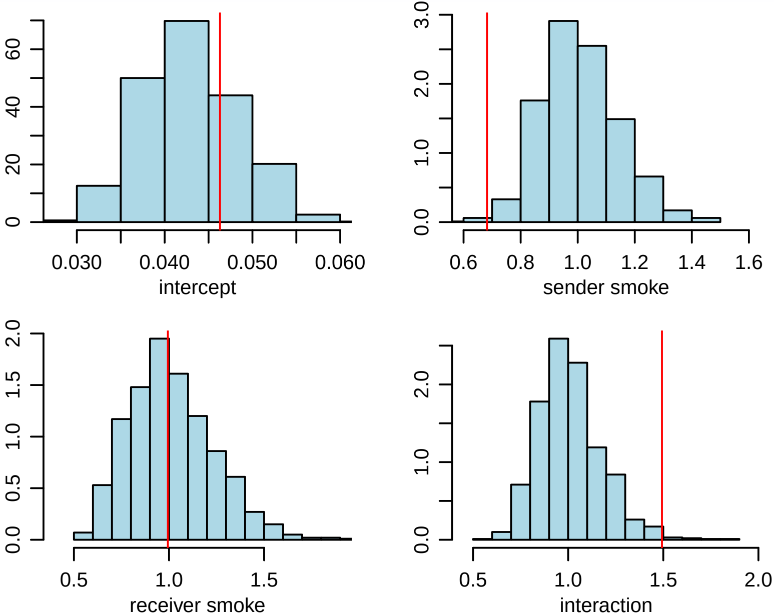
- Simulate $\tilde{\mathbf{Y}}$ conditional on $\tilde{\mathbf{Y}} = \mathbf{Y}$;
- Simulate $\tilde{\mathbf{x}}$ by permuting the entries of $\mathbf{x}$.

Null scenario: The scenario that is being mimicked here is $\mathbf{Y}$ being fixed, $\mathbf{x}$ determined independent of $\mathbf{Y}$.

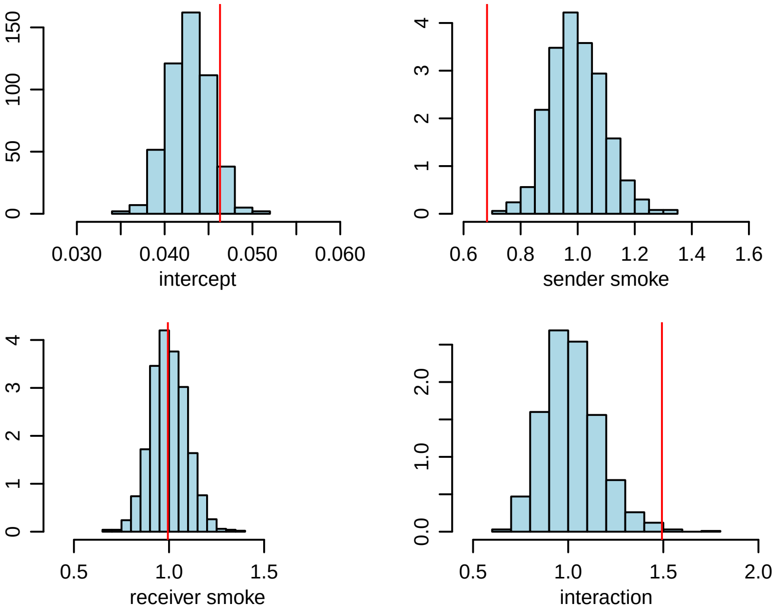
Permutation null distribution

```
EBETA.psim<-NULL
for(s in 1:1000)
{
  xs<-sample(x)
  Xsrow<-outer(xs,rep(1,n)) ; Xscol<-t(Xsrow) ; Xsint<-Xsrow*Xscol

  beta.sim<-glm( c(Y) ~ c(Xsrow) + c(Xscol) + c(Xsint) , family=binomial)$coef
  EBETA.psim<-rbind(EBETA.psim,exp(beta.sim) )
}

head(EBETA.psim)

##      (Intercept)  c(Xsrow)  c(Xscol)  c(Xsint)
## [1,]  0.04724409 0.9921875 0.8137748 0.9586459
## [2,]  0.04855761 0.8565281 0.7188310 1.5099572
## [3,]  0.03814086 1.1900643 1.0959716 0.9323231
## [4,]  0.04500000 0.9289176 0.9335937 1.0558645
## [5,]  0.03447057 1.2187706 1.3475352 0.8806846
## [6,]  0.05006280 0.8813055 0.7721326 1.0520387
```

Permutation null comparisons



SRG null comparisons



Comparison

While the conclusions for this dataset are basically the same, the evidence against the permutation null is generally weaker than against the SRG null.

- The SRG null makes stronger assumptions (that are generally false);
- The permutation null test requires conditioning on much of the data (which lowers power).

Recommendations:
If your goal is just to reject a null, then use the SRG null.

If you'd like to make more meaningful conclusions, use the permutation null.

Limitations: Permutation approaches only test coarse hypotheses:

- can test for **no effect** of a nodal covariate;
- can't test for **homophily**, in the presence of row and column effects.