Resolving GRB Light Curves

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Outline

Introduction

Gamma-ray bursts

LARK for Time Series
  Model
  Sample of Results

LARK for Joint Time and Energy
  Model
  GRB 501
  GRB 493

Discussion
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Discussion
Overview of Lévy Adaptive Regression Kernels

- **Goal:** inference on unknown function $f$
- **Kernel regression** approximates unknown function with weighted sum of functions
- **Adaptive kernel regression** infers the parameters of the kernel instead of using fixed dictionary

\[ f(x) \approx \sum_{j=1}^{J} u_j K(x \mid s_j, \theta_j) \]

where $s_j$ is the location of the kernel on the domain of $f$.

- LARK models the number, weights, and locations of the kernels as the largest jump discontinuities in a Lévy process
- Main advantage: stochastic process gives coherent way to incorporate threshold for approximation
Lévy (Infinitely Divisible) Processes

- For a one-dimensional stochastic process $X(t)$ to be a Lévy Process the increments $X(t_2) - X(t_1)$ for $t_1 < t_2$ must satisfy:
  
  **Stationarity** The distribution of an increment depends only on the length of the interval $(t_2 - t_1)$.
  
  **Independence** Disjoint increments are independent.
  
  - The Lévy-Khinchine theorem states that for an infinitely-divisible random variable $X$, the natural log of the characteristic function
    
    $$
    \log \mathbb{E} \exp \{i\omega' X\} = i\omega' m - \frac{1}{2} \omega' \Sigma \omega + \int_{\mathbb{R}} \left( e^{i\omega' u} - 1 - i\omega' h(u) \right) \nu(du)
    $$
    
    - The $\nu(du)$ corresponding to an infinitely divisible process is called its Lévy measure.
Lévy processes as a Poisson integral (W+Ickstadt, 1998)

- For a measure space \((S, \mathcal{F}, \nu(du))\), the Poisson random measure \(\mathcal{H}(du)\) is a random function from \(\mathcal{F}\) to \(\mathbb{N}\) such that for disjoint \(A_i \in \mathcal{F}\), \(\mathcal{H}(A_i) \sim \text{ind Po}(\nu(A_i))\).

- When \(\nu(du)\) is the Lévy measure for an infinitely-divisible random variable \(X\), then

\[
\mathcal{H}[u] = \int_{\mathbb{R}} u \mathcal{H}(du) = \sum_{j=1}^{\infty} u_j \overset{d}{=} X
\]

for (random) support \(\{u_j\}\) of \(\mathcal{H}\).
Lévy Processes as Poisson Random Fields

- Previous slide: 1D RV can be represented as the countable sum of the support of $\mathcal{H} \sim \text{Po}(\nu(du))$.
- For stochastic process case, we can write

$$\mathcal{H}[1_{\{0<s\leq t\}}] = \int_{\mathbb{R} \times (0,t]} u\mathcal{H}(du \, ds) = \sum \{u_j \mid 0 < s_j \leq t\} \overset{d}{=} X(t)$$

where $\{s_j\}$ are distributed uniformly on $S$ and $u_j$ are distributed proportionally to $\nu(du)$.
- In simulation, only a finite number of pairs $\{(s_j, u_j)\}$ are drawn, creating an approximation to the true process;

$$J_\epsilon \sim \text{Po}(\nu(\epsilon, \infty)), \quad \{u_j \mid u_j > \epsilon\} \overset{iid}{\sim} \nu(du)1_{\{u>\epsilon\}}/\nu(\epsilon, \infty)$$
Lévy Adaptive Regression Kernels

\[ f(t) := \int K(t \mid s, \theta) \mathbf{1}_{\{s \leq t\}} \mathcal{H}(du \, ds \, d\theta) \]

\[ = \sum_{s_j \leq t} u_j K(t \mid s_j, \theta_j) \]

where \((s_j, u_j)\) represent the countable innovations resulting from the random integral representation of a Lévy process. For tractable simulation, we use a truncation parameter \(\epsilon > 0\), and simulate from

\[ f_\epsilon(t) := \sum u_j K(t \mid s_j, \theta_j) \mathbf{1}_{\{s_j \leq t\}} \mathbf{1}_{\{u_j > \epsilon\}} \]

We use functions of this form to do Bayesian inference on unknown functions, such as light curves.
Example of a LARK model

Skewed $\alpha$-Stable: $\alpha = 1.5, \beta = 1, \gamma = 1$

Norris Kernel with $\tau_1 = 0.05, \tau_2 = 0.1$

Sum of Norris kernels with $\tau_1 = 0.05, \tau_2 = 0.1$

Poisson data with LARK model, 10 bins
LARK is coherent under aggregation
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Anatomy of a gamma-ray burst

Photo credit: NASA Goddard Space Center
Scientific Questions

- Number and shape of pulses
- Interaction between time and energy spectra
- Inverse problem from transformation of data by telescope
Pulse Number and Shape

- **Figure:** A variety of GRB photon rate time series (known as *light curves*)
- **Timescale:** 0.5s to 100s
- **Number of pulses:** 1 to 5 or more
Inverse Problem Induced by Detector

- **Figure:** Photons are sorted into 4 energy channels, based on the energy deposited (not the incident energy)
- Channel 1 is lowest energy; Channel 4 is highest
- Energy deposited is less than incident energy; scientific interest is in incident space
Interaction of Time and Energy

BATSE GRB Trigger 501

![Graphs of channel 1, channel 2, channel 3, and channel 4 showing photon counts over time since trigger.](image)
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Modeling a Single Light Curve

Channel 1

Channel 2

Channel 3

Channel 4

photons/s

-10 -5 0 5 10 15 20 25

time since trigger (s)

-10 -5 0 5 10 15 20 25

time since trigger (s)

-10 -5 0 5 10 15 20 25

time since trigger (s)

-10 -5 0 5 10 15 20 25

time since trigger (s)
Model

- Observe counts \((Y_1, \ldots, Y_N)\) in intervals \((t_0, t_1] \ldots (t_{N-1}, t_N]\).
- Model: \(Y_k \sim \text{Po}(\lambda_k)\) where
  \[
  \lambda_k = \int_{t_{k-1}}^{t_k} f(t) \, dt
  \]
  for some uncertain function \(f\).
- We model \(f\) in LARK form: \(f(t) = B + \sum_{j=1}^{J} A_j K(t \mid T_j, \theta_j)\)
- The number of kernels, \(J\), is unknown, as are \(\{(A_j, T_j, \theta_j)\}\) for \(1 \leq j \leq J\)
- We fix the baseline \(B\) and treat it as a known quantity
Kernel Selection

- Fast rise, exponential decay (FRED) shape
- Norris$^+$ (2005):

$$K_N(t \mid T, \tau_1, \tau_2) \propto \exp \left\{ -\frac{\tau_1}{t - T} - \frac{t - T}{\tau_2} \right\} 1\{t > T\}$$

Considered broader "GiG" class but data were inconclusive:

$$K_{GiG}(t \mid T, p, \tau_1, \tau_2) \propto (t - T)^p \exp \left\{ -\frac{\tau_1}{t - T} - \frac{t - T}{\tau_2} \right\} 1\{t > T\}$$
Summary of Poisson Random Field Representation

- Model pulse start times $T_j$'s, and their maximum amplitudes $A_j$'s as the jumps in a Lévy (e.g., Gamma or $\alpha$-Stable) process.
- The Poisson random field representation allows us to see the Gamma process as the sum of countably many jumps.
- whose times ($T_j$) are uniformly distributed on an interval.
- whose heights ($A_j$) larger than any $\epsilon > 0$ are distributed proportionally to
  \[
  \nu_\epsilon(du) = \alpha u^{-1} \exp\{-\beta u\} 1_{\{u > \epsilon\}} \; du.
  \]
- Infinitely many jumps lie below any threshold $\epsilon$, but we only simulate the finitely many jumps larger than $\epsilon$.
- The number $J_\epsilon$ of jumps larger than $\epsilon$ in a time interval of length $L$ has $\text{Po}\left(L \int_\epsilon^\infty \nu(du)\right)$ distribution.
Computation

- **RJ-MCMC** is a Metropolis-Hastings algorithm, but with a trans-dimensional proposal distribution, needed to do inference on our parameter space, the dimension of which is random and variable.

- Our proposal distribution is a mixture of five different proposals: *birth*, *death*, *walk*, *split*, and *merge*.

- **Parallel thinning**, a new variation on parallel tempering which exploits the infinite divisibility of LARK distributions to create interpretable auxiliary chains, also aids in mode-finding and mixing between modes.
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GRB 501– Channel 1
GRB 501 – Channel 1

(a)

(b)
GRB 501– Channel 1

(a)

(b)
GRB 501– Channel 2

Channel 1

Channel 2

Channel 3

Channel 4

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Resolving GRB Light Curves
GRB 501– Channel 2

(a)

(b)
GRB 501– Channel 2

(a)

(b)
GRB 501– Channel 3

Channel 1

Channel 2

Channel 3

Channel 4
GRB 501– Channel 3

(a)

(b)
GRB 501– Channel 3

(a) Number of Pulses vs. Photon Fraction

(b) Fraction Photons using Top J Pulses vs. Number of Pulses Used (J)
GRB 501– Channel 4

Channel 1

Channel 2

Channel 3

Channel 4

photon/s

-10 -5 0 5 10 15 20 25

time since trigger (s)

-10 -5 0 5 10 15 20 25

time since trigger (s)

-10 -5 0 5 10 15 20 25

time since trigger (s)

-10 -5 0 5 10 15 20 25

time since trigger (s)
GRB 501– Channel 4

(a)

(b)
GRB 501 – Channel 4

(a) Number of Pulses

(b) Fraction Photons using Top J Pulses
Models in different channels are incoherent

Number of pulses in each channel

(a): Channel 1  
(b): Channel 2  
(c): Channel 3  
(d): Channel 4
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Model

$i$: time index
$k$: channel index
$j$: pulse index

\[ E(Y_{ik}) = \mu_{ik} \]
\[ \mu_{ik} = \int_{t_i}^{t_{i+1}} B_k \, dt \]
\[ + R_k \left( \int_{t_i}^{t_{i+1}} \sum_{j=1}^{J} V_j \times \Psi(t \mid E, \theta_j) \times \Phi(E \mid \theta_j) \, dt \, dE \right) \]

$R_k$ is the response function for channel $k$
$B_k$ is the empirically-determined baseline photon rate
Kernel Function – Time Spectrum for a Single Pulse

Time fluence conditioned on energy:

\[ \psi(t \mid E, T, \gamma, \lambda) = \psi(t - T + \gamma \ln(E/100))/K \]
\[ \psi(x) = \exp(-\lambda x)1_{\{x > 0\}}. \]

- This model has the pulse arrive instantly (time \( T \), for photons of energy \( E = 100\text{keV} \))
- \( \lambda \): time decay parameter
- \( \gamma \): controls the fact that pulses arrive at different times at different energies (typically, more energetic ones arrive earlier)
- \( K \) is a normalizing constant needed for the interpretation of \( V_j \) as pulse fluence
Kernel Function – Energy Spectrum for a Single Pulse

Comptonized Energy Spectrum:

\[ \Phi(E | \alpha, E_c) = \left( \frac{E}{100} \right)^\alpha \exp\left\{ -\frac{E}{E_c} \right\} \mathbf{1}_{\{E > 0\}} \]

- Energy spectrum decays as a power law with index \( \alpha < 0 \)
- After cutoff \( E_c \), spectrum decays exponentially
- Inference is challenging due to having only 4 channels of DISCLA (Discriminator Large Area) data
How $\lambda$ and $\gamma$ interact to create pulse shape
\[ f(v) = 3 \frac{\eta}{\sqrt{8\pi}} V^{-5/2} 1_{\{V > \epsilon\}} \]

- Volumes of pulses \( > \epsilon \) are modeled by the largest jumps of a fully-skewed \( \alpha \)-Stable process with \( \alpha = 3/2 \)

- \( \alpha = 3/2 \), reflects the observation that photon fluence decays as a power law with index \(-5/2\)

- \( \epsilon = 1 \) photon/cm\(^2\)

- \( \eta = 0.1074 \) so that \( E(J_\epsilon) = 3 \)
Overdispersion

- To compensate for model misspecification, an overdispersion parameter, \( r \), is introduced. This parameter does not depend on the time interval or the energy channel.
- Instead of modeling \( Y_{ij} \) as Poisson,

\[
Y_{ij} \sim \text{NB} \left( n = r\mu_{ij}, p = \frac{r}{1 + r} \right)
\]

\[
\text{E}(Y_{ij}) = \mu_{ij}
\]
\[
\text{V}(Y_{ij}) = \mu_{ij} \frac{1+r}{r}
\]
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GRB 501

Channel 1

Channel 2

Channel 3

Channel 4

time since trigger (s)
GRB 501

![Graphs showing GRB light curves](image)
GRB 501 – Results

Figure: 95% Credible Interval for Mean, GRB 501
GRB 501 – Results

**Figure:** 95% posterior predictive intervals for GRB 501
GRB 501 – Results

Figure: Number of Pulses, GRB 501
GRB 501 – Overdispersion

Figure: Variance inflation
GRB 501 – Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pulse 1</th>
<th>Pulse 2</th>
<th>Pulse 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ (s)</td>
<td>$(-0.542, -0.344)$</td>
<td>$(1.58, 2.44)$</td>
<td>$(-0.602, -0.287)$</td>
</tr>
<tr>
<td>$V$ ($\gamma$/s)</td>
<td>$(10.2, 19.3)$</td>
<td>$(1.02, 6.46)$</td>
<td>$(1.00, 1.45)$</td>
</tr>
<tr>
<td>$\gamma$ (s)</td>
<td>$(0.5923, 0.875)$</td>
<td>$(0.0119, 1.370)$</td>
<td>$(-0.665, -0.154)$</td>
</tr>
<tr>
<td>$\lambda$ ($s^{-1}$)</td>
<td>$(0.344, 0.511)$</td>
<td>$(0.415, 1.19)$</td>
<td>$(1.01, 1.59)$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$(-0.229, 0.0)$</td>
<td>$(-2.09, -0.001)$</td>
<td>$(-1.42, -0.053)$</td>
</tr>
<tr>
<td>$E_c$ (keV)</td>
<td>$(53.1, 71.5)$</td>
<td>$(28.3, 386.0)$</td>
<td>$(79.3, 362.0)$</td>
</tr>
</tbody>
</table>

**Table:** Highest posterior density intervals (95%) for the parameters of the three-pulse models, GRB 501.
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GRB 493 – Doesn’t always work as desired!

Figure: 95% Credible Interval for Mean, GRB 493
GRB 493 – Doesn’t always work as desired!

Figure: Number of Pulses, GRB 493
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Discussion
Our model successfully models the number and shapes of pulses for some GRBs, but not others, due to strict modeling of the rises of pulses as “instantaneous” in incident space.

This modeling approach does not depend on fixed-length time or energy bins.

Our approach successfully incorporates knowledge of the photon detector and corrects for systematic bias in photon energy assignment.
Future work

- Flexible model for better rise-time modeling
- Improve sampling for better effective sample sizes
- Incorporate 16-channel data (MER) instead of 4 (DISCLA) for improved recovery of spectral parameters
- Share information among bursts to infer population parameters
Thanks for your attention!

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