Combining Deterministic & Stochastic Models for Hazard Assessment

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Introduction
- The Problem
- Emulation
- Catastrophe Threshold

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The Stochastic Model
- PF Volumes
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- PF Times
- Summary

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The Caribbean Island of Montserrat

Our Goal:

- Goal: quantify the hazard from Pyroclastic Flows (PFs)—
- For each location "x" on Montserrat, we would like to find the probability of a "catastrophic event" (*e.g.*, inundation to $\geq$1m) within T years— for $T = 1$, $T = 5$, $T = 10$, $T = 20$ years. Isoprobability curves $\Rightarrow$ "Hazard Maps".
- Reflecting all that is uncertain, including:
  - How often will PFs of various volumes occur?
  - What initial direction will they take?
  - How will the flow evolve?
  - How are things changing, over time?
- This is *not* what is needed for short-term crisis and event management— here we consider only long-term hazard.

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Pieces of the Puzzle

Our team (geologist/volcanologist, applied mathematicians, statisticians, a stochastic processor) breaks the problem down into two parts: two Stochastic, one Deterministic.

- Try to model the volume, frequency, and initial direction of PFs at SHV, based on 15 years' data, using Prob & EVT;
- For specific volume V and initial direction θ (and other uncertain things like the basal friction angle ϕ), try to model the evolution of the PF, and max depth $M(x)$ at x, using PDE-based TITAN-2D
- Try to model what TITAN-2D *would* say at untried locations, using GaSP Emulator

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A Brute Force Bayesian Method

- Draw $V_j,\ t_j,\ \theta_j,\ \phi_j$ from some dist'n;
- Compute inundation height $M(x;\ V_j, \theta_j, \phi_j)$ at site x using TITAN-2D for each site x of interest;
- Use M/H to decide if you like the draw;
- Repeat about 10^6 times
- Consuming about 10^6 hours.

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- Faster now, but

  Still missing low-probability high-consequence tail behavior.

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Finally, a New Idea

- Key idea: If some volume V with initiation angle θ inundates location x, then any higher volume $V' > V$ will too.
- SO: For each angle $0 \leq \theta < 2\pi$, identify the *smallest* volume $\Psi(\theta; x)$ that will reach x.
- Then: Find the probability of at least one $(V_j,\ t_j,\ \theta_j)$ with:

$$V_j \geq \Psi(\theta_j, x), \qquad t_0 < t_j \leq t_0 + T$$

- How do we find $\Psi(\theta, x)$?

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Finally, a New Idea That Just Might Work

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- How do we find $\Psi(\theta, x)$?

Emulation

We could pick off $\Psi(\theta; x)$ from a grid of TITAN-2D outputs at each of:

- 100 Initiation Angles θ_i;
- 100 Volumes V_j;
- 100 Friction Angles ϕ_k;

Which would entail maybe $100 \times 100 \times 100 = 1\,000\,000$ runs of TITAN-2D...but we still don't have $1\,000\,000$ hours. Our Solution:

Build an Emulator for the slow T2D PDE flow model.

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A GaSP Emulator is a (very fast):

- statistical model (based on Gaussian Processes) for our
- computer model (based on PDE solver) of the
- volcano.

- The emulator can predict (in seconds) what TITAN-2D would return (in hours), *with* an estimate of its accuracy;
- Based on Gaussian Stochastic Process (GaSP) Model. We:
  - Pick a few hundred "design points" $(V_j, \theta_j, \phi_j, ...)$ (LHD);
  - Run TITAN-2D at each of them to find TITAN-2D Model Output $M(x;\ V_j, \theta_j, \phi_j)$;
  - Train the GaSP to return a statistical estimate

$$\mathrm{E}\left[M(x:\ V, \theta, \phi) \mid \{M(x:\ V_j, \theta_j, \phi_j)\}\right]$$

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A Reminder— What do we Emulate, and Why?

With this we try to evaluate a "threshold function" for each site x in Montserrat:

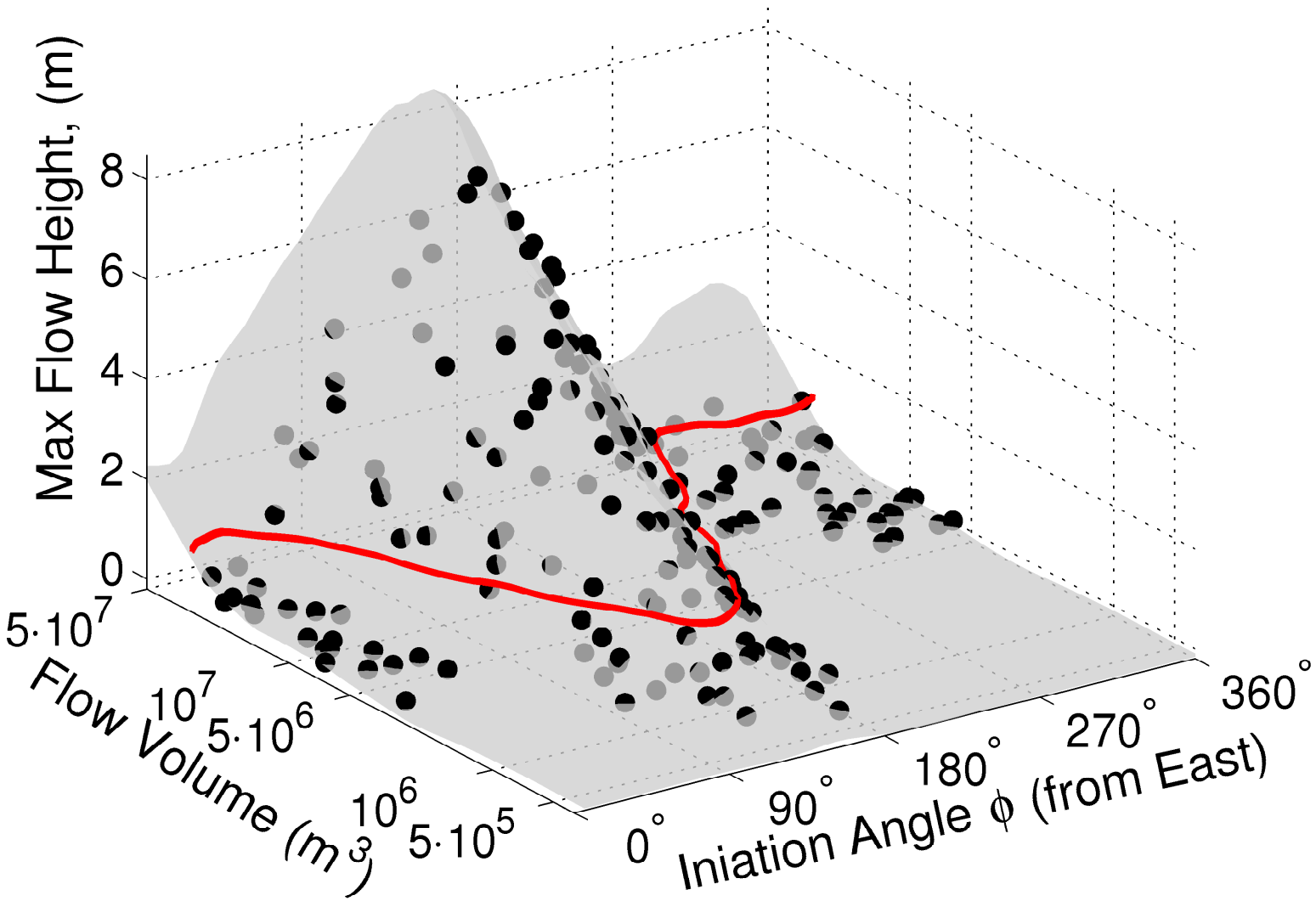
$$\Psi(\theta; x) = \text{Smallest volume } V \text{ that would inundate site } x \text{ if flow begins in direction } \theta$$

for each possible angle θ (0–2π, with $0 =$ East and $\pi/2 =$ North). Then we Quantify the Uncertain Hazard at x for T years as

$$\Pr\left\{ V_i \geq \Psi(\theta_i; x) \text{ for some PF } (V_i, \theta_i) \text{ in time } [t_0, t_0 + T] \right\}$$

which in turn we study with the probability models.

Finding $\Psi(\theta; x)$ In Pictures



Designs and Sub-designs

- We train the Emulator at "Design Points" $\{(V_i, \theta_i, \phi_i)\}$—
- *Not* random variables from a prior distribution— rather,
- as widely-dispersed as we can manage in a 3-dim space.
- We use a maxi-min Latin Hypercube Design (LHD), with 2048 points in $[10^5, 10^9] \times [0, 2\pi) \times (8^o, 12^o)$
- . . . but we can't invert 2048×2048 matrices, so we look for special SUBDESIGNS ($\approx$ 40–50 pts) for each x.

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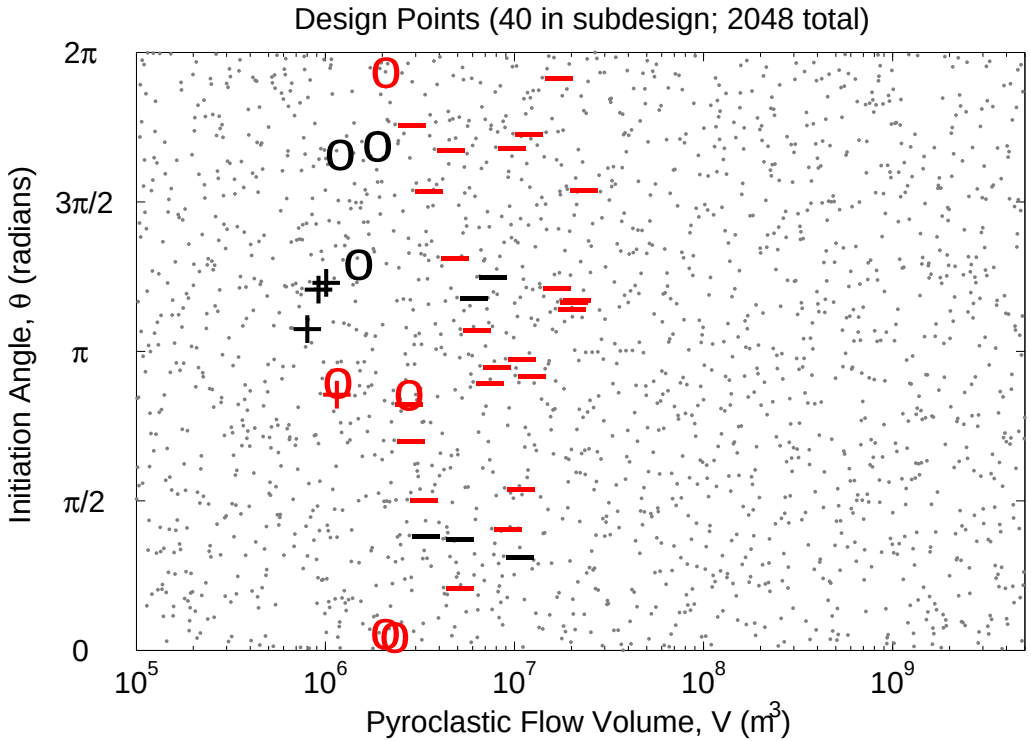
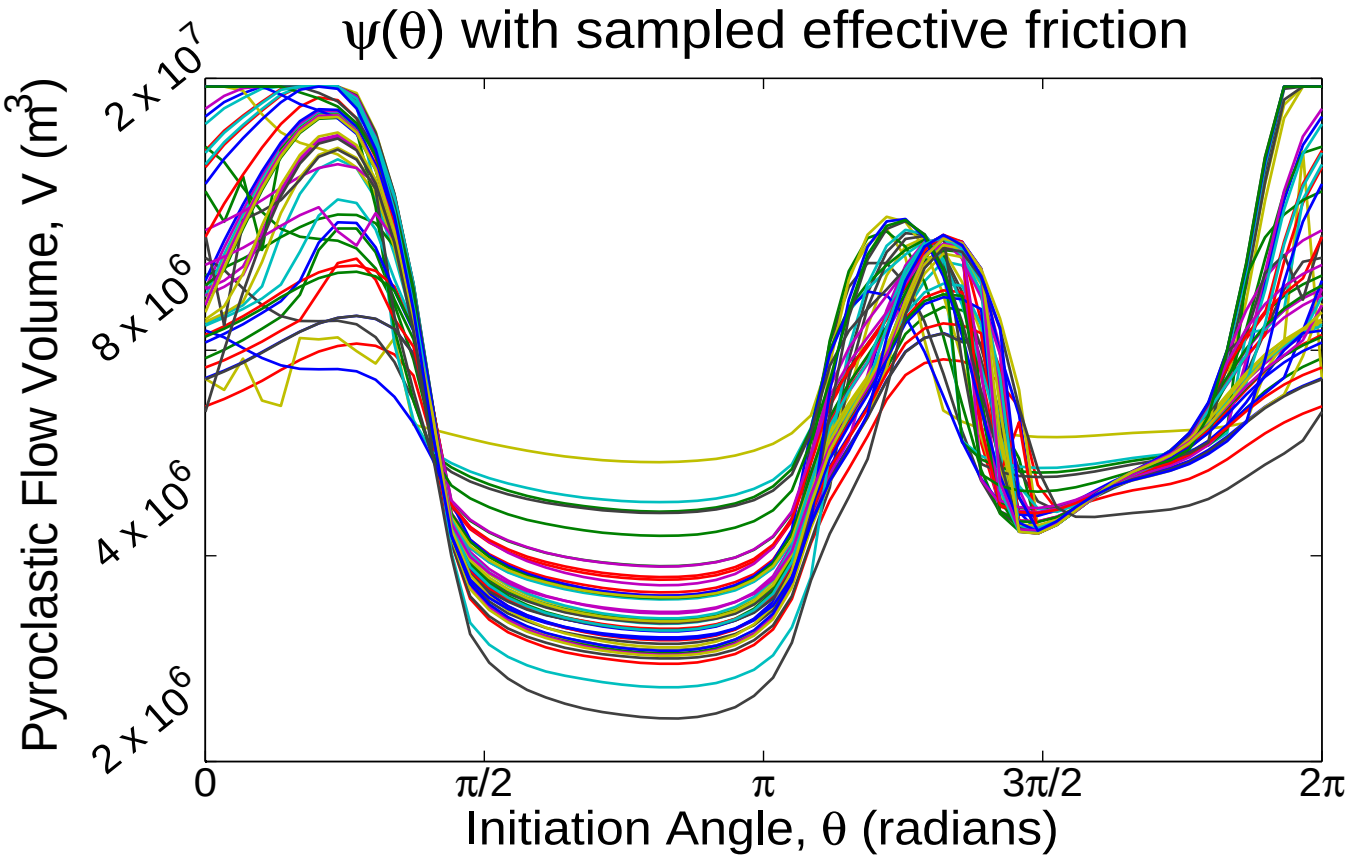
Design points V, θ



2048 Design Points total; 40 used for determining Ψ at $x=$
Level 4 Trigger Point: Dyers River Valley (head of Belham valley) .
Black $\Longleftrightarrow M(x;\ V,\theta,\phi) = 0$ Red $\Longleftrightarrow M(x;\ V,\theta,\phi) > 0$;

$-$: $8^o < \phi < 8.5^o$; $\bigcirc$: $8.5^0 < \phi < 10.5^o$; $+$: $10.5^o < \phi < 12^o$.

50 Simulations of $\Psi(\theta; x)$ for TP4, Dyers RV



Combining Det & Stoch Models

We have tried to separate our challenge into

- The Deterministic problem of identifying those "input values" V, θ, ϕ that lead to a catastrophe at x, using T2D, and
- The Stochastic problem of finding the probability of such an event.
- Use Det Mod to identify the Hazard Region in parameter space;
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SO, we need a joint model for points $\{(V_j, \theta_j, t_j)\} \subset \mathbb{R}^3$.
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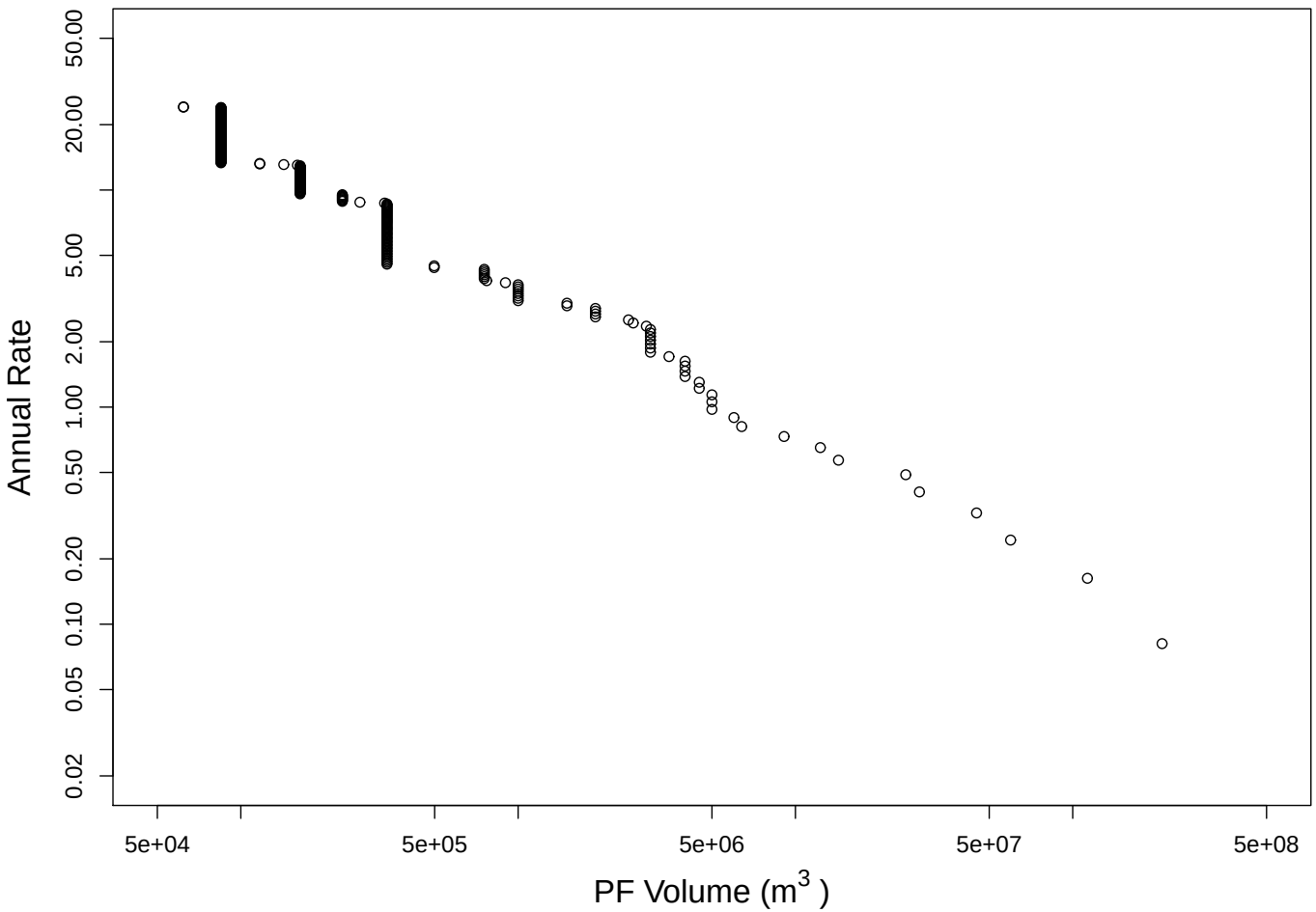
PF Volumes



Seems kind of linear, on log-log scale... remember that. rtn

PF Frequencies (1990's–2010)

- PF's exceeding 10^4 m^3: Daily, 0–1.6 km runout
- PF's exceeding 10^5 m^3: Fortnightly, 2–3.0 km runout
- PF's exceeding 10^6 m^3: Quarterly, 4–6.0 km runout
- PF's exceeding 10^7 m^3: Yearly, 6.0+ km runout
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How much *is* $10^9\,\mathbf{m}^3 = 1\,\mathbf{km}^3$ **of flow?**

V	Cube Width	Radius 1 m deep
10^3	10 m (32.8')	18 m
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10^9	1000 m (0.6 mi)	17.8 km

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Remember how the log-log Freq. *vs.* Volume plot was nearly linear? Linear log-log plots of Magnitude *vs.* Frequency are a hallmark of the Pareto probability distribution $\mathrm{Pa}(\alpha, \epsilon)$,

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The Pareto Distribution

The negative slope seems to be about $\alpha \approx 0.64$ or so.
The Pareto distribution with $\alpha < 1$ has:

- Heavy tails;
- Infinite mean $\mathrm{E}[V] = \infty$, infinite variance $\mathrm{E}[V^2] = \infty$;
- No Central Limit Theorem for sums (α-Stable instead);
- Significant chance that, in the future, we will see volumes larger than any we have seen in the past. Like $V > 10^9 \mathrm{m}^3$.
- The Pareto comes up all the time in the Peaks over Threshold (PoT) approach to the Statistics of Extreme Events— related to Fisher/Tippett Three Types Theorem.

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- Infinite mean $\mathrm{E}[V] = \infty$, infinite variance $\mathrm{E}[V^2] = \infty$;
- No Central Limit Theorem for sums (α-Stable instead);
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- The Pareto comes up all the time in the Peaks over Threshold (PoT) approach to the Statistics of Extreme Events— related to Fisher/Tippett Three Types Theorem.

The Pareto Distribution

The negative slope seems to be about $\alpha \approx 0.64$ or so.
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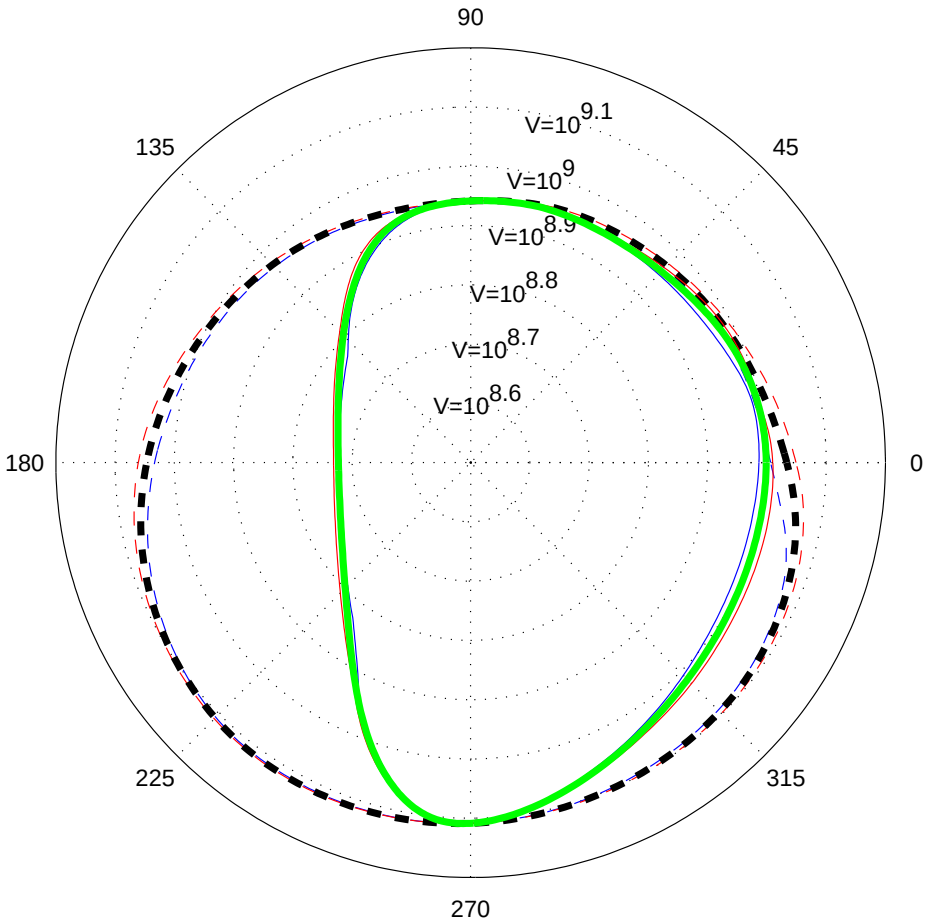
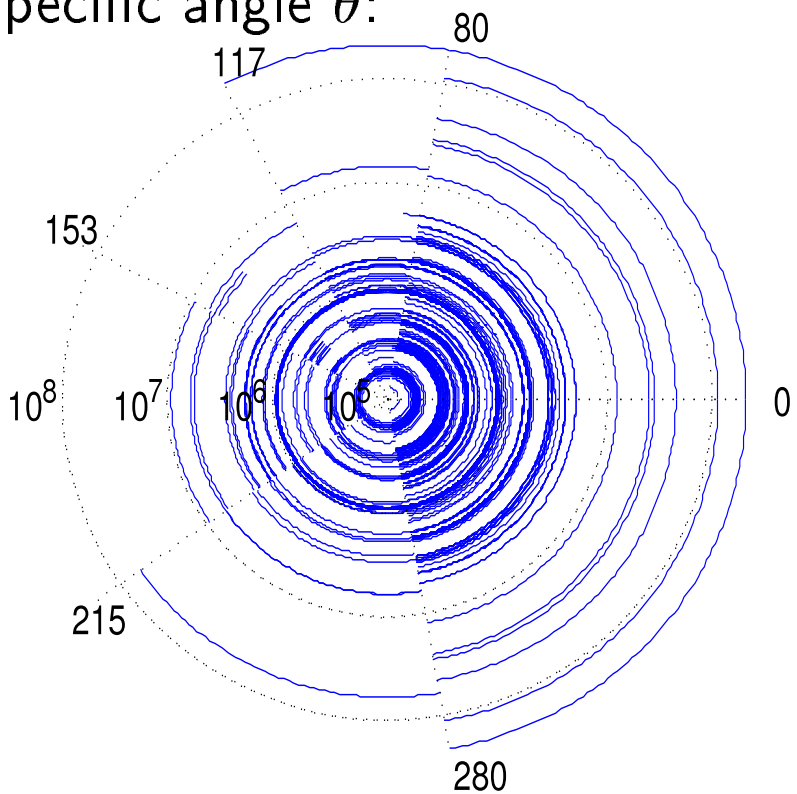
How big is bad?



Kinda depends which way it goes...

PF Initiation Angles

Our data on angles is quite vague— we only know which of 7 or 8 *valleys* were reached by a given PF, from which we can infer a *sector* but not a specific angle θ:



Any nonuniformity for θ? Any dependence of θ on Volume V?

Angle/Volume Data (cont)

We need a *joint* density function

$$f(V,\ \theta) = f(V)\ f(\theta \mid V)$$

given by:

$$V, \theta \sim \alpha \epsilon^{\alpha}\ V^{-\alpha-1} \mathbf{1}_{\{V>\epsilon\}} \pi_{\kappa}(\theta)$$

where $\pi_{\kappa}(\theta)$ is the von Mises distribution with pdf

$$f(\theta \mid \kappa, \mu) = \frac{e^{\kappa \cos(\theta-\mu)}}{2\pi I_0(\kappa)},$$

centered at $\theta \approx \mu$ close to zero (East) with concentration κ that might depend on V if the data support that.

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PF Times

If we observe λ PFs per year of volume $V > \epsilon$, then what is the probability that such a PF will occur in the next 24 hours?

For short-term predictions, it may be important to note this can depend on:

- How high is the dome just now?
- Any seismic activity suggesting dome growth and instability?
- Has it rained recently?
- How long an interval since last PF?

But, for long-term predictions all these factors average out and we assume that:

- PF occurrences in disjoint time intervals are independent
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Under those assumptions (which we will revisit), the number of PFs in any time interval of length Δt (like 1/365, for a single day) has an expected number of $\lambda \Delta t$ PFs and a *probability* of

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The data suggest a (provisional!) model in which:

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These have the beautiful and simplifying consequence that the number of PFs in *any* region of three-dimensional $(V \times \theta \times t)$ space has a Poisson probability distribution— so, we can evaluate:

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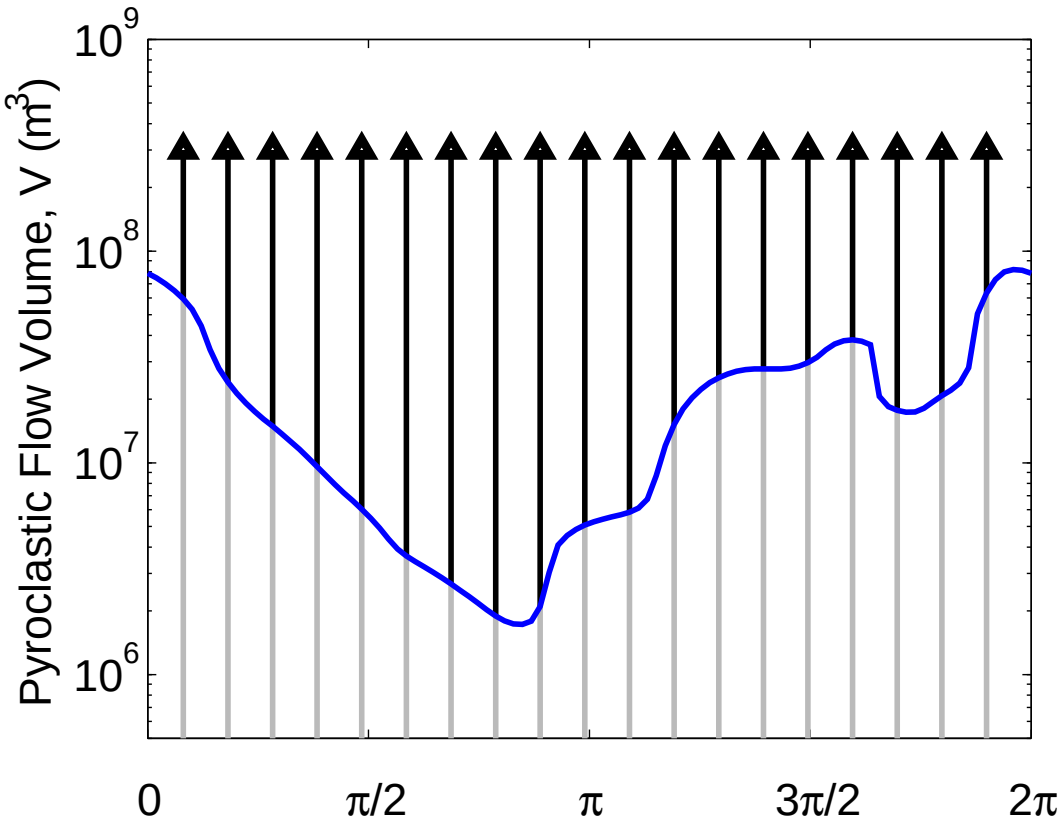
Hazard (in formulas)

The hazard prediction we wish to make is

$$
\begin{aligned}
&\mathrm{P}[\text{ Catastrophy within } t \text{ Years }] \\
&\quad = 1 - \mathrm{P}[Y_t = 0] \qquad \text{(where } Y_t \text{ is the } \textit{number} \text{ of catastrophes)} \\
&\quad = 1 - \exp\left(- \mathrm{E}\, Y_t \right) \\
&\quad = 1 - \exp\left(-t\, \lambda_\epsilon \int_{V > \Psi(\theta; x)} \epsilon^\alpha \alpha V^{-\alpha-1}\, dV\, \pi_\kappa(d\theta) \right)
\end{aligned}
$$

Which we can compute pretty easily on a computer:

Hazard (in pictures)



If we knew the (uncertain) values of the Pareto parameter $\alpha \approx 0.64$, rate parameter $\lambda_\epsilon \approx 22/yr$, basal friction $\phi \approx 8$–11^o.

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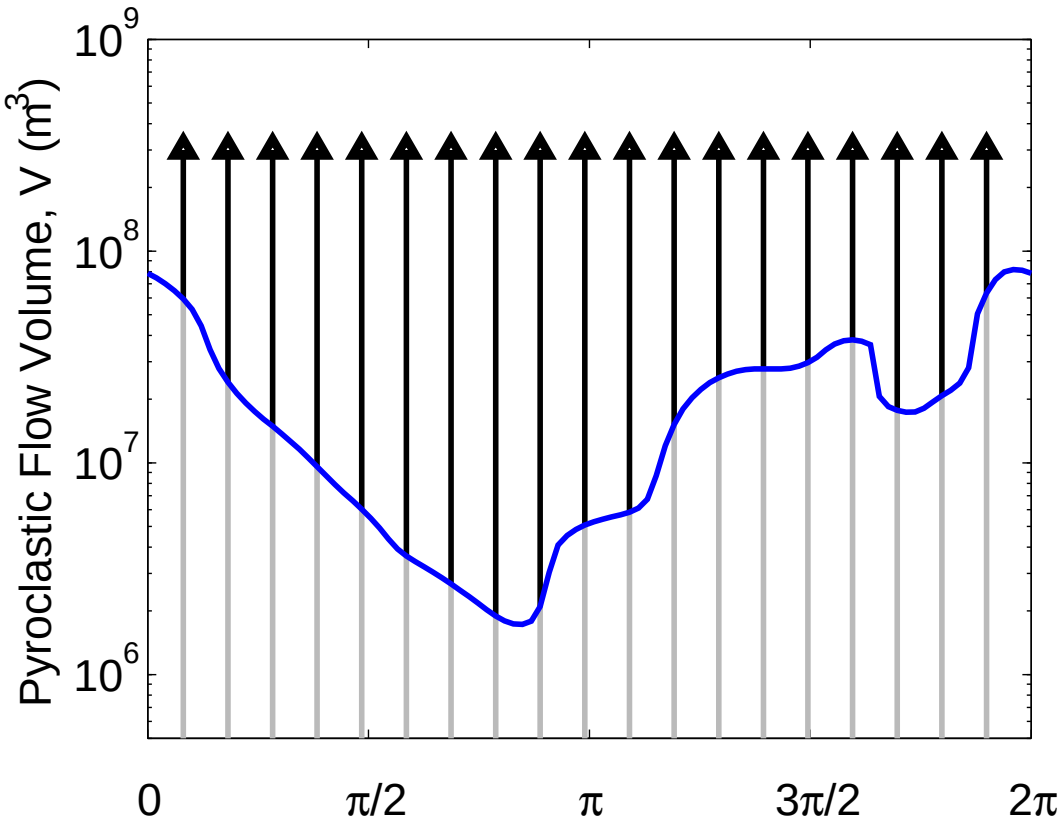

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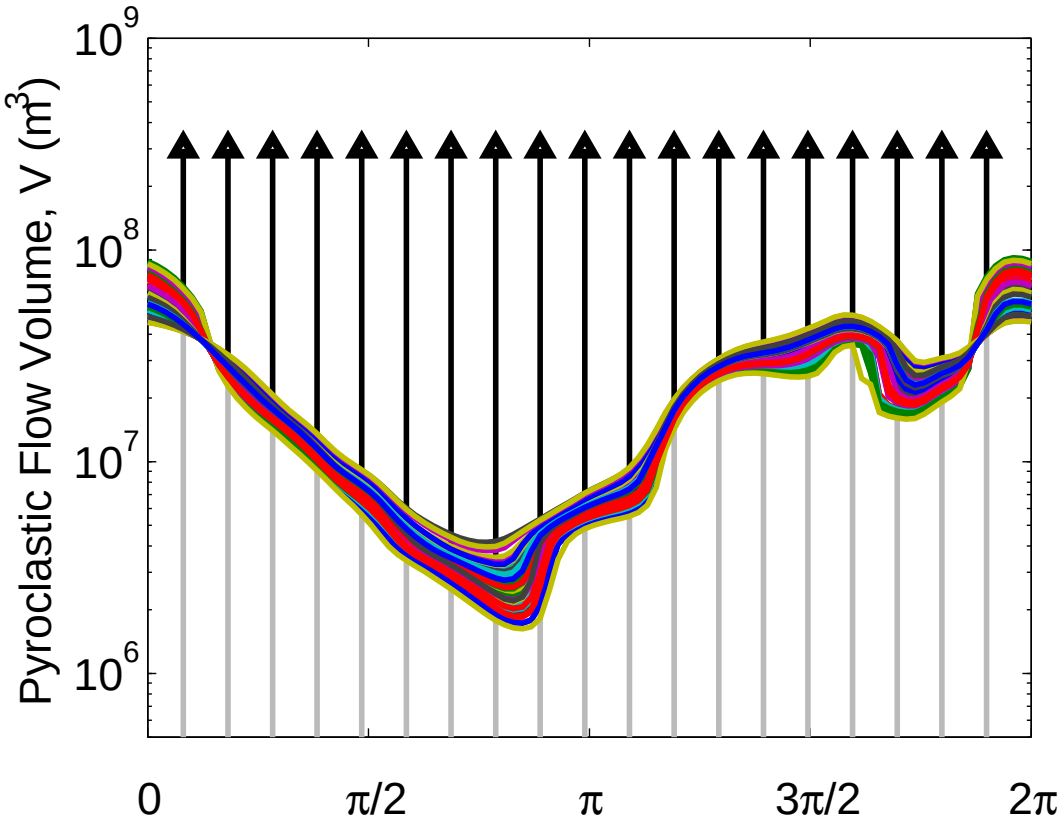

The necessary integrals in V and λ_ϵ are available in closed form; we accommodate (posterior uncertainty) about α and ϕ using MCIS.

Computing the Posterior

$$\text{P[Catastrophy within } t \text{ Years } \mid \lambda_\epsilon,\, \alpha,\, \phi]$$

$$= 1 - \exp\left(-t\,\lambda_\epsilon \int_{V>\Psi(\theta;x)} \epsilon^\alpha \alpha V^{-\alpha-1}\, dV\, \pi_\kappa(d\theta) \right)$$

$$= 1 - e^{-t\,\lambda_\epsilon I_\epsilon(\alpha,\phi)} \quad \text{where} \quad I_\epsilon(\alpha,\phi) = \int_0^{2\pi} \{\Psi(\theta;x)/\epsilon\}^{-\alpha}\, \pi_\kappa(d\theta)$$

$$\text{P[No Catastrophy } \mid \text{ Data]}$$

$$= Z^{-1} \int e^{-t\,\lambda_\epsilon I_\epsilon(\alpha,\phi)}\, L(\alpha,\lambda,\phi)\, \pi(d\lambda\, d\alpha\, d\phi)$$

$$= Z^{-1} \int \left\{1 + \frac{t}{Tb} I_\epsilon(\alpha,\phi)\right\}^{-J_\epsilon - a} L(\alpha,\phi)\, \pi(d\alpha\, d\phi)$$

$$\approx \frac{1}{N} \sum \left\{1 + \frac{t}{Tb} I_\epsilon(\alpha_i,\phi_i)\right\}^{-J_\epsilon - a}$$

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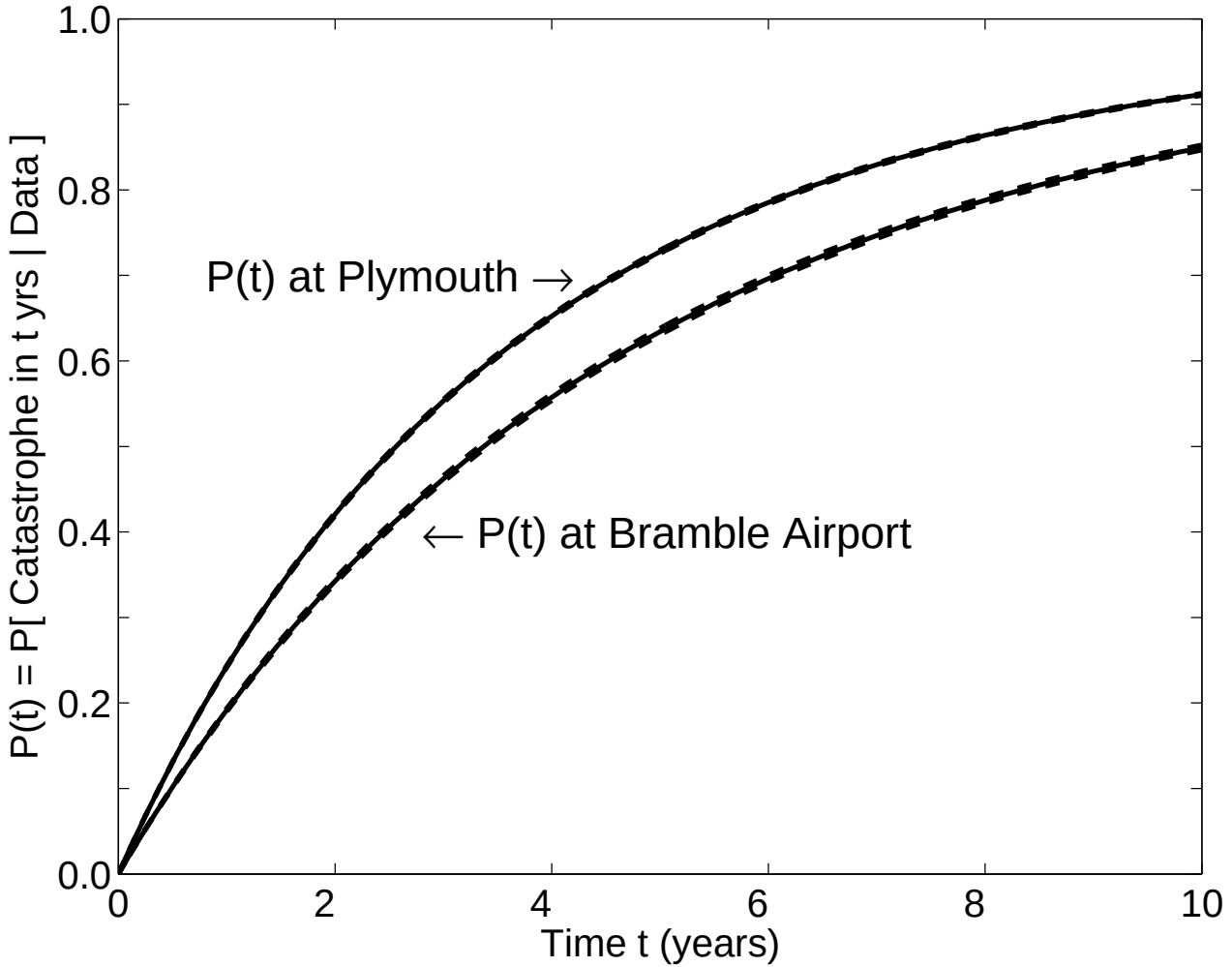
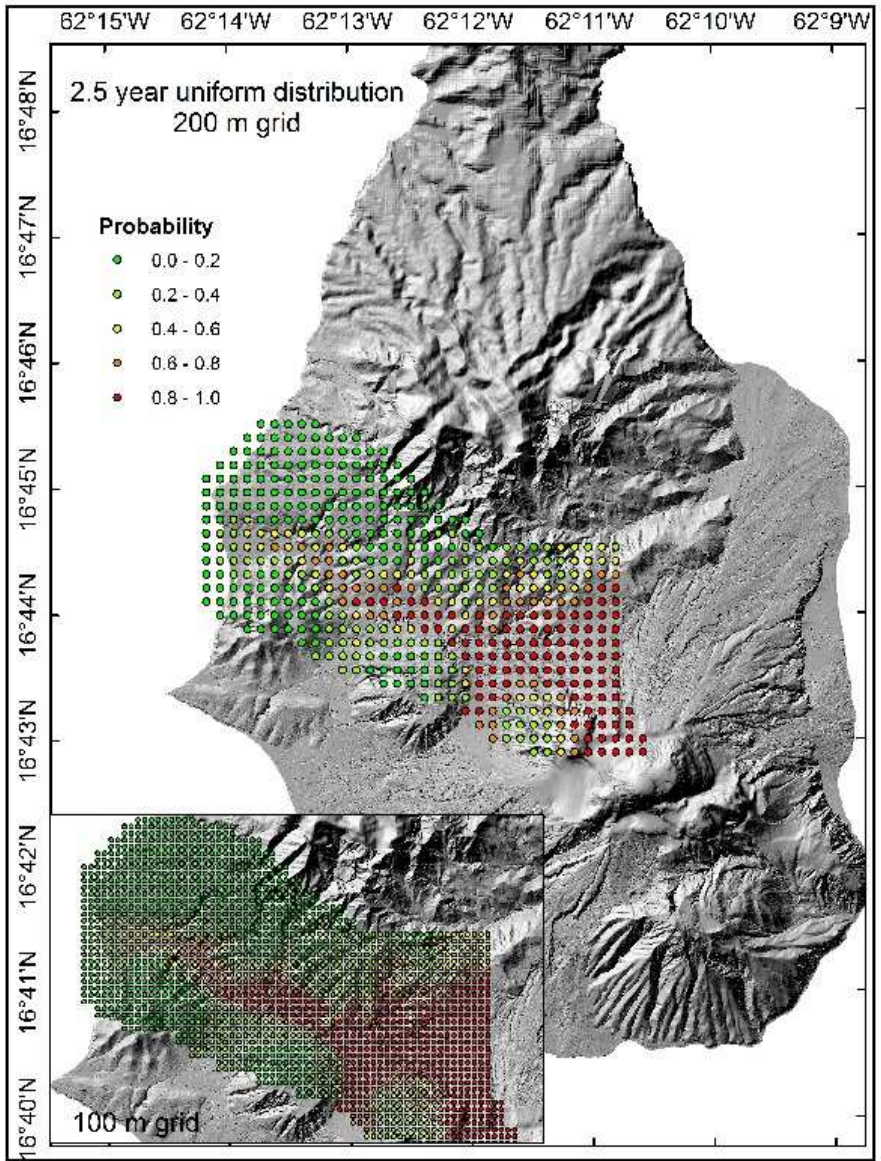
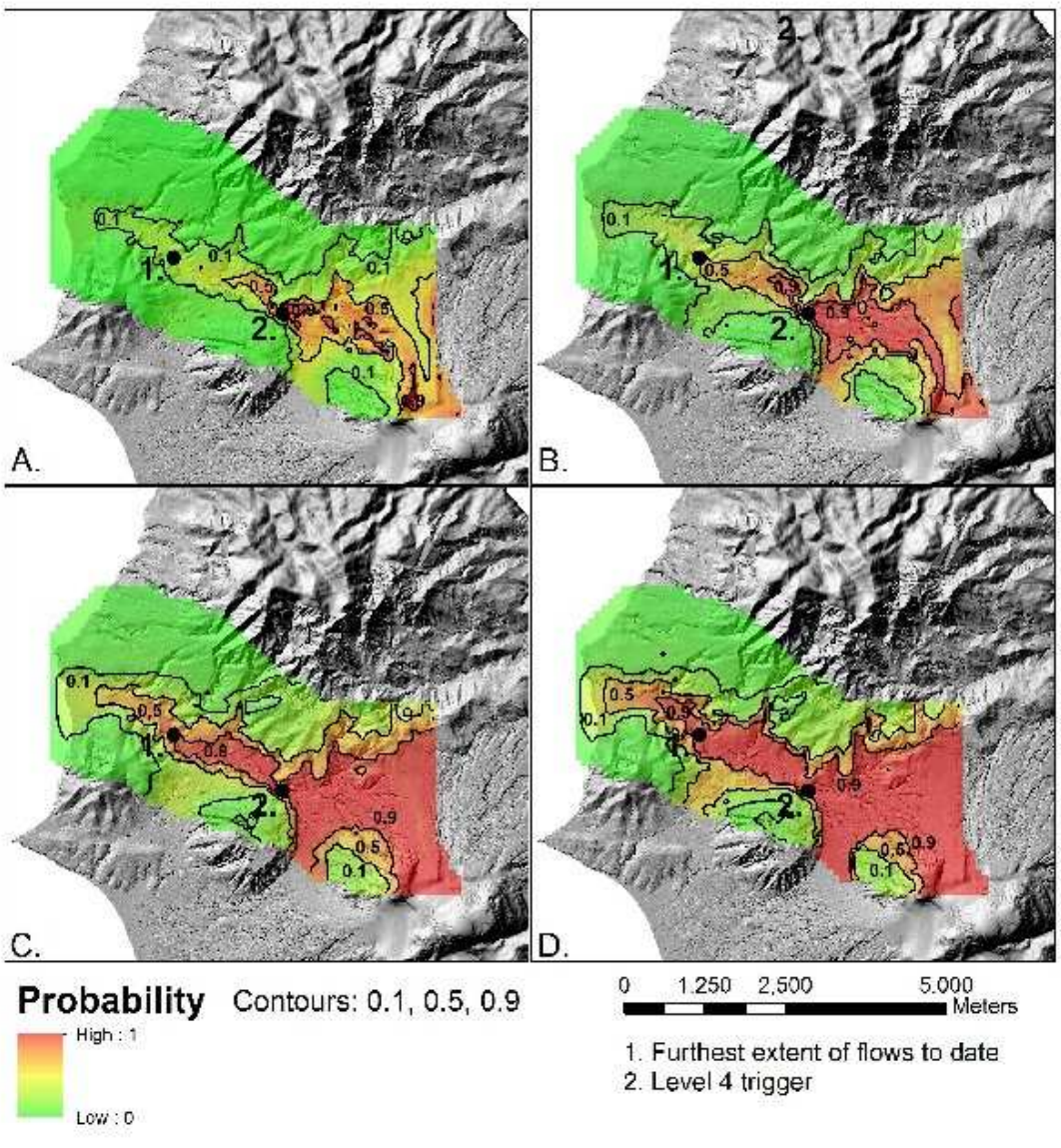
Hazard Over Time at Plymouth & Bramble



Hazard Over Space, for 2.5 Yrs



A few more Hazard Maps



Current Hazard Maps...

Extensions & Related matters...

Some challenges along the way:

- TITAN-2D relies on Digital Elevation Models for its flow predictions. How can we quantify uncertainty in the DEM and its effect on hazard?
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In fact the rate λ_t varies over time, and sometimes seems to vanish for a while at times. We're building dynamic models for this, which feature possibility of $\lambda_t = 0$ for some time intervals.

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There's a slight suggestion of a downturn in the Freq-vs-Vol log-log plot; perhaps a better model is Tempered Pareto, with

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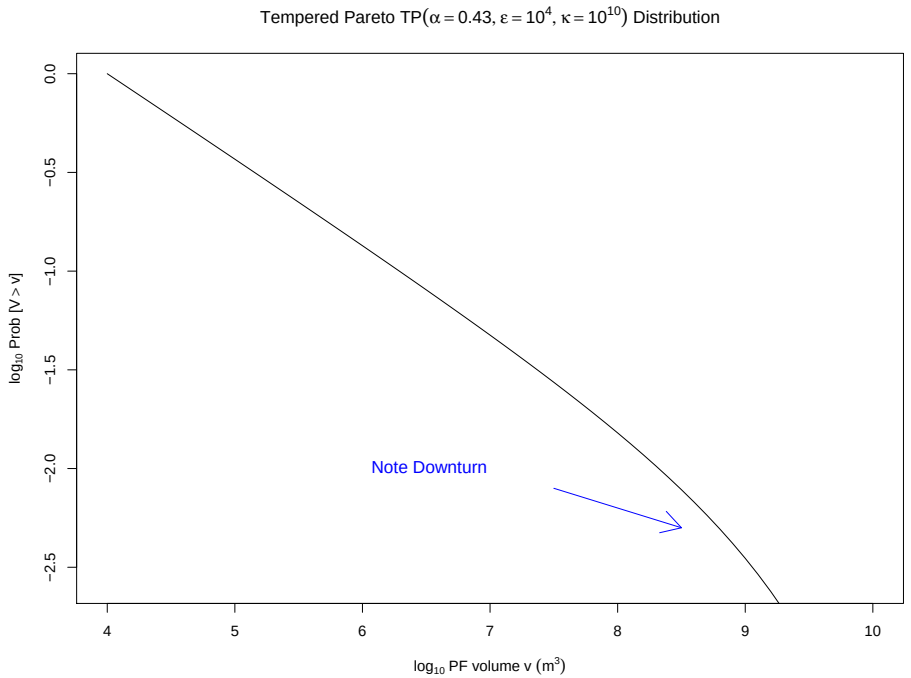

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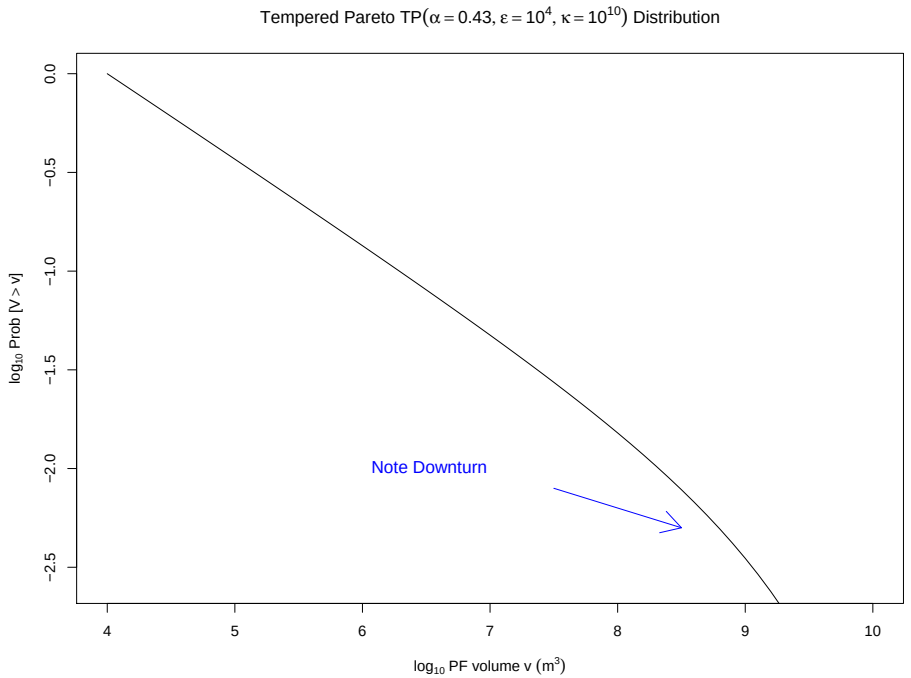

See the drop-off near 10^8 m^3 or 10^9 m^3? That's Good news!

Uncertainty

MLE and reference Bayesian posterior means ($\pm$ SD's) for the three parameters of this model are:

Shape	$\alpha \approx 0.64$	± 0.04	
Rate	$\lambda \approx 22.42$	± 1.3	/year
Bend	$\beta \approx 9.22 \times 10^{-9}$	$\pm 6.0 \times 10^{-9}$	/ m^3

Note how uncertain β is... only 1.5 SD above zero.
How does this affect hazard predictions?
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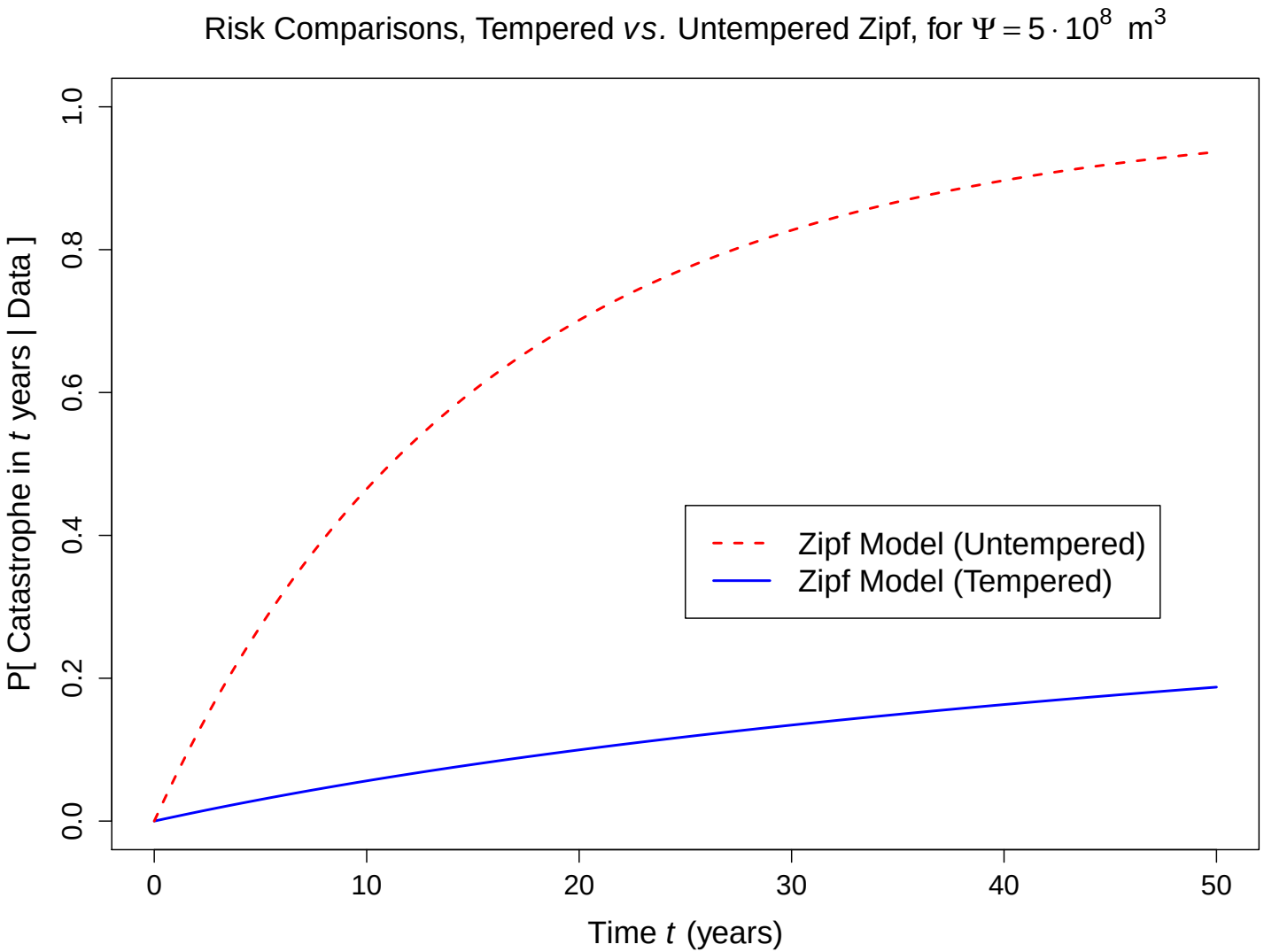
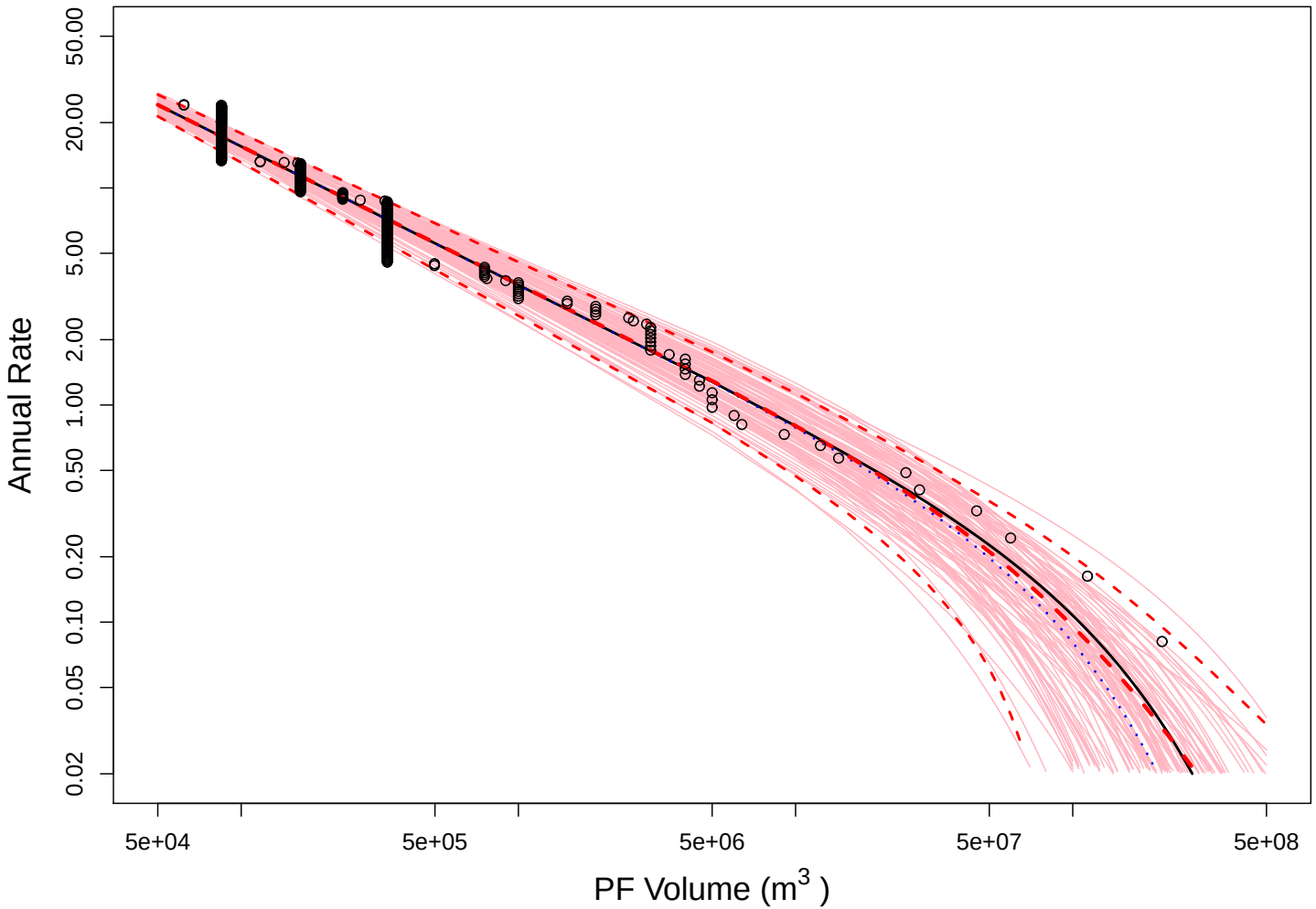
Hazard for Fifty Years of $V > 10^8\,\mathrm{m}^3$

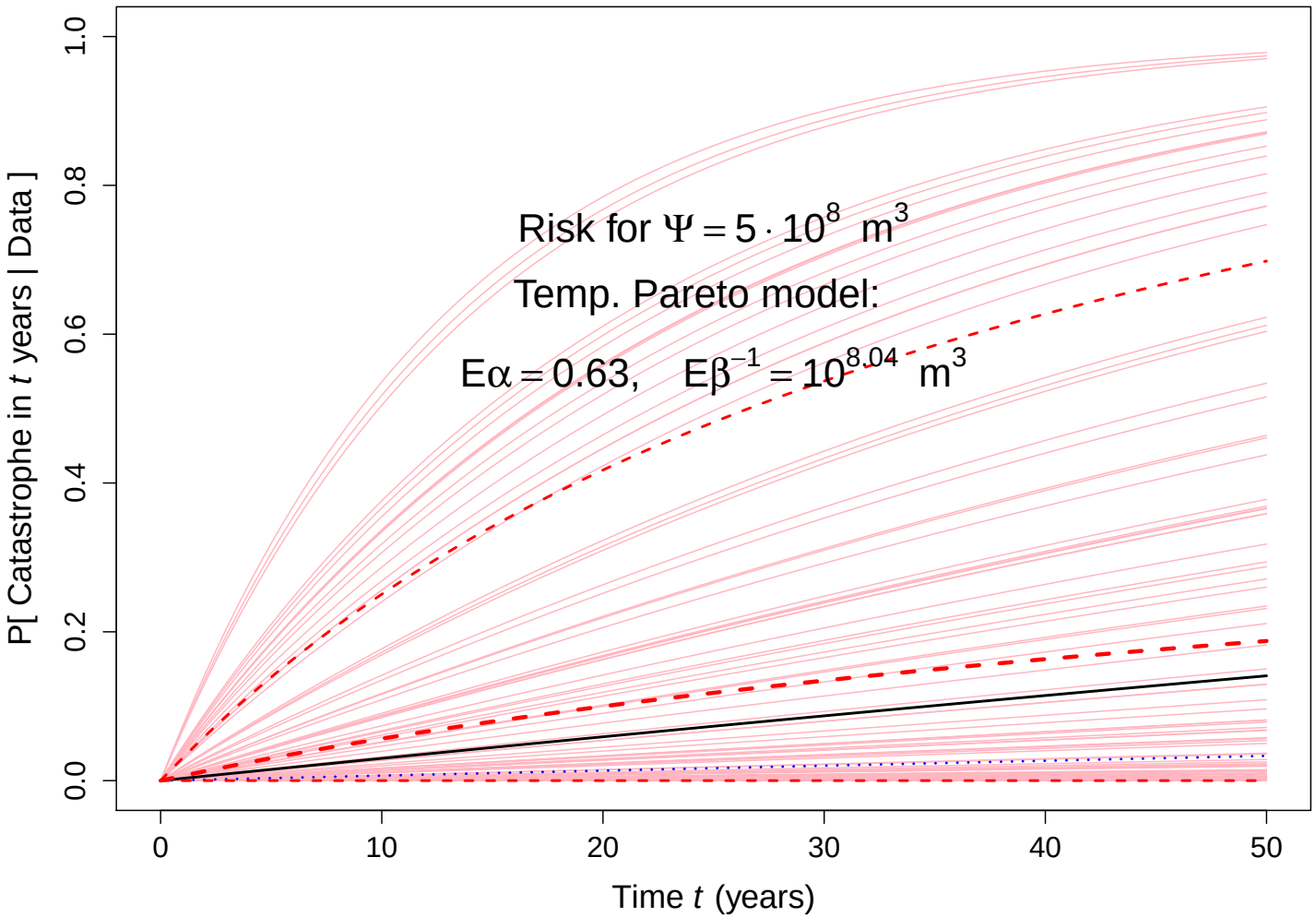
Hazard for Fifty Years of $V > 5 \times 10^8\,\mathrm{m}^3$



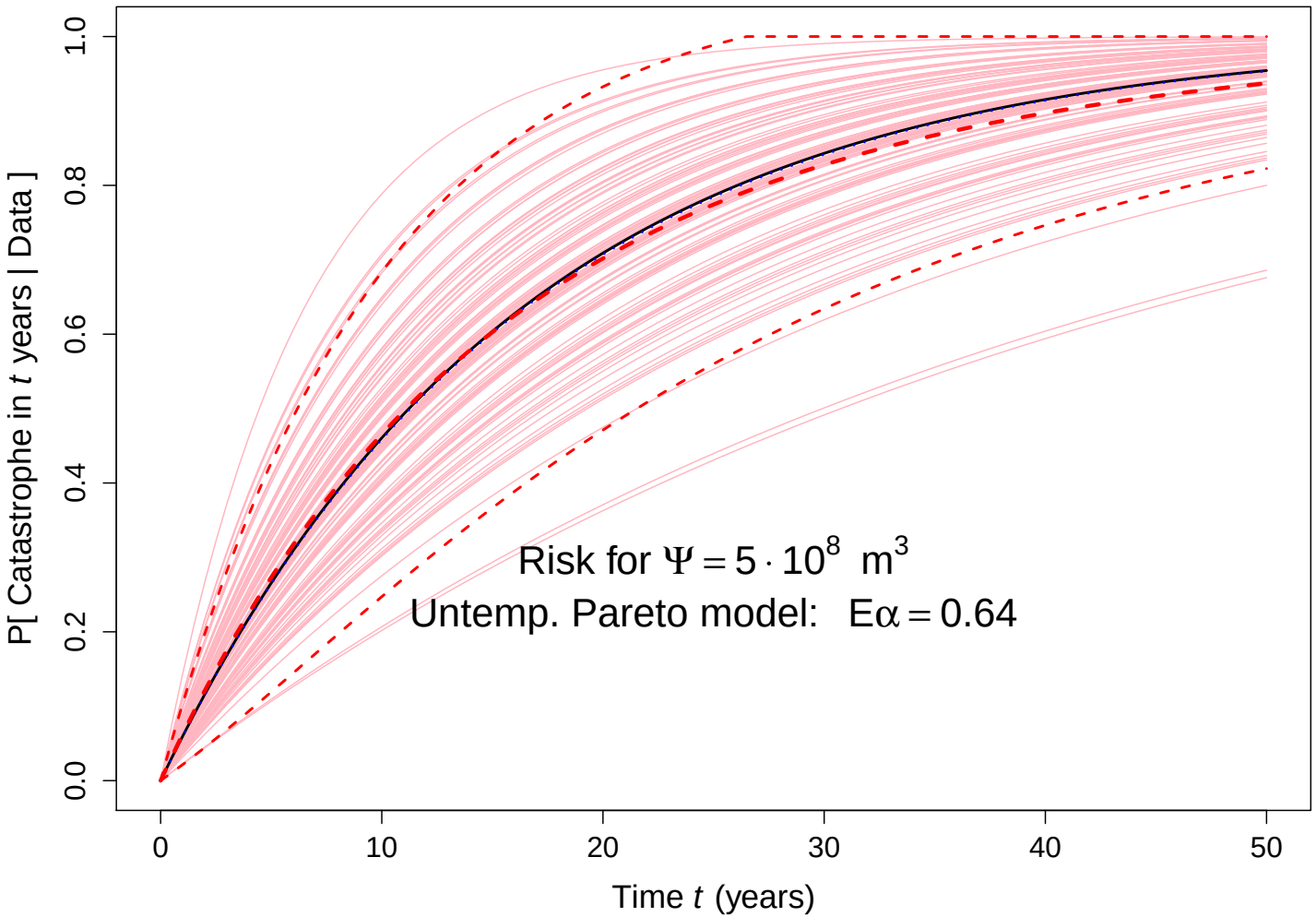
Why the wide uncertainty?



How does that affect estimated hazard?

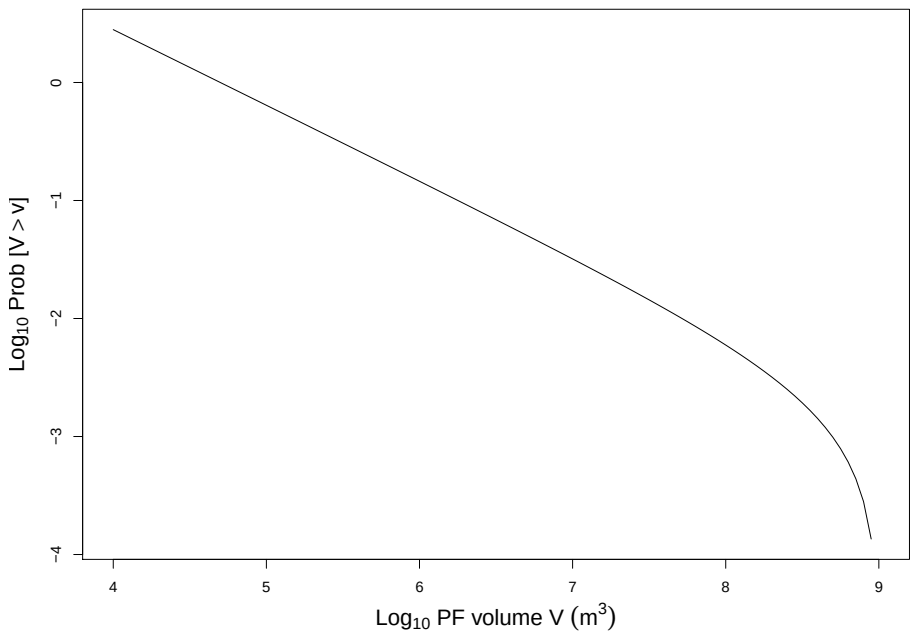


What if there is no tempering?



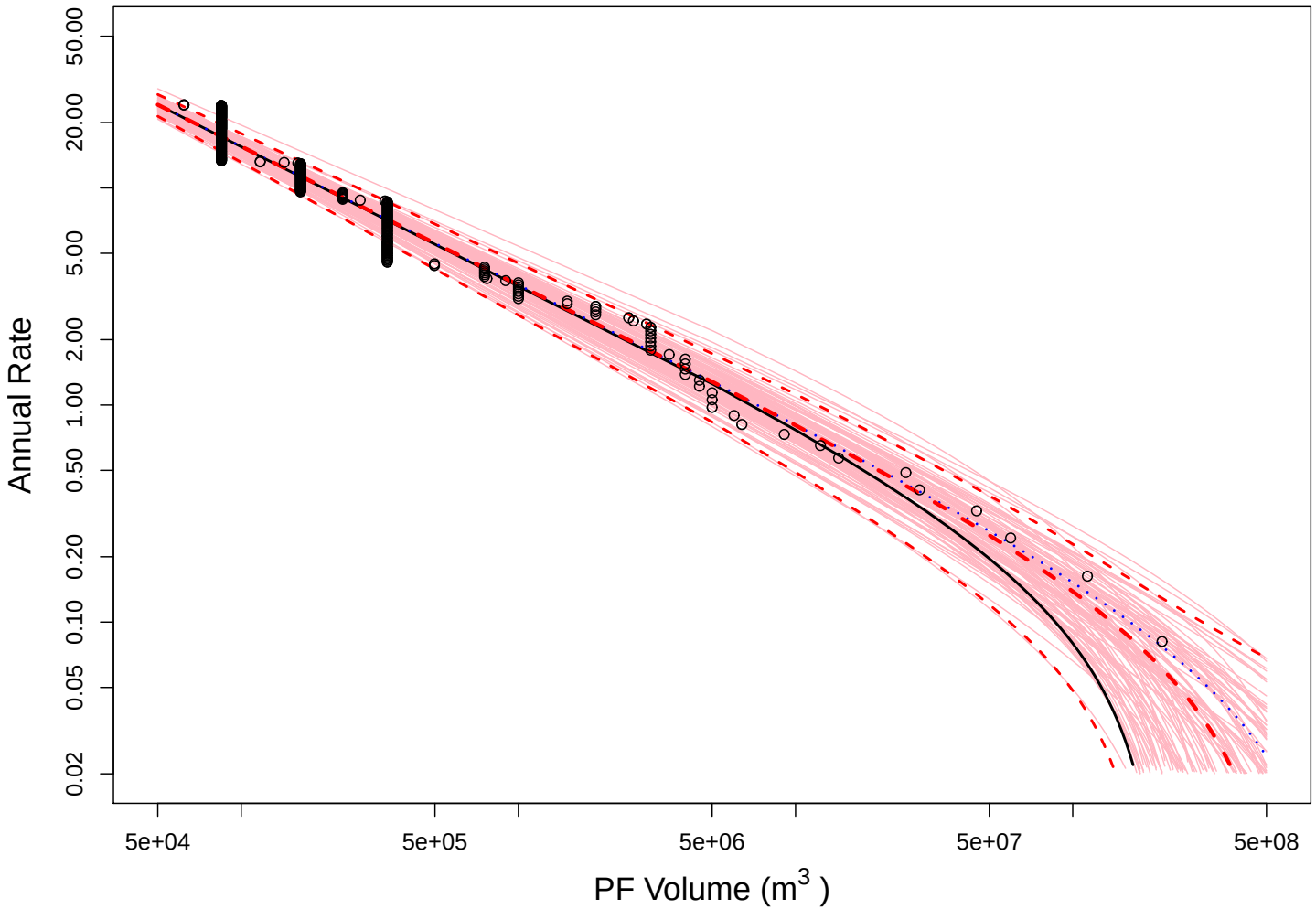
A little more extreme— Truncated Pareto

$$P[V > v] = \frac{v^{-\alpha} - L^{-\alpha}}{\epsilon^{-\alpha} - L^{-\alpha}} \qquad \epsilon \leq v \leq L :$$



Now the drop-off approaching 10^9 m^3 is fast!

A little more extreme— Truncated Pareto



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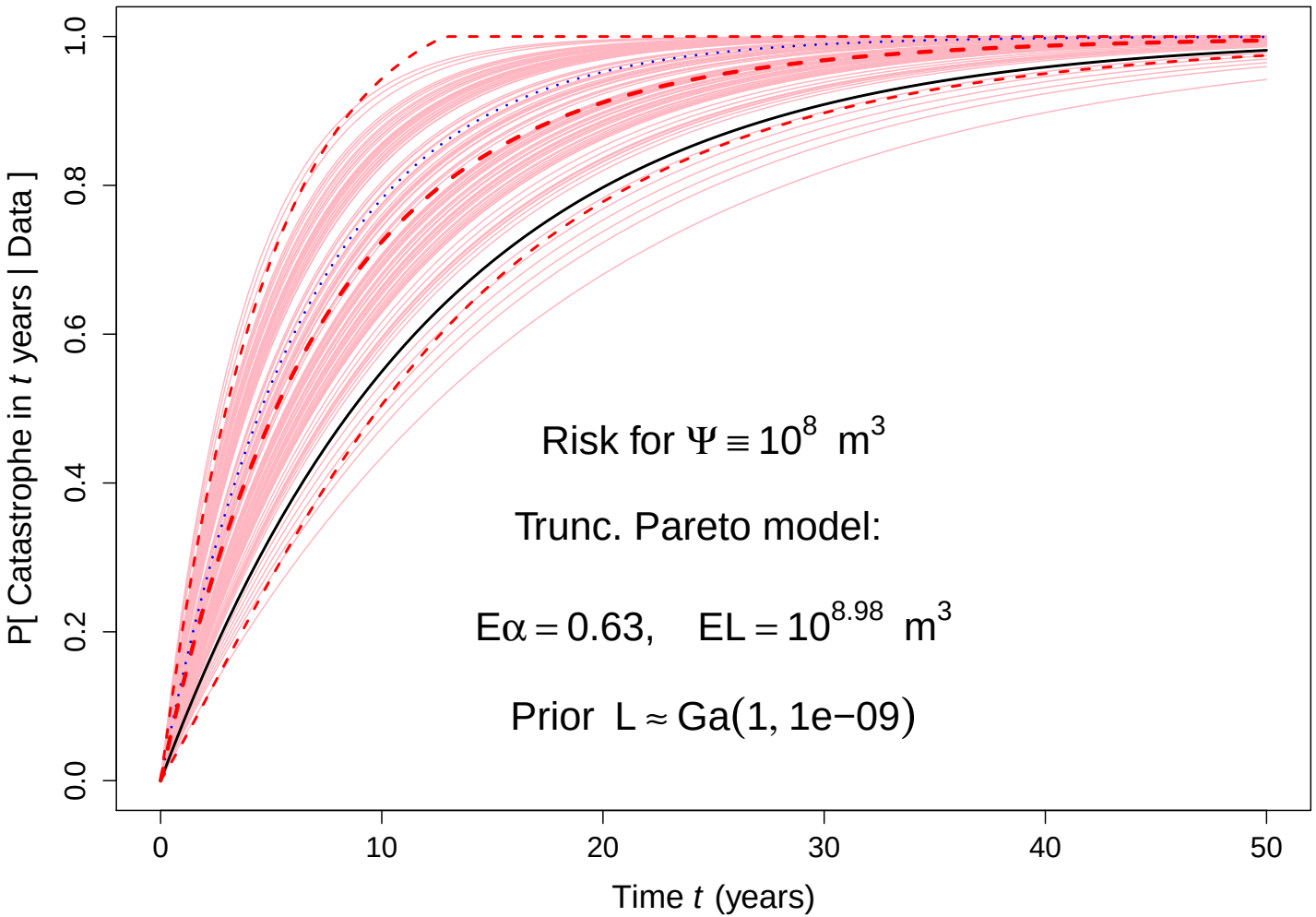

Morals

- It's hard to tell about tail behaviour (extremes) from data;
- Clearer picture of tempering would help—
  working to elicit expert opinion about physical limits
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Conclusions

- Probability, Statistics, Computational PDE, and Geology are a good mix!
- Useful to Separate the Stochastic & Deterministic aspects
  - Use Deterministic models to identify region $\mathcal{R}_C$ of input parameter space for Catastrophes
  - Use Stochastic models to identify posterior probability

$$\mathrm{P}[\mathcal{R}_C \mid \text{ Data }]$$

Ongoing Work & Extensions

- Studying dynamic models for variable rate of eruption process and changing direction of flows ($\Rightarrow$ short-term predictions);
- Exploring dependence between angle θ and flow volume V;
- Estimating possible "tempered Pareto" models,

$$P[V > v] = (v/\epsilon)^{-\alpha} e^{-\beta(v-\epsilon)}, \qquad v > \epsilon$$

- Non-stationary variants;
- Finite lifetimes ("extinction").

A Collaborative Effort

Thanks!

More details (references, this talk in .pdf, related work) are available from "Froggy" website

www.stat.duke.edu/research/projects/frog/

or on request from

rlw@duke.edu.

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[1] DMS-0757549, DMS-0757367, DMS-0757527, DMS-1228317, DMS-1228217, DMS-1228265