Sta-230: Probability **Spring 2017**

Key ideas for topic 3: January 26, 31

You should be able to think of distributions in two ways

(1) As a **sampling procedure or experiment**, for example flipping coins and recording the number of heads.

(2) As a **probability distribution function**

Bernoulli distribution: This is the coin flip distribution $X = \{0, 1\}$

$$\mathbf{P}(X = 1) = p, \quad \mathbf{P}(X = 0) = 1 - p, \quad \mathbf{P}(x) = p^x(1 - p)^{1-x}.$$

Binomial distribution: The random variable is the sum of n independent and identical (iid) trials of the Bernoulli distribution $X = \{0, 1, ..., n\}$ and

$$\mathbf{P}(x) = \binom{n}{x} p^x(1 - p)^{n-x}.$$

One can also think of this as an experiment of sampling with replacement. Consider a bin with 10^9 balls of which $10^9 \times p$ are blue and the remaining balls are red. Draw a ball from the bin and record the color of the ball and return the ball to the bin, replace this procedure n times. The distribution of the number of blue balls drawn is the Binomial distribution.

Cumulative distribution function: For the binomial distribution it is

$$F(z) = \mathbf{P}(z \leq x) = \sum_{x=0}^{z} \binom{n}{x} p^x(1 - p)^{n-x}.$$

In general for a discrete distribution

$$F(z) = \mathbf{P}(z \leq x) = \sum_{x=-\infty}^{z} \mathbf{P}(x).$$

Normal approximation: The DeMoivre-Laplace central limit theorem says if n is large and p is not near 0 or 1 then the following distribution approximates the binomial

$$\mathbf{P}(x) \approx ce^{-(x-np)^2/(2np(1-p))}, \quad c = \frac{1}{2\pi np(1 - p)}, \quad x = \{0, 1, ..., n\},$$

which we can rewrite as

$$\mathbf{P}(x) \approx \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}, \quad \mu = np, \quad \sigma^2 = np(1 - p).$$

Poisson distribution: A random variable Y is Poisson with parameter $\lambda > 0$ if

$$\mathbf{P}(Y = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, ...$$

Poisson approximation: If n is large, p is small, and $np = \lambda$ then the binomial can be approximated by a Poisson distribution

$$\mathbf{P}(x) \approx \frac{e^{-\lambda}\lambda^{x}}{x!} \quad k = 0, 1, 2, ..., n.$$