Sta-230: Probability **Spring 2017**

Key ideas for topic 7: March 2, 7, 9

Joint distribution:
For the discrete case of two variables x, y the following are joint probabilities

$$\begin{aligned} \mathbf{P}(X = x, Y = x) &= P(x, y) \\ \mathbf{P}((X, Y) \in B) &= \sum_{(x,y)\in B} P(x, y), \end{aligned}$$

with

$$P(x, y) \geq 0, \quad \sum_x \sum_y P(x, y) = 1.$$

For the continuous case of two variables x, y the following is a joint probability density

$$\mathbf{P}(X \in dx, Y \in dy) = f(x, y)dx\, dy$$

with

$$f(x, y) \geq 0, \quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)dx\, dy = 1,$$

and

$$\mathbf{P}((X, Y) \in B) = \int_{(x,y)\in B} f(x, y)\, dx\, dy,$$

Marginals:
For discrete distributions

$$\begin{aligned} \mathbf{P}(X = x) &= \sum_y P(x, y), \\ \mathbf{P}(Y = y) &= \sum_x P(x, y). \end{aligned}$$

For continuous distributions

$$\begin{aligned} f_X(x) &= \int_{y=-\infty}^{\infty} f(x, y)dy, \\ f_Y(y) &= \int_{x=-\infty}^{\infty} f(x, y)dx. \end{aligned}$$

Independence:
For discrete distributions

$$P(x, y) = P(X = x)P(Y = y).$$

For continuous distributions

$$f(x, y) = f_X(x)f_Y(y).$$

Covariance:
The covariance of two random variables X and Y is

$$\mathrm{Cov}(X,Y) = \mathbb{E}[(X-\mu_X)(Y-\mu_Y)], \quad \mu_X = \mathbb{E}[X], \quad \mu_Y = \mathbb{E}[Y],$$

and

$$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X,Y).$$

Correlation:
The correlation between two random variables X and Y is

$$\mathrm{Corr}(X < Y) = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y}.$$