These shortcomings of PSSMs set the stage for a new kind of profile, based on Markov chains, called Hidden Markov models (HMMs)

- modeling positional dependencies
- recognizing pattern instances with indels
- modeling variable length patterns
- detecting boundaries


## Markov chains

Markov chains are stochastic processes that undergo transitions between a finite series of states in a chainlike manner.


The system transverses states with probability

$$
p\left(x_{1}, x_{2}, x_{3}, \ldots\right)=p\left(x_{1}\right) p\left(x_{2} \mid x_{1}\right) p\left(x_{3} \mid x_{2}\right) p\left(x_{4} \mid x_{3}\right) \ldots
$$

i.e. Markov chains are memoryless: the probability that the chain is in state $x_{i}$ at time $t$, depends only on the state at the previous time step and not on the past history of the states visited before time $t-1$.

This specific kind of "memorylessness" is called the Markov property.

The Markov property states that the conditional probability distribution for the system at the next step (and in fact at all future steps) depends only on the current state of the system, and not additionally on the state of the system at previous steps.

## Markov chains...

Markov chains, and their extension hidden Markov models (HMMs), are commonly represented by state diagrams, which consist of states and connecting transitions

E.g., A general Markov chain modeling DNA. Note that any sequence can be traced through the model by passing from one state to the next via the transitions.

A transition probability parameter $\left(a_{i j}\right)$ is associated with each transition (arrow) and determines the probability of a certain state $\left(S_{j}\right)$ following another state $\left(S_{i}\right)$.

A Markov chain is defined by:

- a finite set of states, $S_{1}, S_{2} \ldots S_{N}$
- a set of transition probabilities: $a_{i j}=P\left(q_{t+1}=S_{j} \mid q_{t}=S_{i}\right)$
- and an initial state probability distribution, $\pi_{i}=P\left(q_{0}=S_{i}\right)$


## Simple Markov chain example for $\mathrm{x}=\{\mathrm{a}, \mathrm{b}\}$

Observed sequence: $\mathrm{x}=\mathrm{aba} a \mathrm{ababba}$

## Model:

| transition | Prev | Next | Prob |
| :---: | :---: | :---: | :---: |
| probabilities | $i$ | $j$ | $a_{i j}$ |
|  | $\mathbf{a}$ | $\mathbf{a}$ | $\mathbf{0 . 7}$ |
|  | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{0 . 3}$ |
|  | $\mathbf{b}$ | $\mathbf{a}$ | $\mathbf{0 . 5}$ |
|  | $\mathbf{b}$ | $\mathbf{b}$ | $\mathbf{0 . 5}$ |


| $\underset{\text { probs }}{\text { Start }}{ }_{\text {prem }}$ |  | a 0.5 |
| :---: | :---: | :---: |
|  |  | b 0.5 | distribution

$$
P(x)=0.5 \times 0.3 \times 0.5 \times 0.7 \times 0.7 \times 0.3 \times 0.5 \times 0.3 \times 0.5 \times 0.5 \times 0.7
$$

Q. Can you sketch the state diagram with labeled transitions for this model?

## Typical questions we can ask with Markov chains include:

- What is the probability of being in a particular state at a particular time? (By time here we can read position in our query sequence)
- What is the probability of seeing a particular sequence of states? (I.e., the score for a particular query sequence given the model)
Q. What do Markov chains add over the traditional PSSM approach? In particular how do Markov chains deal with the following PSSM weaknesses?

1. Positional dependencies
2. Pattern instances containing insertions or deletions
3. Variable length patterns, and
4. The detection boundaries (i.e. segmentation of sequences)

## Markov chains: 1. Positional dependencies

The connectivity or topology of a Markov chain can easily be designed to capture dependencies and variable length motifs.

WEIRD WEIRD WEIQH WEIRD


Recall that a PSSM for this motif would give the sequences WEIRD and WEIRH equally good scores even though the RH and QR combinations were not observed

## Markov chains: 2. Insertions and deletions

To address pattern instances with gaps and variable length motifs, we can construct a Markov chain to recognize a query sequences with insertions (via an extra insertion state) and deletions (via extra transitions (edges))

## WETIRD

 WE-IRD WETIQH WE-IRD WE--QH

## Markov chains: 3. Boundary detection

Giving a sequence we wish to label each symbol in the sequence according to its class (e.g. transmembrane regions or extracellular/cytosolic)

tend to be hydrophobic in composition

Given a training set of labeled sequences we can begin by modeling each amino acid as hydrophobic (H) or hydrophilic (L)
i.e. reduce the dimensionality of the 20 amino acids into two classes
E.g., A peptide sequence can be represented as a sequence of Hs and Ls. e.g. HHHLLHLHHLHL...

## Markov chains: boundary detection...

A simpler question: is a given sequence a transmembrane sequence?
A Markov chain for recognizing transmembrane sequences


- States: $\mathrm{S}_{\mathrm{H}}, \mathrm{S}_{\mathrm{L}}$
- $\Sigma=\{\mathrm{H}, \mathrm{L}\}$
- $\pi(\mathrm{H})=0.6, \pi(\mathrm{~L})=0.4$

Question: Is sequence hнLHH a transmembrane protein?
$P(H H L H H)=0.6 \times 0.7 \times 0.7 \times 0.3 \times 0.7 \times 0.7=0.043$
Problem: need a threshold,
threshold must be length dependent

## Markov chains: boundary detection

We can classify an observed sequence $\left(\mathrm{O}=\mathrm{O}_{1}, \mathrm{O}_{2}, \ldots\right)$ by its log odds ratio

## transmembrane model



Transmembrane (TM)

- $\pi(\mathrm{H})=0.6, \pi(\mathrm{~L})=0.4$
null model


Extracellular/cytosolic (E/C)

- $\pi(\mathrm{H})=0.5, \pi(\mathrm{~L})=0.5$

$$
\frac{P(H H L H H \mid T M)}{P(H H L H H \mid E C)}=\frac{0.6 \times 0.7 \times 0.7 \times 0.3 \times 0.7 \times 0.7}{0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5}=\frac{0.043}{0.016}=2.69
$$

In other words, it is more than twice as likely that HHLHH is a transmembrane sequence. The log-odds score is: $\log _{2}(2.69)=1.43$

## Side note: Parameter estimation

Both initial probabilities $(\pi(i))$ and transition probabilities $\left(a_{i j}\right)$ are determined from known examples of transmembrane and non-transmembrane sequences.


- initial probabilities $\pi(H), \pi(L)$
- transition probabilities: $a_{H}, a_{H L}, a_{L H}$ and $a_{L L}$.

Given labeled sequences (TM and E/C), we determine the initial probabilities $\pi(i)$ by counting the number of sequences that begin with residue $i$.

To determine transition probabilities, $\mathrm{a}_{i j}$, we first determine $\mathrm{A}_{i j}$ (the number of transitions from state $i$ to $j$ in the training data, i.e. count the number of $i j$ pairs in the training data).

$$
a_{i j}=\frac{A_{i j}}{\sum_{j^{\prime}} A_{i j^{\prime}}}
$$ Then normalize by the number of $i^{*}$ pairs.

## Side note：Parameter estimation．．．

Both initial probabilities（ $\pi(i)$ ）and transition probabilities $\left(a_{i j}\right)$ are determined from known examples of transmembrane and non－transmembrane sequences．

$\pi(H)=\#$ of sequences that begin with $H$ ， normalized by the total \＃of training sequences
－$\pi(H)=0.6, \pi(\mathrm{~L})=0.4$

HHHL工HHHL工LHLHL工HL工工HLHHHL HHHLHHLHL工L工LHHHHL工LHHHHHL HH ．．．（ $\mathrm{A}_{H L}=12, \mathrm{~A}_{H^{*}}=40$ ）

$$
a_{H L}=\frac{A_{H L}}{\sum_{i} A_{H i}} \quad \frac{\text { \#HL pairs }}{\# \mathrm{H}^{\star} \text { pairs }} \quad \frac{12}{40}
$$

## Boundary detection challenge

Given sequence of Hs and Ls, find all transmembrane regions:

To approach this question we can construct a new four state model by adding transitions connecting the TM and E/C models


Transitions between the $M$ states and the $E / C$ states indicate boundaries between membrane regions and cytosolic or extracellular regions.

However this is no longer a standard Markov chain!

## Boundary detection challenge...

In a Markov chain, there is a one-to-one correspondence between symbols and states, which is not true of our new merged four state, two symbol model.

For example, both $H_{M}$ and $H_{E / C}$ are associated with hydrophilic residues.

- This four-state transmembrane model is a hidden Markov model.



## So whats hidden?

We will distinguish between the observed parts of the problem and the hidden parts

- In the Markov models we have considered previously it is clear which states account for each part of the observed sequence
Due to the one-to-one correspondence between symbols and states
- In our new model, there are multiple states that could account for each part of the observed sequence
i.e. we don't know which state emitted a given symbol from knowledge of the sequence and the structure of the model
- This is the hidden part of the problem


For our Markov models

- Given HLLH..., we know the exact state sequence ( $\mathrm{q}_{0}=\mathrm{S}_{\mathrm{H}}, \mathrm{q}_{1}=\mathrm{S}_{\mathrm{L}}, \mathrm{q} 2=\mathrm{S}_{\mathrm{L}}, \ldots$ )


## For our HMM

- Given HLLH..., we must infer the most probable state sequence
- This HMM state sequence will yield the boundaries between likely TM and E/C regions


```
HM, LM, LM, HM
HM, LM, LM, HE/C
HM, LM, LH/C, HM
HM, LM, LH/C, HE/C
HM, LE/C, LM, HM
HM, LE/C, LM, HE/C
HM, LE/C, LH/C, HM,
HM, LE/C, LH/C, HE/C,
HE/C, LM, LM, HM
HE/C, LM, LM, HE/C
HE/C, LM, LH/C, HM
HE/C, LM, LH/C, HE/C
HE/C, LE/C, LM, HM
HE/C, LE/C, LM, HE/C
HE/C, LE/C, LH/CM, HM
HE/C, LE/C, LH/CM, HE/C
```


## Side note: HMM states as sequence emitters

It's useful to imagine HMM states emitting symbols each time they are visited

In this way, transversing the model will "generate" a sequence with a certain probability (i.e. "score").

This probability is a product of the state path taken through the model
That is, it depends on initial probabilities, transition probabilities and emission probabilities (the probability that a visited state emits a particular symbol) along the path

There may be many possible paths that can generate the same sequence

An HMM is a full probabilistic model - the model parameters $\theta$ and the overall sequence "scores" $P(x, S \mid H M M, \theta)$ are all probabilities. As a result, we can use standard Bayesian probability theory to manipulate these numbers in powerful ways, including optimizing parameters, calculating confidence in predictions, and interpreting the statistical significance of scores.

## Hidden Markov models (HMMs)

## Markov Chains

- States: $S_{1}, S_{2} \ldots S_{N}$
- Initial probabilities: $\pi_{i}$
- Transition probabilities: $a_{i j}$

Hidden Markov Models

- States: $S_{1}, S_{2} \ldots S_{N}$
- Initial probabilities: $\pi_{i}$
- Transition probabilities: $a_{i j}$
- Alphabet of emitted symbols, $\boldsymbol{\Sigma}$
- Emission probabilities: $e_{i}(a)$ probability state $i$ emits symbol a

Symbol may be emitted by more than one state

Similarly, a state can emit more than one symbol

## Example three state HMM

In this example we will use only one state for the transmembrane segment $(M)$ and use emission probabilities to distinguish between H and L residues. We will also add separate $E \& C$ states with distinct emission probabilities.


## Side note: Parameter estimation

As in the case of Markov chains, the HMM parameters can be learned from labeled training data

Note that we now have to learn the initial probabilities, transition probabilities and emission probabilities

$$
a_{i j}=\frac{A_{i j}}{\sum_{j^{\prime}} A_{i j^{\prime}}}
$$

$$
e_{i}(x)=\frac{E_{i}(x)}{\sum_{x}^{\prime} E_{i}\left(x^{\prime}\right)}
$$



|  | $E$ | $M$ | $C$ |
| :---: | :---: | :---: | :---: |
| $\pi_{i}$ | 0 | 0 | 1 |
| $e_{i}(H)$ | 0.2 | 0.9 | 0.3 |
| $e_{i}(L)$ | 0.8 | 0.1 | 0.7 |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ |  |  |  |  |  |
| $M$ |  |  |  |  |  |
| $C$ |  |  |  |  |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 |  |  |  |  |
| $M$ | $0 \times 0.9$ <br> $=0$ |  |  |  |  |
| $C$ | $1 \times 0.3$ <br> $=0.3$ |  |  |  |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 |  |  |  |  |
| $M$ | $0 \times 0.9$ <br> $=0$ |  |  |  |  |
| $C$ | $1 \times 0.3$ <br> $=0.3$ |  |  |  |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - |  |  |  |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ |  |  |  |
| $C$ | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 \times 0.3$ <br> $=0.063$ |  |  |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - |  |  |  |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ |  |  |  |
| C | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 \times 0.3$ <br> $=0.063$ |  |  |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $=0.016$ |  |  |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.08 \mathrm{I}$ <br> $=0.04$ |  |  |
| $C$ | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.081$ <br> $=0.014$ |  |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $7=0.016$ |  |  |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.08 \mathrm{I}$ <br> $=0.04$ |  |  |
| C | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.081$ <br> $=0.014$ |  |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $7=0.016$ | $0.7 \times 0.8 \times 0.016$ <br> $=0.009$ |  |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.08 \mathrm{I}$ <br> $=0.04$ | $0.3 \times 0.1 \times 0.016$ <br> $=0.0005$ |  |
| C | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.08 \mathrm{I}$ <br> $=0.014$ | - |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $7=0.016$ | $0.7 \times 0.8 \times 0.016$ <br> $=0.009$ |  |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.08 \mathrm{I}$ <br> $=0.04$ | $0.3 \times 0.1 \times 0.016$ <br> $=0.0005$ |  |
| C | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.08 \mathrm{I}$ <br> $=0.014$ | - |  |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | 工 | 工 | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $\begin{gathered} 0 \times 0.2 \\ =0 \end{gathered}$ | - | $\begin{gathered} 0.25 \times 0.8 \times 0.081 \\ =0.016 \end{gathered}$ | $\begin{gathered} 0.7 \times 0.8 \times 0.016 \\ =0.009 \end{gathered}$ | $\begin{gathered} 0.7 \times 0.2 \times 0.009 \\ =0.001 \end{gathered}$ |
| $M$ | $\begin{gathered} 0 \times 0.9 \\ =0 \end{gathered}$ | $\begin{aligned} & 0.3 \times 0.9 \times 0.3 \\ & =0.081 \end{aligned}$ | $\begin{gathered} 0.5 \times 0.1 \times 0.081 \\ =0.04 \end{gathered}$ | $\begin{gathered} 0.3 \times 0.1 \times 0.016 \\ =0.0005 \end{gathered}$ | $\begin{gathered} 0.3 \times 0.9 \times 0.009 \\ =0.002 \end{gathered}$ |
| C | $\Rightarrow \begin{aligned} & 1 \times 0.3 \\ & =0.3 \end{aligned}$ | $\begin{gathered} 0.7 \times 0.3 .0 .3 \\ =0.063 \end{gathered}$ | $\begin{gathered} 0.25 \times 0.7 \times 0.081 \\ =0.014 \end{gathered}$ | - | - |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $\pi=0.016$ | $0.7 \times 0.8 \times 0.016$ <br> $=0.009$ | $0.7 \times 0.2 \times 0.009$ <br> $=0.001$ |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.08 \mathrm{I}$ <br> $=0.04$ | $0.3 \times 0.1 \times 0.016$ <br> $=0.0005$ | $0.3 \times 0.9 \times 0.009$ <br> $=0.002$ |
| C | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.08 \mathrm{I}$ <br> $=0.014$ | - | - |
| START |  |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $7=0.016$ | $0.7 \times 0.8 \times 0.016$ <br> $=0.009$ | $0.7 \times 0.2 \times 0.009$ <br> $=0.001$ |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.08 \mathrm{I}$ <br> $=0.04$ | $0.3 \times 0.1 \times 0.016$ <br> $=0.0005$ | $0.3 \times 0.9 \times 0.009$ <br> $=0.002$ |
| $C$ | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.081$ <br> $=0.014$ | - | - |
| START | $C$ |  |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> $=0$ | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $\pi=0.016$ | $0.7 \times 0.8 \times 0.016$ <br> $=0.009$ | $0.7 \times 0.2 \times 0.009$ <br> $=0.001$ |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.08 \mathrm{I}$ <br> $=0.04$ | $0.3 \times 0.1 \times 0.016$ <br> $=0.0005$ | $0.3 \times 0.9 \times 0.009$ <br> $=0.002$ |
| $C$ | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.08 \mathrm{I}$ <br> $=0.014$ | - | - |
| START | C | M |  |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> 0 | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $=0.016$ | $0.7 \times 0.8 \times 0.016$ <br> $=0.009$ | $0.7 \times 0.2 \times 0.009$ <br> $=0.001$ |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.081$ <br> $=0.04$ | $0.3 \times 0.1 \times 0.016$ <br> $=0.0005$ | $0.3 \times 0.9 \times 0.009$ <br> $=0.002$ |
| $C$ | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.081$ <br> $=0.014$ | - | - |
| START | C | M | E |  |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> $=0$ | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $\pi=0.016$ | $0.7 \times 0.8 \times 0.016$ <br> $=0.009$ | $0.7 \times 0.2 \times 0.009$ <br> $=0.001$ |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.08 \mathrm{I}$ <br> $=0.04$ | $0.3 \times 0.1 \times 0.016$ <br> $=0.0005$ | $0.3 \times 0.9 \times 0.009$ <br> $=0.002$ |
| C | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.08 \mathrm{I}$ <br> $=0.014$ | - | - |
| START | C | M | E | E |  |



Query Sequence

| States | H | H | L | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $0 \times 0.2$ <br> $=0$ | - | $0.25 \times 0.8 \times 0.08 \mathrm{I}$ <br> $7=0.016$ | $0.7 \times 0.8 \times 0.016$ <br> $=0.009$ | $0.7 \times 0.2 \times 0.009$ <br> $=0.001$ |
| $M$ | $0 \times 0.9$ <br> $=0$ | $0.3 \times 0.9 \times 0.3$ <br> $=0.081$ | $0.5 \times 0.1 \times 0.081$ <br> $=0.04$ | $0.3 \times 0.1 \times 0.016$ <br> $=0.0005$ | $0.3 \times 0.9 \times 0.009$ <br> $=0.002$ |
| $C$ | $1 \times 0.3$ <br> $=0.3$ | $0.7 \times 0.3 .0 .3$ <br> $=0.063$ | $0.25 \times 0.7 \times 0.081$ <br> $=0.014$ | - | - |
| START | $C$ | $M$ | $E$ | $E$ | $M$ |



Query Sequence

| States | H | H | 工 | L | H |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ | $\begin{gathered} 0 \times 0.2 \\ =0 \end{gathered}$ | - | $\begin{gathered} 0.25 \times 0.8 \times 0.081 \\ =0.016 \end{gathered}$ | $\begin{gathered} 0.7 \times 0.8 \times 0.016 \\ =0.009 \end{gathered}$ | $\begin{gathered} 0.7 \times 0.2 \times 0.009 \\ =0.001 \end{gathered}$ |
| M | $\begin{gathered} 0 \times 0.9 \\ =0 \end{gathered}$ | $\begin{gathered} 0.3 \times 0.9 \times 0.3 \\ =0.081 \end{gathered}$ | $\begin{gathered} 0.5 \times 0.1 \times 0.081 \\ =0.04 \end{gathered}$ | $\begin{gathered} 0.3 \times 0.1 \times 0.016 \\ =0.0005 \end{gathered}$ | $\begin{gathered} 0.3 \times 0.9 \times 0.009 \\ =0.002 \end{gathered}$ |
| $C$ | $\begin{aligned} & 1 \times 0.3 \\ & =0.3 \end{aligned}$ | $\begin{gathered} 0.7 \times 0.3 .0 .3 \\ =0.063 \end{gathered}$ | $\begin{gathered} 0.25 \times 0.7 \times 0.081 \\ =0.014 \end{gathered}$ | - | - |
| START | C | M <br> Mos | $\begin{gathered} \text { E } \\ \text { robable State } \end{gathered}$ | $\begin{gathered} E \\ \text { equence } \end{gathered}$ | M |

## We have just used the Viterbi algorithm

The Viterbi algorithm finds the most probable "state path" (S*) (i.e. sequence of hidden states) for generating a given sequence ( $\mathrm{x}=\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots \mathrm{x}_{\mathrm{N}}$ )

$$
S^{*}=\operatorname{argmax} P(x, S)
$$

This process is often called decoding because we "decode" the sequence of symbols to determine the hidden sequence of states HMMs were original developed in the field of speech recognition, where speech is "decoded" into words or phonemes to determine the meaning of the utterance

Note that we could have used brute force by calculating $P(x \mid S)$ for all paths but this quickly becomes intractable for longer sequences or HMMs with a large number of states

The Viterbi algorithm is guaranteed to find the most probable state path given a sequence and an HMM

## Three key HMM algorithms

- Viterbi algorithm

Given observed sequence $x$ and an HMM $M$, composed of states $S$, calculate the most likely state sequence, $S^{*}$

- $S^{*}=\operatorname{argmax} P(x, S)$
- Forward algorithm

Given observed sequence $x$ and an HMM composed of states S, calculate the probability of the sequence for the HMM, $P(x \mid M)$

- $P(x)=\sum_{S} P(x, S)$
- Baum-Welch algorithm

Given many observed sequences, estimate the parameters of the HMM

- heuristic expectation maximization method to optimize of $a_{i j}$ and $e_{i}(a)$


## The forward algorithm

Another important question is how well does a given sequence fit the HMM?

To answer this question we must sum over all possible state paths that are consistent with the sequence in question (Because we don't know which path emitted the sequence)

The number of paths can quickly become intractable. The forward algorithm is a simple dynamic programing solution that makes use of the Markov property so that we don't have to explicitly enumerate every path.

The forward algorithm basically replaces the maximization step of the Viterbi algorithm with sums to calculate the probability of the sequence given a HMM.

$$
P(x)=\sum_{S} P(x, S)
$$

## The Baum-Welch algorithm

The Baum-Welch algorithm is an heuristic optimization algorithm for learning probabilistic models in problems that involve hidden states

If we know the state path for each training sequence (i.e. no hidden states with respect to the training sequences), then learning the model parameters is simple (just like it was for Markov chain models)

- count how often each transition and emission occurs
- normalize to get probabilities

If we don't know the path for each training sequence, we can use the BaumWelch algorithm, an expectation maximization method, which estimates counts by considering every path weighted by its probability

- start from a given initial guess for the parameters
- perform a calculation which is guaranteed to improve the previous guess
- run until there is little change in parameters between iterations

For sequence profile-HMMs we train from a MSA and hence we can estimate our probabilities from the observed sequences

## Segmentation/boundary detection

Given: A test sequence and a HMM with different sequence classes
Task: Segment the sequence into subsequences, predicting the class of each subsequence
Question: What is the most probable "path" (sequence of hidden states) for generating a given sequence from the HMM?
Solution: Use the Viterbi algorithm

## Classification/sequence scoring

Given: A test sequence and a set of HMMs representing different sequence classes
Task: Determine which HMM/class best explains the sequence
Question: How likely is a given sequence given a HMM?
Solution: Use the Forward algorithm

## Learning/parameterization

Given: A model, a set of training sequences
Task: $\quad$ Find model parameters that explain the training sequences
Question: Can we find a high probability model for sequence characterization
Solution: Use the Forward backward algorithm

## Segmentation/boundary detection

Question: What is the most probable "path" (sequence of hidden states) for generating a given sequence from the HMM?
HMMER: hmmalign - align sequences to our HMM

## Classification/sequence scoring

Question: How likely is a given sequence given a HMM?
HMMER: hmmsearch - find sequences that match our HMM

Learning/parameterisation
Question: Can we find a high probability model for sequence characterization HMMER: hmmbuild - setup our HMM parameters

## Half time break...

## Questions:

For what kinds of motifs are PSSMs not well suited?
What is the Markov property?
In what important ways do HMMs differ Markov chains?
What is the Viterbi algorithm used for?
How does the Forward algorithm differ from the Viterbi algorithm?

## For what kinds of motifs are PSSMs not well suited?

PSSMs are not well suited to pattern instances containing insertions or deletions, variable length patterns and those with positional dependencies.

## What is the Markov property?

The Markov property states that the conditional probability distribution for the system at the next step (and in fact at all future steps) depends only on the current state of the system, and not additionally on the state of the system at previous steps.

In what important ways do HMMs differ Markov chains?
HMMs differ from Markov chains in a number of ways:

- In HMMs, the sequence of states visited is hidden. Unlike Markov Chains, there is no longer a one-to-one correspondence between states and output symbols.
- In a HMM the same symbol may be emitted by more than one state.
- In a HMM a state can emit more than one symbol.


## What is the Viterbi algorithm used for?

The Viterbi algorithm is used to find the most probable state path given a sequence and an HMM

## HMM network structure is hand tailored to the problem

No algorithm for the prediction of optimal HMM network structure and probabilities has yet been able to beat simple hand-built topologies

These topologies are tailored to the problem at hand - exon/intron detection, transmembrane regions, secondary structure elements, protein families...


## GenScan - gene-prediction HMM



Here, each circle or square represents a functional unit (a state) of a gene on its forward strand (for example, E Einit is the 5 ' coding sequence (CDS) and Eterm is the $3^{\prime}$ CDS, and the arrows represent the transition probability from one state to another. The GenScan HMM is trained by pre-computing the transition probabilities from a set of known gene structures.

See: Zhang et al. (2002) Nature Reviews Genetics 3, 698-709

## TMHMM - transmembrane protein topology prediction



Each box corresponds to one or more states in the HMM. Cyt. represents the cytoplasmic side of the membrane and non-cyt. the other side. (b) The detailed structure of the inside and outside loop models and helix cap models. (c) The structure of the model for the helix core modeling lengths between 5 and 25 , which translates to helices between 15 and 35 when the caps are included.

See: Krogh et al. (2001) JMB 305, 567-580

## SAMTOOLS - SNP calling in NextGen sequencing data



Application of HMMs in the area of SNP discovery from NextGen sequencing data, to greatly reduce false SNP calls caused by misalignments around insertions and deletions (indels). The central concept is per-Base Alignment Quality, which accurately measures the probability of a read base being wrongly aligned.

See: Li et al. (2011) Bioinformatics 27, 1157-1158

## HMMER - protein homology detection and alignment



Profile HMM architecture used in HMMER2, SAM and PFTOOLS protein homology detection and alignment packages. Match states carry position-specific emission probabilities for scoring residues at each consensus position. Insert states emit residues with emission probabilities identical to a background distribution. We will describe this in more detail shortly...

See: Eddy (1998) Bioinformatics 14, 755-763

## Building sequence profile-HMMs: Match states

How do the above HMMs relate to profiles? Let's see how we can use the HMM framework to build profile HMMs that describe families of related sequences.

In the last lecture, we built a profile for the alignment:

| s1 | LEVK |
| :--- | :--- |
| s2 | LDIR |
| s3 | LEIK |
| s4 | LDVE |



Ignoring the "background" frequencies for now, a profile for this alignment can be viewed as a simple HMM with one "match" state for each column, where consecutive match states are separated by transitions of probability 1.
Q. Why is this not a Markov chain?

## Building profile-HMMs: Insert states

Introduce insert states ( l ), which will model inserts after the jth column in our alignment.


Typically, the output probabilities for insert states are set equal to the background probabilities. Note that we can have different probabilities for entering different insert states, and this models the fact that insertions may be less well-tolerated in certain portions of the alignment.

## Building profile-HMMs: Insert states + affine gaps

For any particular insert state, we may have different transition probabilities for entering it for the first time vs. staying in the insert state; this models affine gap penalties.


## Building profile-HMMs: Delete states

One could model deletions with additional transitions between match states. However, arbitrarily long gaps would introduce lots of transitions in the model. Instead, we will introduce delete states that do not emit any symbols

| s1 | LEVK |
| :--- | :--- |
| s2 | LDIR |
| s3 | LEIK |
| s4 | LDVE |
| query3 | L-VK |



## Building profile-HMMs

Putting it all together we get a complete profile HMM topology with match, insert and delete states.


However we still need to decide how many states our HMM has, what the transition probabilities are, etc.

## Example profile-HMM building

- How do we pick the length of the HMM?

Common heuristic is to include only those columns that have > 50\% occupancy

- How do we pick emission probabilities for

$$
\begin{aligned}
& \mathrm{b}_{\mathrm{m} 1}(\mathrm{~V})=5 / 7 \\
& \mathrm{~b}_{\mathrm{m} 1}(\mathrm{~F})=1 / 7 \\
& \mathrm{~b}_{\mathrm{m} 1}(\mathrm{I})=1 / 7
\end{aligned}
$$



How do we pick transition probabilities?

- We let the transition probability of going from state $i$ to state $j, a_{i j}$ be equal to:

```
No. of transitions from state i to state j
    ам2м3 (V) = 6/7 No. of matches (=6)
    am2d3 (F) = 1/7 No. of gaps (=1)
    am2I2(I) = 0/7 No. of insertions (=0)
```



## Side note: Weighting the training sequences

If there is a high degree of redundancy in our initial MSA (i.e. it contains a large group of very closely related sequences and a small number of more distantly related sequences) the resulting HMM will over represent the similar sequences and adversely effect our ability to detect distantly related sequences when searching databases

Sequences weighting attempts to compensate for this sequence sampling bias by differentially weighting sequences to reduce redundancy prior to model building

By default HMMER uses a sequence clustering tree as a guide to weight each sequence by its distance to other sequences. This approach will effectively downweight the influence of redundant sequences.

A number of other approaches have been developed (Voronoi algorithm, maximum entropy, etc.)

## Side note: Pseudocounts and Dirichlet distributions

Unfortunately, for alignments containing a small number of sequences the observed counts may not be representative of the family as a whole.

In such cases we must adjust the probabilities to account for our under-sampling (i.e. unobserved residues)

One common approach is to add pseudocounts to the observed counts so that no zero probabilities can occur.

Simplest approach is to just add one to all counts. More accurate adjustments consider prior knowledge about the behavior of sequence families adjusting counts according to pre-tabulated Dirichlet distributions - which are rather like protein comparison matrixes used in profile methods

Such information is often called prior information, indicating that it is known before any sequence data is seen

See: Durbin et al. "Biological Sequence Analysis"

## Generating multiple sequence alignments

Large MSAs can be generated very quickly by using the Viterbi algorithm to find the most likely path through the HMM for a set of unaligned sequences

This is the basis of the PFAM database which uses the HMMER software package Namely, HAMMER's hmmalign from the results of hmmsearch

MSA produced by HMMs are not true MSAs in the way that those produced by ClustalW are. ClustalW compares every sequence to every other sequence, whereas HMM aligning compares every sequence to the model independently so that the alignment between sequences is by proxy. Adding new sequences to the ClustalW alignment will add new information which may alter the alignment of existing sequences; adding new sequences to the HMM alignment never changes the alignment of any sequences relative to each other.

As an alternative to HMMER, you can use the Sequence Alignment and Modeling Software System (SAM)
http://compbio.soe.ucsc.edu/sam.html

## HMM sequence searching performance



Recent speed benchmarks indicate that HMMER3 is approaching BLAST speed
Each point represents a speed measurement for one search with one query against target sequences. Both axes are logarithmic, for speed in millions of dynamic programming cells per second ( $\mathrm{Mc} / \mathrm{s}$ ) on the y -axis and query length on the x -axis.

See: Eddy (2011) PLoS Comp Biol 7(10): e1002195

## HMM sequence searching performance...



However HMMER3 has a much higher search sensitivity and specificity
In each benchmark, true positive subsequences have been selected to be no more than $25 \%$ identical to any sequence in the query alignment ... (see paper for details).

See: Eddy (2011) PLoS Comp Biol 7(10): e1002195

## HMM limitations

HMMs are linear models and are thus unable to capture higher order correlations among positions (e.g. distant cysteins in a disulfide bridge, RNA secondary structure pairs, etc).

Another flaw of HMMs lies at the very heart of the mathematical theory behind these models. Namely, that the probability of a sequence can be found from the product of the probabilities of its individual residues.

This claim is only valid if the probability of a residue is independent of the probabilities of its neighbors. In biology, there are frequently strong dependencies between these probabilities (e.g. hydrophobic residues clustering at the core of protein domains).

These biological realities have motivated research into new kinds of statistical models. These include hybrids of HMMs and neural nets, dynamic Bayesian nets, factorial HMMs, Boltzmann trees and stochastic context-free grammars.

See: Durbin et al. "Biological Sequence Analysis"

## PFAM: Protein Family Database of Profile HMMs

Comprehensive compilation of both multiple sequence alignments and profile HMMs of protein families.

## http://pfam.sanger.ac.uk/

PFAM consists of two databases:

- Pfam-A is a manually curated collection of protein families in the form of multiple sequence alignments and profile HMMs. HMMER software is used to perform searches.
- Pfam-B contains additional protein sequences that are automatically aligned. Pfam-B serves as a useful supplement that makes the database more comprehensive.
- Pfam-A also contains higher-level groupings of related families, known as clans

Pfam 25.0 (March 2011, 12273 families)
The Pfam database is a large collection of protein families, each represented by multiple sequence alignments and hidden Markov models (HMMs). More...

| QUICK LINKS | YOU CAN FIND DATA IN PFAM IN VARIOUS WAYS... |
| ---: | :--- |
| SEQUENCE SEARCH | Analyze your protein sequence for Pfam matches |
| VIEW A PFAM FAMILY | View Pfam family annotation and alignments |
| VIEW A CLAN | See groups of related families |
| VIEW A SEQUENCE | Look at the domain organisation of a protein sequence |
| VIEW A STRUCTURE | Find the domains on a PDB structure |
| KEYWORD SEARCH | Query Pfam by keywords |
| JUMP TO | enter any accession or ID Go Example |
|  | Enter any type of accession or ID to jump to the page for a Pfam family or <br> clan, UniProt sequence, PDB structure, etc. |
|  | Or view the help pages for more information |

## Citing Pfam

If you find Pfam useful, please consider citing the reference that describes this work:

The Pfam protein families databasem: R.D. Finn, J. Mistry, J. Tate, P. Coggill, A. Heger, J.E. Pollington, O.L. Gavin, P.
Gunesekaran, G. Ceric, K. Forslund, L. Holm, E.L.
Sonnhammer, S.R. Eddy, A. Bateman

## Mirrors

The following are official Pfam mirror sites:

W WTSI, UK ㅓㅕ
: SBC, Swedenm

- JFRC, USAN










Pfam：Family：Kinesin（PF00225）

HOME｜SEARCH｜BROWSE｜FTP｜HELP｜ABOUT
Pfom
keyword search Go

## Family：Kinesin（PF00225）



248 species
114 structures

## Structures

For those sequences which have a structure in the Protein DataBankw，we use the mapping between UniProtw，PDB and Pfam coordinate systems from the PDBer² group，to allow us to map Pfam domains onto UniProt sequences and three－dimensional protein structures．The table below shows the structures on which the Kinesin domain has been found．

| UniProt entry | UniProt residues | $\begin{aligned} & \text { PDB } \\ & \text { ID } \end{aligned}$ | PDB chain ID | PDB residues | View |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A8BKD1 GIALA | 11－335 | $\underline{2 v v g}$ | A | 11－335 | Jmol AstexViewer SPICE® |
|  |  |  | B | 11－335 | Jmol AstexViewer SPICE 心 |
| CENPE HUMAN | 12－329 | 1t5c | A | 12－329 | Jmol AstexViewer SPICE心 |
|  |  |  | B | 12－329 | Jmol AstexViewer SPICE떵 |
| KAR3 YEAST | 392－723 | $1 \mathrm{f9t}$ | A | 392－723 | Jmol AstexViewer SPICE® |
|  |  | $1 \mathrm{f9u}$ | A | 392－723 | Jmol AstexViewer SPICE ${ }^{\text {c／}}$ |
|  |  | $1 \mathrm{f9v}$ | A | 392－723 | Jmol AstexViewer SPICE |
|  |  | 1f9w | A | 392－723 | Jmol AstexViewer SPICE® |
|  |  | 19\％ | B | 392－723 | Jmol AstexViewer SPICE® |
|  |  | 3kar | A | 392－723 | Jmol AstexViewer SPICE® |
| KI13B HUMAN | 11－352 | 3 gbj | A | 11－352 | Jmol AstexViewer SPICE［ |
|  |  |  | B | 11－352 | Jmol AstexViewer SPICE心 |
|  |  |  | C | 11－352 | Jmol AstexViewer SPICE |
|  |  | $1 \mathrm{ii6}$ | A | 24－359 | Jmol AstexViewer SPICE ${ }^{\text {c／}}$ |
|  |  |  | B | 24－359 | Jmol AstexViewer SPICE心 |
|  |  | 1a0b | A | 24－359 | Jmol AstexViewer SPICE® |
|  |  |  | B | 24－359 | Jmol AstexViewer SPICE |
|  |  | 1×88 | A | 24－359 | Jmol AstexViewer SPICE［ |
|  |  |  | B | 24－359 | Jmol AstexViewer SPICE［ |
|  |  |  | A | 24－359 | Jmol AstexViewer SPICE® |



## HMMER3: a new generation of sequence homology search software

HMMER is used for searching sequence databases for homologs of protein sequences, and for making protein sequence alignments. It implements methods using probabilistic models called profile hidden Markov models (profile HMMs).

Compared to BLAST, FASTA, and other sequence alignment and database search tools based on older scoring methodology, HMMER aims to be significantly more accurate and more able to detect remote homologs because of the strength of its underlying mathematical models. In the past, this strength came at significant computational expense, but in the new HMMER3 project, HMMER is now essentially as fast as BLAST.

As part of this evolution in the HMMER software, we are committed to making the software available to as many scientists as possible. Earlier releases of HMMER were restricted to command line use. To make the software more accessible to the wide scientific community, we now provide servers that allow sequence searches to be performed interactively via the Web.

The current version is HMMER $\mathbf{3 . 0}$ (28 March 2010) and can be downloaded from the software section of the site. Previous versions of the HMMER software can be obtained from the archive section.

If you have used the HMMER website, please consider citing the following reference that describes this work:

HMMER web server: interactive sequence similarity searching 중
R.D. Finn, J. Clements, S.R. Eddy

Nucleic Acids Research (2011) Web Server Issue 39:W29-W37. PDF飞

## Download HMMER

Get the latest version
v3.0
Release notes (28 March 2010)


Alternative Download Options
Source

## Search

Perform an interactive search now.
Search

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phmmer hmmscan hmmsearch

## protein sequence vs protein sequence database

Paste in your sequence or use the example
>sp|Q14807|KIF22_HUMAN

MAAGGSTQQRRREMAAASAAAISGAGRCRLSKIGATRRPPPARVRVAVRLRPFVDGTAGA SDPPCVRGMDSCSLEIANWRNHQETLKYQFDAFYGERSTQQDIYAGSVQPILRHLLEGQN ASVLAYGPTGAGKTHTMLGSPEQPGVIPRALMDLLQLTREEGAEGRPWALSVTMSYLEIY QEKVLDLLDPASGDLVIREDCRGNILIPGLSQKPISSFADFERHFLPASRNRTVGATRLN QRSSRSHAVLLVKVDQRERLAPFRQREGKLYLIDLAGSEDNRRTGNKGLRLKESGAINTS LFVLGKVVDALNQGLPRVPYRDSKLTRLLQDSLGGSAHSILIANIAPERRFYLDTVSALN FAARSKEVINRPFTNESLQPHALGPVKLSQKELLGPPEAKRARGPEEEEIGSPEPMAAPA SASQKLSPLQKLSSMDPAMLERLLSLDRLLASQGSQGAPLLSTPKRERMVLMKTVEEKDL EIERLKTKQKELEAKMLAQKAEEKENHCPTMLRPLSHRTVTGAKPLKKAVVMPLQLIQEQ AASPNAEIHILKNKGRKRKLESLDALEPEEKAEDCWELQISPELLAHGRQKILDLLNEGS ARDI RSI ORIC.PKKAOIIVGWRFI HC.PFSOVFRI FRVFGITCKOMFSFI KANII CI AACO

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- Job: 9924F9AC-FEB5-11E0-A304-2B0C998A7913
- Started: 2011-10-24 23:01:15
- Algorithm: phmmer
- HMMER Options: -E 1 --domE 1 --incE 0.01 --incdomE 0.03 --mx BLOSUM62 --pextend 0.4 --popen 0.02 --seqdb nr
v Format







That's it!

