Chapter 19

The Diversity of Samples from the Same Population

19.1 Setting the Stage

Working Backward from Samples to Populations

• Start with question about population.
• Collect a sample from the population, measure variable.
• Answer question of interest for sample.
• With statistics, determine how close such an answer, based on a sample, would tend to be from the actual answer for the population.

Understanding Dissimilarity among Samples

• Suppose most samples are likely to provide an answer that is within 10% of the population answer.
• Then the population answer is expected to be within 10% of whatever value the sample gave.
• So, can make a good guess about the population value.

Possible Samples

Sample 1: Proportion with gene = 12/25 = 0.48 = 48%
Sample 2: Proportion with gene = 9/25 = 0.36 = 36%
Sample 3: Proportion with gene = 10/25 = 0.40 = 40%
Sample 4: Proportion with gene = 7/25 = 0.28 = 28%

• Each sample gave a different answer.
• Sample answer may or may not match population answer.
Conditions for Rule for Sample Proportions

1. **There exists an actual population with fixed proportion** who have a certain trait. *Or*
   There exists a repeatable situation for which a certain outcome is likely to occur with fixed probability.

2. **Random sample** selected from population (so probability of observing the trait is same for each sample unit). *Or*
   Situation repeated numerous times, with outcome each time independent of all other times.

3. **Size of sample** or number of repetitions is relatively large – large enough to see at least 5 of each of the two possible responses.

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Example 1: Election Polls
Pollster wants to estimate proportion of voters who favor a certain candidate. **Voters** are the population units, and **favoring candidate** is opinion of interest.

Example 2: Television Ratings
TV rating firm wants to estimate proportion of households with television sets tuned to a certain television program. Collection of **all households with television sets** makes up the population, and **being tuned to program** is trait of interest.

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Defining the Rule for Sample Proportions
If numerous samples or repetitions of the same size are taken, the frequency curve made from proportions from various samples will be approximately bell-shaped.

**Mean** will be true proportion from the population.

**Standard deviation** will be:

\[
\sqrt{\frac{\text{true proportion} \times (1 - \text{true proportion})}{\text{sample size}}}
\]
Example 5: Using Rule for Sample Proportions

Suppose 40% of all voters in U.S. favor candidate X. Pollsters take a sample of 2400 people. What sample proportion would be expected to favor candidate X?

The sample proportion could be anything from a bell-shaped curve with mean 0.40 and standard deviation:

\[
\sqrt{\frac{(0.40)(1-0.40)}{2400}} = 0.01
\]

For our sample of 2400 people:
- 68% chance sample proportion is between 39% and 41%
- 95% chance sample proportion is between 38% and 42%
- almost certain sample proportion is between 37% and 43%

19.3 What to Expect of Sample Means

- Want to estimate average weight loss for all who attend national weight-loss clinic for 10 weeks.
- Unknown to us, population mean weight loss is 8 pounds and standard deviation is 5 pounds.
- If weight losses are approximately bell-shaped, 95% of individual weight losses will fall between –2 (a gain of 2 pounds) and 18 pounds lost.

Possible Samples

| Sample 1: | 1,1,2,3,4,4,5,6,7,7,7,8,8,9,9,11,11,13,13,14,14,15,16,16 |
| Sample 2: | –2,0,0,3,4,4,4,5,6,6,8,8,9,9,9,9,10,11,11,12,13,13,16 |
| Sample 3: | –4,–4,2,3,4,5,7,8,9,9,9,9,9,10,10,11,11,12,12,13,14,16,18 |
| Sample 4: | –5,–3,–2,0,1,2,2,2,2,4,4,4,4,5,7,7,7,9,9,10,10,10,11,11,12,12,14,14,14,14,19 |

Results:
- Sample 1: Mean = 8.32 pounds, std dev = 4.74 pounds
- Sample 2: Mean = 6.76 pounds, std dev = 4.73 pounds
- Sample 3: Mean = 8.48 pounds, std dev = 5.27 pounds
- Sample 4: Mean = 7.16 pounds, std dev = 5.93 pounds

- Each sample gave a different sample mean, but close to 8.
- Sample standard deviation also close to 5 pounds.
Conditions for Rule for Sample Means

1. Population of measurements is bell-shaped, and a random sample of any size is measured.

OR

2. Population of measurements of interest is not bell-shaped, but a large random sample is measured. Sample of size 30 is considered “large,” but if there are extreme outliers, better to have a larger sample.

Example 6: Average Weight Loss

Weight-loss clinic interested in average weight loss for participants in its program. Weight losses assumed to be bell-shaped, so Rule applies for any sample size. Population is all current and potential clients, and measurement is weight loss.

Example 7: Average Age at Death

Researcher is interested in average age at which left-handed adults die, assuming they have lived to be at least 50. Ages at death not bell-shaped, so need at least 30 such ages at death. Population is all left-handed people who live to be at least 50 years old. The measurement is age at death.

Defining the Rule for Sample Means

If numerous samples or repetitions of the same size are taken, the frequency curve of means from various samples will be approximately bell-shaped.

Mean will be same as mean for the population.

Standard deviation will be:

\[
\frac{\text{population standard deviation}}{\sqrt{\text{sample size}}}
\]
Example 9: Using Rule for Sample Means

Weight-loss example, population mean and standard deviation were 8 pounds and 5 pounds, respectively, and we were taking random samples of size 25.

Potential sample means represented by a bell-shaped curve with mean of 8 pounds and standard deviation:

\[
\frac{5}{\sqrt{25}} = 1 \text{ pound}
\]

For our sample of 25 people:

- 68% chance sample mean is between 7 and 9 pounds
- 95% chance sample mean is between 6 and 10 pounds
- almost certain sample mean is between 5 and 11 pounds

Increasing the Size of the Sample

Weight-loss example: suppose a sample of 100 people instead of 25 was taken.

Potential sample means still represented by a bell-shaped curve with mean of 8 pounds but standard deviation:

\[
\frac{5}{\sqrt{100}} = 0.5 \text{ pounds}
\]

For our sample of 100 people:

- 68% chance sample mean is between 7.5 and 8.5 pounds
- 95% chance sample mean is between 7 and 9 pounds
- almost certain sample mean is between 6.5 and 9.5 pounds

19.4 What to Expect in Other Situations

- So far two common situations – (1) want to know what proportion of a population fall into one category of a categorical variable, (2) want to know the mean of a population for a measurement variable.
- Many other situations and similar rules apply to most other situations
Two Basic Statistical Techniques

- **Confidence Intervals**
  Interval of values the researcher is fairly sure covers the true value for the population.

- **Hypothesis Testing**
  Uses sample data to attempt to reject the hypothesis that nothing interesting is happening—that is, to reject the notion that chance alone can explain the sample results.

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**Case Study 19.1: Do Americans Really Vote When They Say They Do?**

Reported in *Time* magazine (Nov 28, 1994):
- Telephone poll of 800 adults (2 days after election) – 56% reported they had voted.
- Committee for Study of American Electorate stated only 39% of American adults had voted.

*Could it be the results of poll simply reflected a sample that, by chance, voted with greater frequency than general population?*

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Suppose only 39% of American adults voted. We can expect sample proportions to be represented by a bell-shaped curve with mean 0.39 and standard deviation:

\[
\sqrt{\frac{0.39(1-0.39)}{800}} = 0.017
\]

For our sample of 800 adults, we can be almost certain to see a sample proportion between 33.9% and 44.1%. The reported 56% is far above 44.1%.

The standard score for 56% is: \((0.56 - 0.39)/0.017 = 10\). Virtually impossible to see a standard score of 10 or more.
20.1 Confidence Intervals

**Confidence Interval:** an interval of values computed from sample data that is almost sure to cover the true population number.

- Most common level of confidence used is 95%. So willing to take a 5% risk that the interval does not actually cover the true value.
- We never know for sure whether any one given confidence interval covers the truth. However, …
- In long run, 95% of all confidence intervals tagged with 95% confidence will be correct and 5% of them will be wrong.
20.2 Three Examples of Confidence Intervals from the Media

Most polls report a margin of error along with the proportion of the sample that had each opinion.

Formula for a 95% confidence interval:

\[
\text{sample proportion} \pm \text{margin of error}
\]

20.3 Constructing a Confidence Interval for a Proportion

Recall Rule for Sample Proportions:
If numerous samples or repetitions of same size are taken, the frequency curve made from proportions from various samples will be approximately bell-shaped.

Mean will be true proportion from the population.
Standard deviation will be:

\[
\sqrt{\frac{\text{true proportion} \cdot (1 - \text{true proportion})}{\text{sample size}}}
\]

In 95% of all samples, the sample proportion will fall within 2 standard deviations of the mean, which is the true proportion.

A Confidence Interval for a Proportion

In 95% of all samples, the true proportion will fall within 2 standard deviations of the sample proportion.

Problem: Standard deviation uses the unknown “true proportion”.

Solution: Substitute the sample proportion for the true proportion in the formula for the standard deviation.

\[
\text{A 95% confidence interval for a population proportion:}
\]

\[
\text{sample proportion} \pm 2(\text{S.D.})
\]

Where S.D. = \[
\sqrt{\frac{\text{true proportion} \cdot (1 - \text{true proportion})}{\text{sample size}}}
\]

A technical note: To be exact, we would use 1.96(S.D.) instead of 2(S.D.). However, rounding 1.96 off to 2.0 will not make much difference.
Example 4: Wife Taller than the Husband?

In a random sample of 200 British couples, the wife was taller than the husband in only 10 couples.

- sample proportion = \( \frac{10}{200} = 0.05 \) or 5%
- standard deviation = \( \sqrt{\frac{(0.05)(1 - 0.05)}{200}} = 0.015 \)
- confidence interval = .05 ± 2(0.015) = .05 ± .03 or 0.02 to 0.08

Interpretation: We are 95% confident that of all British couples, between .02 (2%) and .08 (8%) are such that the wife is taller than her husband.

Other Levels of Confidence

- 68% confidence interval: sample proportion ± 1(S.D.)
- 99.7% confidence interval: sample proportion ± 3(S.D.)
- 90% confidence interval: sample proportion ± 1.645(S.D.)
- 99% confidence interval: sample proportion ± 2.576(S.D.)

How the Margin of Error was Derived

Two formulas for a 95% confidence interval:

- sample proportion ± \( \frac{1}{\sqrt{n}} \) (from conservative m.e. in Chapter 4)
- sample proportion ± 2(S.D.)

Two formulas are equivalent when the proportion used in the formula for S.D. is 0.50.

Then 2(S.D.) is simply \( \frac{1}{\sqrt{n}} \), which is our conservative formula for margin of error – called conservative because the true margin of error is actually likely to be smaller.
For Those Who Like Formulas

Notations for Population and Sample Proportions
Sample size = n
Population proportion = \( p \)
Sample proportion = \( \hat{p} \)

Notation for the Multiplier for a Confidence Interval
For means that will become clear in later chapters, we specify the level of confidence for a confidence interval as \( 1 - \alpha \) (read “Alpha”), e.g., 0.95. For example, for a 95% confidence interval, \( \alpha = 0.05 \). Let \( z_{\alpha/2} \) = standard normal score with area \( \alpha/2 \) closest to 0. For example, when \( \alpha = 0.05 \), \( z_{0.025} = 1.96 \), or about 2.

Formula for \( 100(1 - \alpha) \% \) Confidence Interval for a Proportion
\[
\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]

Common Values of \( z_{\alpha/2} \)
1.00 for 100% confidence interval
1.96 for 95% confidence interval
1.645 for 99% confidence interval
2.576 for 99.9% confidence interval
3.0 for 99.99% confidence interval

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