1. A political researcher interviewed 100 randomly chosen students at a college campus on whether they believed Hillary Clinton will run for US President in 2016. Based on her records of how many said “yes she’ll run” (the rest said “no”) she produces a Beta(43; 63) posterior distribution for \( p \) = proportion of students who believed Hillary will run. She later found out that 5 of her records were wrongly logged as “no” when the actual answers were “yes”. What should be the correct posterior distribution for \( p \)? Justify your answer. [6 points]

2. Let \( C_0, C_1, \cdots \) denote the US population counts (in millions) in the census years since 1910 (so, \( C_0 \) is the count of 1910, \( C_1 \) is the count of 1920 and so on). Suppose these counts are modeled by the growth equation \( C_t = C_{t-1} e^{X_t} \), \( t = 1, 2, \cdots \) and the log-growth rates \( X_1, X_2, \cdots \) are modeled as \( X_t \sim \text{Normal}(\mu, \sigma^2) \) where \( \sigma \) is fixed at 0.03 and \( \mu \in (-\infty, \infty) \) is assigned the Jeffreys’ prior \( \xi(\mu) = 1, \mu \in (-\infty, \infty) \). Give a 95% posterior predictive range for the population count at the next census (in 2020) based on the following observations. [Hint: You might find normal convolution result from Appendix, page 2, useful.] [10 points]

3. A machine goes through 4 hazard levels \( \theta \), coded 0 through 3 (from low hazard to high hazard) with use over time. The hazard level of a day can be measured by the frequency of hazardous incidents \( X \) of that day, again coded 0 through 3 (low frequency to high frequency). Suppose \( X \) is modeled with pmfs \( f(x|\theta) \), \( \theta \in \Theta = \{0, 1, 2, 3\} \) as given by the rows of the following table. [10 points]
Frequencies of hazardous incidents from different days are assumed to be conditionally independent of each other given the underlying hazard levels of those days.

Suppose $X$ denotes today’s measurement of the machine’s frequency of hazardous incidents, while $X^*$ denotes the frequency of hazardous incidents measured on the day exactly a month from now. Suppose today’s hazard level $\theta$ is assigned a discrete uniform prior pmf $\xi(0) = \xi(1) = \xi(2) = \xi(3) = 1/4$. Assuming the hazard level goes up one unit with probability 0.8 or remains the same with probability 0.2 in a month’s time (unless it is already 3, in which case it simply remains at 3), calculate the posterior predictive probability $P(X^* \geq 1|X = 1)$. [Hint. It might help to label the hazard level underlying $X^*$ as $\theta^*$.] [14 points]