1. In the NSW study, out of $n = 185$ subjects receiving job training $X = 120$ reported a substantial increase in income (defined as their 1978 annual income being at least 25% more than their 1975 annual income). From the control arm of the study, it was determined that there is a 56% chance of a substantial income increase for people with similar socio-economic background who did not receive any training. Consider the model: $X \sim \text{Binomial}(n, p)$, $p \in (0, 1)$ and answer the following. 

(a) Report a 95% ML confidence interval for $p$.  

(b) If 0.56 is below the left boundary of the above interval, what can you say about the p-value for testing $H_0 : p \leq 0.56$ against $H_1 : p > 0.56$ based on ML tests?  

(c) Consider a formal Bayes testing of $H_0 : p = 0.56$ (treatment has no additional effect) against $H_1 : p = 0.65$ (treatment gives a 50% boost to the odds of substantial increase). From binomial pmf formula, 

$$
\frac{P(X = 120|n = 185, p = 0.65)}{P(X = 120|n = 185, p = 0.54)} = 20.3
$$

Which decision will you take under the loss:

<table>
<thead>
<tr>
<th>Decision</th>
<th>Truth</th>
<th>$p = 0.56$</th>
<th>$p = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No effect</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>50% boost</td>
<td>25</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

and prior: $P(H_0) = 1/2 = P(H_1)$?

2. In a high energy physics experiment, 10 energy channels are searched for a signal that may or may not exist. If the signal exists, then it must show up in exactly one of the 10 channels. Let $X_1, \ldots, X_{10}$ denote the channel measurements. If channel $j$ has the signal then $X_j \sim \text{Normal}(3, 1)$, otherwise $X_j \sim \text{Normal}(0, 1)$. Consider testing 

$$
H_0 : \text{the signal does not exist} \quad vs. \quad H_1 : \text{the signal exists}
$$

based on the channel measurements data and answer the following. [4 + 4 = 8 points]
(a) For each channel $j$ consider the pair of hypotheses

$H_{0j} :$ channel $j$ does not have the signal vs. $H_{1j} :$ channel $j$ has the signal.

For this pair, any ML test rejects for large values of $X_j$ and the ML tests based p-value is precisely $1 - \Phi(X_j)$ where $\Phi$ is the standard normal CDF. We could combine these ML tests for all channels to test the overall $H_0$ stated earlier. For carrying out this multiple testing, what would be the most appropriate error rate to control for: the “family-wise error rate” or the “false discovery rate”? Justify your answer.

(b) Whichever error rate you pick in part (a), control it at 10% level and decide whether to reject the overall $H_0$ when the recorded p-values from the 10 channels are:

$0.30 \ 0.91 \ 0.32 \ 0.72 \ 0.92 \ 0.58 \ 0.013 \ 0.54 \ 0.56 \ 0.003$

3. In an online survey of $n = 60$ students randomly chosen from all students registered in all statistics courses in Fall 2013, $X_1 = 25$ reported satisfaction with homework assignments, $X_2 = 23$ reported dissatisfaction and $X_3 = 12$ did not respond. Assume there are only two types of students: ‘satisfied’ and ‘dissatisfied’ with regards to homework assignments. Also assume truthful reporting. Consider the model $X \sim \text{Multinomial}(n,p), p \in \Delta_3$. We are interested in testing the null hypothesis $H_0 : a$ third of all students are dissatisfied with homework assignments.

Answer the following [3 + 4 + 2 + 3 = 12 points]

(a) Assume that chance of responding is NOT related to satisfaction level. Show that $H_0$ corresponds to the null subset

$$\Delta^0_3 = \{(\frac{2}{3}a, \frac{1}{3}a, 1 - a) : a \in (0,1)\}$$

and give an interpretation of the free parameter ‘$a$’.

(b) Argue (with logic and precise calculations) that the expected cell counts under the null is (32, 16, 12). [Hint: For any two non-negative numbers $r$ and $s$, the function $g(a) = a^r(1-a)^s$ over $a \in (0,1)$ is maximized at $\hat{a} = \frac{r}{r+s}$].

(c) From (b), the Pearson’s chi-square test statistic value is 4.59 and the p-value = 0.03. So in carrying out a 5% level testing, you will reject $H_0$. What would you conclude about the actual proportion of dissatisfied students? Does it appear larger or smaller than a third?

(d) What would have been the expected counts under $H_0$ and the p-value had we assumed ‘dissatisfied’ students were two times as likely to respond to the survey than ‘satisfied’ students? Select an option from below and give a short justification.

(i) (32, 16, 12) and p-value = 0.03 (ii) (16, 32, 12) and p-value < 0.03

(iii) (24, 24, 12) and p-value > 0.03 (iv) (40, 8, 12) and p-value < 0.03.