Each year the news magazine, *US News and World Report*, puts out a special issue on college rankings. In this issue they present information on many schools. Some of the variables include the college's acceptance rate (proportion of applicants that are admitted), freshmen retention rate (proportion of students that stay past their first year), and student quality (proportion of admitted students that are from the top 10% of their high school class). For this exam assume this is a simple random sample of all colleges in the U.S. Numbers have been changed to make the math simple. You must show work to support your answers.

1) Suppose that in the population of all colleges in the U.S. that 20% percent are public and 80% are private.
   a) Getting exactly 2 public colleges in a sample of 10 colleges. 
   b) Getting exactly 20 public colleges in a sample of 100 colleges.
   c) Getting 3 or fewer public colleges in a sample of 10 colleges.
   d) Getting 30 or fewer public colleges in a sample of 100 colleges.

2) Assume there are 5,000 colleges in the U.S., and again assume that 20% are public and 80% are private. We are interested in how much variability we could get in the proportion of private schools in a sample of size 36.
   a) Draw a box model for this problem. Be sure to include (if possible) the number of tickets in the box, the numbers on the tickets, and the number of draws.

   b) Suppose 36 schools are selected at random with replacement from all U.S. colleges. What is the expected value for the proportion of private colleges in the sample?

   c) Suppose 36 schools are selected at random with replacement from all U.S. colleges. What is the SE for the proportion of private colleges in the sample?

   d) In all possible samples of size 36, what proportion will have fewer than 24 private colleges?
\[ b \left( \frac{\sum x^2 - n \overline{x}^2}{n-1} \right) = b \left( \frac{5^2 - 5 \cdot 0.65^2}{4} \right) = 0.45 \]
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The p-value tells us the probability these two variables are independent.

- The observed p-value is closest to 1%, 5%, 10%, 90%, 95%, 99%, (circle one).

How many degrees of freedom are there for the distribution of this statistic?

- \( \frac{35}{(\bar{x} - 35)^2} + \frac{35}{(\bar{x} - 35)^2} + \frac{35}{(\bar{x} - 35)^2} + \frac{35}{(\bar{x} - 35)^2} = \frac{2}{(\bar{x} - 35)^2} = 2.9 \)

Calculate the test statistic for the null hypothesis that these two variables are independent.

- \( \frac{35}{(40 - 35)^2} = 35 \)

4. If these two variables are independent, what is the expected number of private colleges that have more than half of their admissions from the top 10% of their high school class and whether the college is public or private.

5. Suppose there were 140 colleges included in the report, and schools are randomly selected from public and private schools separately, so then 70 are public and 70 are private. The table below shows the relationship between percentage of students from the top 10% of their high school class and whether the college is public or private.