1. Suppose $f(x,y) = c$ for all $(x, y)$ that lie on the perimeter of the circle $(x - 4)^2 + (y - 2)^2 \leq 3$. The following questions can be answered with minimal calculation, just geometry.

0.09 What is the value of $c$?

The perimeter has length $2\pi \sqrt{3}$, so $c$ is the inverse of that, or $0.0919$.

4 What is the value of $\mu_X$?

By symmetry, 4.

8 What is the expected value of the product $XY$?

8, since the correlation has to be 0. The correlation is unchanged by recentering the circle to $(0, 0)$, and there the symmetry shows that every positive product is cancelled by a negative product.

0 What is the correlation?

0

No Are $X$ and $Y$ independent?

Explain your answer regarding independence.

Knowing $X$ gives lots of information about $Y$. If you are the largest possible $x$-value, then $y$ has to 2.

What is $g_2(y|x)$?
It is equally likely to be $2 + \sqrt{3 - (x - 4)^2}$ or $2 - \sqrt{3 - (x - 4)^2}$.

2. Suppose $f(x, y) = 6x$ for $x + y \leq 1$ with both $x$ and $y$ restricted to be between 0 and 1.

What is the marginal density of $X$? (Indicate support.)

The region of support is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$. So

$$f_1(x) = \int_0^{1-x} 6x \, dy = 6x(1 - x)$$

for $0 \leq x \leq 1$.

$$f_1(x) = \begin{cases} 
0 & \text{if } x < 0 \\
6x(1-x) & \text{if } 0 \leq x \leq 1 \\
0 & \text{if } x > 1 
\end{cases}$$

0.5 What is the expected value of $X$?

$$\mathbb{E}[X] = \int_0^1 x f_1(x) \, dx = \int_0^1 6x^2(1 - x) \, dx = 1/2.$$ 

What is the conditional density of $Y$ given $X = x$? (Indicate support.)

$$g_2(y \mid x) = f(x, y)/f_1(x) = 6x/6x(1 - x) = (1 - x)^{-1} \text{ for } 0 \leq y \leq 1 - x.$$ 

$$g_2(y \mid x) = \begin{cases} 
0 & \text{if } y < 0 \\
\frac{1}{1-x} & \text{if } 0 \leq y \leq 1 - x \\
0 & \text{if } y > 1 - x 
\end{cases}$$

0.1 What is $\mathbb{E}[XY]$?

$$\int_0^1 \int_0^{1-y} xyf(x, y) \, dxdy = \int_0^1 y[2x^3]_{0}^{1-x} \, dy$$
\[
\int_0^1 y \cdot 2(1 - y)^3 \, dy = 0.1
\]