STA 104 Name:____________________________

MTH 135

Probability Midterm Exam

2:10-3:30 pm Thursday, October 5, 1995

This is a closed-book examination, so please do not refer to your notes, the text, or to any other books. You will find a copy of the text's normal table (Appendix 5) below. If you don't understand something in one of the questions feel free to ask Jane, but please do not talk to each other. Please sign the Duke Honor Code: **I have neither given nor received aid on this examination: __________________________ .**

You must **show** some **work** to get credit—*unsupported answers are not acceptable.* Attach any necessary work sheets to the exam before returning it; be sure to put your name on each page. It is to your advantage to write your solutions as clearly as possible, since I cannot give you credit for solutions I do not understand. Good luck.

Probability Midterm Exam

1. A bag contains eight wooden tiles, each with a single letter on it: two show A's, two B's, two C's, and two show D's. We try to make a word by drawing three letters from the bag; the dictionary lists only seven three-letter words with these letters: ABC, ADD, BAD, CAB, CAD, DAB, and DAD.

a. What is the probability that our three letters, drawn **in order** and **without replacement**, spell the word BAD?

$\Pr[BAD] =$

b. What is the probability that our three letters, still drawn in order and without replacement, spell the word ADD?

$\Pr[ADD] =$

c. What is the probability that our three letters, still drawn in order and without replacement, spell one of the seven three-letter words?

$\Pr[\text{any word}] =$

d. If instead the letters are drawn *with* replacement, what is the probability that we spell one of the seven words?

$\Pr[\text{any word}] =$

2. In the hope of increasing the chance of rolling a "seven," an unscrupulous gambler has added weights to the six on one die and to the ace on the other, leading to the following probabilities:

Face:	1	2	3	4	5	6
First Die:	0.5	0.1	0.1	0.1	0.1	0.1
Second Die:	0.1	0.1	0.1	0.1	0.1	0.5

The faces shown by the two dice are independent. If the gambler rolls this pair of dice,

a. Find the probability that the sum of the two faces shown is seven:

$\Pr[X_1 + X_2 = 7] =$

b. Find the probability that the two dice show the same face:

$\Pr[X_1 = X_2] =$

c. One of the dice is chosen at random, and rolled, yielding a six. What is the probablity that it is the "First Die"?

$\Pr[I = 1 | X_I = 6] =$

3. In a certain (imaginary) lake, the number X_t of fish caught in t hours has the Poisson distribution with mean $\mu = 4t$,

$$\Pr[X_t = k] = e^{-4t}\frac{(4t)^k}{k!}$$

for $k = 0, 1, ...$

a. What is the probability of catching two (or more) fish in the first quarter hour? Give your answer as a decimal approximation, accurate to at least two decimal places, or as a *finite* sum.

$\Pr[X_{.25} \geq 2] =$

b. What is the probability that the length of time T_1 until catching the first fish is more than one-half hour? (HINT: Try to express this *event* in terms of $X_{.5}$)

$\Pr[T_1 > 0.5] =$

Note: $e^{-.25} = 0.7788$, $e^{-.5} = 0.6065$, $e^{-1} = 0.3679$, and $e^{-2} = 0.1353$.

4. A bowl contains six red poker-chips and four black ones. All ten chips are drawn from the bowl, one at a time, and their color is noted. Chips are not replaced once they have been drawn.

a. What is the probability that the first three chips are all the same color?

   Pr[Match] =

b. What is the probability that the *last* three chips are all the same color?

   Pr[Match] =

c. Given that the first three are the same color, what is the probability that the color is Red?

   Pr[Red|Match] =

5. Remove all the face-cards (J,Q,K) from a deck of cards, leaving forty cards $A, 2, 3, 4, 5, 6, 7, 8, 9, 10$. Let X denote the number of aces in a draw of N cards from this deck.

a. If we draw $N = 4$ cards *without* replacement, what is the exact probability of drawing at least one ace?

   $P[X \geq 1] =$

b. If we draw $N = 400$ cards *with* replacement, what is the **exact** probability of drawing at least 50 aces? (Give an expression, not a number)

   $\Pr[X \geq 50] =$

c. Give a **numerical approximation** to the probability in b. above, still with $N = 400$ and *with* replacement, accurate to at least three decimal places:

   $\Pr[X \geq 50] \approx$