STA 104 Name: ______________________________
MTH 135

# Probability Second Test

2:10-3:30 pm Thursday, November 16, 1995

This is a closed-book examination, so please do not refer to your notes, the text, or to any other books. If you don't understand something in one of the questions feel free to ask Jane or me, but please do not talk to each other. Please sign the Duke Honor Code: **I have neither given nor received aid on this examination: ________________________ .**

You must **show** some **work** to get credit—*unsupported answers are not acceptable.* Attach any necessary work sheets to the exam before returning it; be sure to put your name on each page. It is to your advantage to write your solutions as clearly as possible, since I cannot give you credit for solutions I do not understand. Good luck.

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1. A fair die with $d$ sides is one that shows the numbers $1, \ldots, d$ with equal probability, independently on consecutive rolls. Let $X_i$ be the number appearing on the $i^{th}$ roll of an ordinary fair 6-sided die, and let $Y_i$ be the number showing on the $i^{th}$ roll of a fair (dodecahedral) 12-sided die. For each of the problems below, give a correct expression for half-credit, or find the exact answer (or decimal approximation to at least 3 correct significant digits) for full credit:

a. What is the probability that $X_1 = Y_1$?

$\Pr[X_1 = Y_1] =$ ____________

b. What is the probability that $X_1 + X_2 = Y_1$? You may use the well-known fact that $\Pr[X_1 + X_2 = k]$ is given by $\frac{1}{36}, \frac{2}{36}, \frac{3}{36}, \frac{4}{36}, \frac{5}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{3}{36}, \frac{2}{36}, \frac{1}{36}$ for $k = 2, \ldots, 12$.

$\Pr[X_1 + X_2 = Y_1] =$ ____________

c. Let $Z_6$ be the number of rolls needed to see the first Ace (or one) with the 6-sided die, and let $Z_{12}$ be the number of rolls needed for the 12-sided die. If we roll each die again and again until one (or both) show an Ace, what is the probability that the 12-sided die shows an ace first, or at least ties?

$\Pr[Z_{12} \leq Z_6] =$ ____________

2. Let $U$ be a random variable with the Uniform distribution on $(0,1]$ (so it has a probability density function $f(u) = \begin{cases} 1 & 0 < u \le 1 \\ 0 & \text{else} \end{cases}$), and set $X = U/2$, $Y = 2/U$:

a. Find the cumulative distribution function (CDF) for $X = U/2$:

$F_X(x) =$ ____________

b. Find the cumulative distribution function (CDF) for $Y = 2/U$:

$F_Y(y) =$ ____________

c. Find the probability density function (pdf) for $Y = 2/U$:

$f_Y(y) =$ ____________

d. Are $X$ and $Y$ independent? Justify your answer.

Circle one: Y N Reasoning:

3. The lifetimes times (in months) before failing for frubistors are random variables with survival function $\Pr[T > t] = e^{-t^2}$, for $t > 0$.

a. Johnny has two frubistors— a brand new one, and a one-month old one. Which has a better chance of lasting for one more month? Why? Show your work.

Circle one: New Old Reasoning:

b. Find the probability density function for frubistor life:

$f_T(t) =$ ____________

c. Find the probability density function for $X = \sqrt{T}$:

$f_X(x) =$ ____________

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4. Garrison Kieler says that in Lake Wobegon, "All the children are above average." Let $X$ be a random variable with mean $\mathsf{E}[X] = \mu$.

a. Is it possible to have $\Pr[X > \mu] = 1$? Circle one: Y N Why?

b. If $\mu = \mathsf{E}[X] = 0$ and $\mathsf{E}[X^2] = 1$, then what is the largest value $\Pr[X^2 > 4]$ can have, and why?

$\Pr[X^2 > 4] \leq$ ____________

c. Show by example that it *is* possible to have a random variable that only takes on two possible values $x_1$ and $x_2$, has mean $\mu = 0$, and has $\Pr[X > 0] = 0.99$. Give the distribution of such a random variable; show your work:

| $i$ | $x_i$ | $\Pr[X = x_i]$ |
|---|---|---|
| 1 | | |
| 2 | | |

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5. Let $X_1$ and $X_2$ be two points drawn independently and uniformly from $(0,1)$, and let $Y \equiv \max(X_1, X_2)$. Find the:

a. Cumulative distribution function (CDF) for $Y$:

$F(y) = $ ____________

b. Density function (pdf) for $Y$:

$f(y) = $ ____________

c. Expectation of $Y$:

$\mathsf{E}[Y] = $ ____________

d. Expectation of $e^{Y^2}$:

$\mathsf{E}[e^{Y^2}] = $ ____________