STA 104 Name:______________________________
MTH 135

Probability Second Test

2:15-3:30pm Tuesday, 26 November, 1996

This is a closed-book examination, so please do not refer to your notes, the text, or to any other books. A normal distribution table and a blank worksheet are attached to the exam. If you don't understand something in one of the questions feel free to ask me, but please do not talk to each other. Please acknowledge the Duke Honor Code:

I have neither given nor received aid on this examination: ________________ .

You must **show** some **work** to get credit—*unsupported answers are not acceptable*, even if they are correct. It is to your advantage to write your solutions as clearly as possible, since I cannot give you credit for solutions I do not understand. Good luck.

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2.	/20
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5.	/20
	/100

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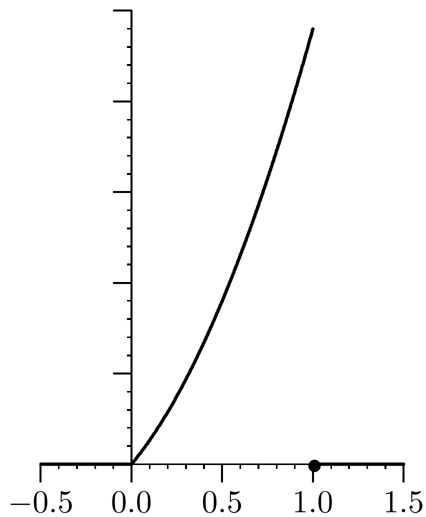
Probability Second Test

1. For some $c > 0$, a function is defined by $f(x) = \begin{cases} c(x + x^2) & \text{for } 0 < x < 1 \\ 0 & \text{for other } x. \end{cases}$

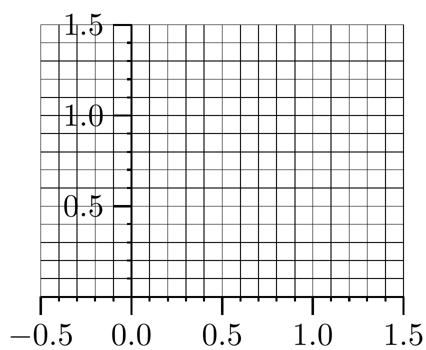
a. Find the constant c which makes the function $f(x)$ a density function, and label the y axis of the plot of the pdf $f(x)$:

$c =$ ____________________



b. Find the corresponding c.d.f. $F(x)$ and sketch its graph. Be careful to give (and plot) the CDF correctly at *all* points $x \in \mathbb{R}$.

$F(x) =$ ____________________



c. Find the expectation μ and standard deviation σ of a random variable X with this distribution. Mark the points μ, $\mu + \sigma$ on your graphs.

$\mu =$ _______________ $\sigma =$ _______________

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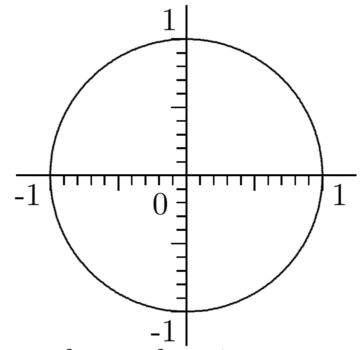
2. Let X and Y be the coordinates of a point drawn uniformly from the unit ball in two dimensions, $\mathcal{B} = \{(x, y) : x^2 + y^2 < 1\}$.

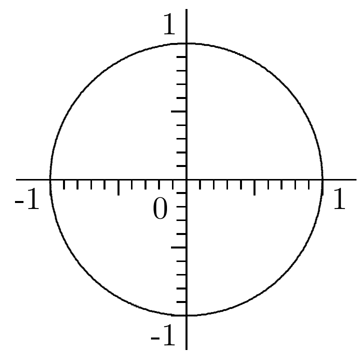
a. What is the probability that the point would lie inside the circle of radious 1/2? Why? (Hint: Draw a picture— here, I'll help:)

$\mathsf{P}[\sqrt{X^2 + Y^2} < 1/2] =$ ____________________



b. What is the probability that the point would lie inside the polygonal region $\{(x, y) : |x| + |y| < 1\}$? Why? (Hint: Be an artist.)

$\mathsf{P}[|X| + |Y| < 1] =$ ____________________



c. Are X and Y independent? Circle one:

Yes No Why?

d. What is the joint density function for X and Y? Be sure to give it correctly for all $(x, y) \in \mathbb{R}^2$.

$f(x, y) =$ ____________________

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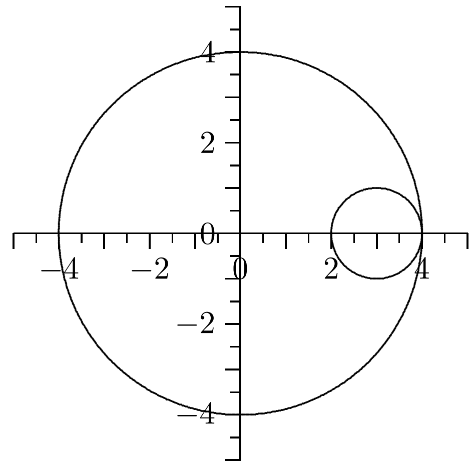
3. A U.S. quarter, with diameter about 2cm, is dropped into a large Hunts tomato-sauce can, with diameter about 8cm.

a. What's the chance the coin covers the center of the can bottom?

P[Center covered] = ____________________



b. What assumptions did you make for a. above?

c. What is the distribution function (CDF) for the distance R from the center of the can to center of the quarter?

$F(r) =$ ____________________

d. What is the distribution function (CDF) for the distance S from the center of the can to the *closest* part of the quarter (*not* to its *center* this time)?

$F(s) =$ ____________________

4. The random variable X has the standard exponential distribution, with probability density function and cumulative distribution function

$$f(x) = \begin{cases} 0 & \text{for } x \le 0 \\ e^{-x} & \text{for } x > 0 \end{cases} \qquad F(x) = \begin{cases} 0 & \text{for } x \le 0 \\ 1 - e^{-x} & \text{for } x > 0 \end{cases}$$

Find the density functions for the following random variables at **all** $y \in \mathbb{R}$:

a. $Y_1 = e^{-X}$:

$f_1(y) =$ ____________________

b. $Y_2 = 2\,X$:

$f_2(y) =$ ____________________

c. $Y_3 = \ln(X)$:

$f_3(y) =$ ____________________

d. $Y_4 = (X - 1)^2$:

$f_4(y) =$ ____________________

5. Let X and Y be normally-distributed random variables with means $\mu_x = 1$ and $\mu_y = 2$, both with variance $\sigma^2 = 1$, but with covariance (and also correlation coefficient) $\rho = 0.8$.

a. What is the probability that X is positive?

$\mathsf{P}[X > 0] =$ ____________________

b. What is the probability that X exceeds Y?

$\mathsf{P}[X > Y] =$ ____________________

c. If Y vanishes, would you expect X to be bigger, smaller, or about equal to its mean, $\mu_x = 1$? Explain.

Bigger Same Smaller Why?

d. What is the (conditional) probability that X is positive, if Y vanishes? (Hint: Express X in terms of Y and an independent random variable $Z \sim N(0, 1)$.)

$\mathsf{P}[X > 0|Y = 0] =$ ____________________

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Extra worksheet, if needed

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Normal Distribution Table

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt:$$

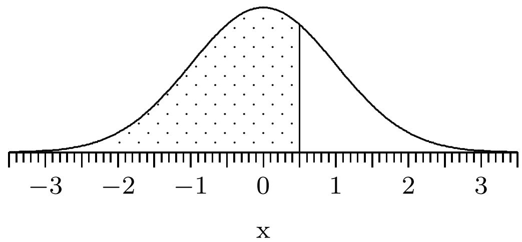

Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998