STA 213

Problem 5.8
Let $\mu_i = E(X_i)$. Then

$$
\begin{aligned}
Var\left(\sum_{i=1}^{n} X_i\right) &= Var(X_1 + X_2 + \cdots + X_n) \\
&= E[(X_1 + X_2 + \cdots + X_n) - (\mu_1 + \mu_2 + \cdots + \mu_n)]^2 \\
&= E[(X_1 - \mu_1) + (X_2 - \mu_2) + \cdots + (X_n - \mu_n)]^2 \\
&= \sum_{i=1}^{n} E(X_i - \mu_i)^2 + 2 \sum_{1 \le i < j \le n} E[(X_i - \mu_i)(X_j - \mu_j)] \\
&= \sum_{i=1}^{n} Var(X_i) + 2 \left(\sum_{1 \le i < j \le n} Cov(X_i, X_j) \right)
\end{aligned}
$$