STA 213

Problem 3.2
Let X = number of defective parts in the sample.
Then $X \sim$ hypergeometric$(N = 100, M, K)$ where
M = number of defectives in the lot and K = sample size.

(a) If there are 6 or more defectives in the lot, then the probability that the lot is accepted ($X = 0$) is at most

$$\Pr(X = 0 \mid N = 100, M = 6, K) = \frac{\binom{6}{0}\binom{94}{K}}{\binom{100}{K}} = \frac{(100-K)\cdots(100-K-5)}{100\cdots 95}$$

By trial we find $\Pr(X = 0) = 0.10056$ for $K = 31$ and $\Pr(X = 0) = 0.09182$ for $K = 32$. So the sample size must be at least 32.

(b) Now Pr(accept lot) = $\Pr(X = 0 \text{ or } 1)$, and, for 6 or more defectives, the probability is at most

$$\Pr(X = 0 \text{ or } 1 \mid N = 100, M = 6, K) = \frac{\binom{6}{0}\binom{94}{K}}{\binom{100}{K}} + \frac{\binom{6}{1}\binom{94}{K-1}}{\binom{100}{K}}$$

By trial we find $\Pr(X = 0 \text{ or } 1) = 0.10220$ for $K = 50$ and $\Pr(X = 0 \text{ or } 1) = 0.09331$ for $K = 51$. So the sample size must be at least 51.