STA 213

Problem 4.50

(a) The probability that a test is negative is $(1-p)$. So the probability that the test for a pooled sample of k people is negative is $(1-p)^k$, since the test results for different people are statistically independent. The probability that the test for a pooled sample of k people is positive is $1-(1-p)^k$.

(b) Let X = "number of blood tests necessary under plan (ii)" and Y = "number of blood tests for a single pooled sample of k people". From part (a) we derive:

$$\begin{aligned}\Pr(Y=1) &= (1-p)^k \\ \Pr(Y=k+1) &= 1-(1-p)^k \\ E(Y) &= k+1-k(1-p)^k\end{aligned}$$

Then

$$E(X) = E(mY) = mk + m - mk(1-p)^k$$

(c) Under plan (i) $N = mk$ blood tests need to be performed on the N people. Under plan (ii) the expected number $E(X)$ of blood tests is $mk + m - mk(1-p)^k$; if p is close to 0, then $E(X)$ is close to m. Therefore plan (ii) is preferred.